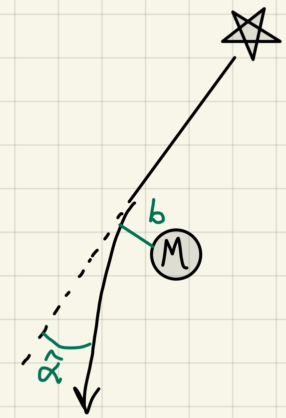
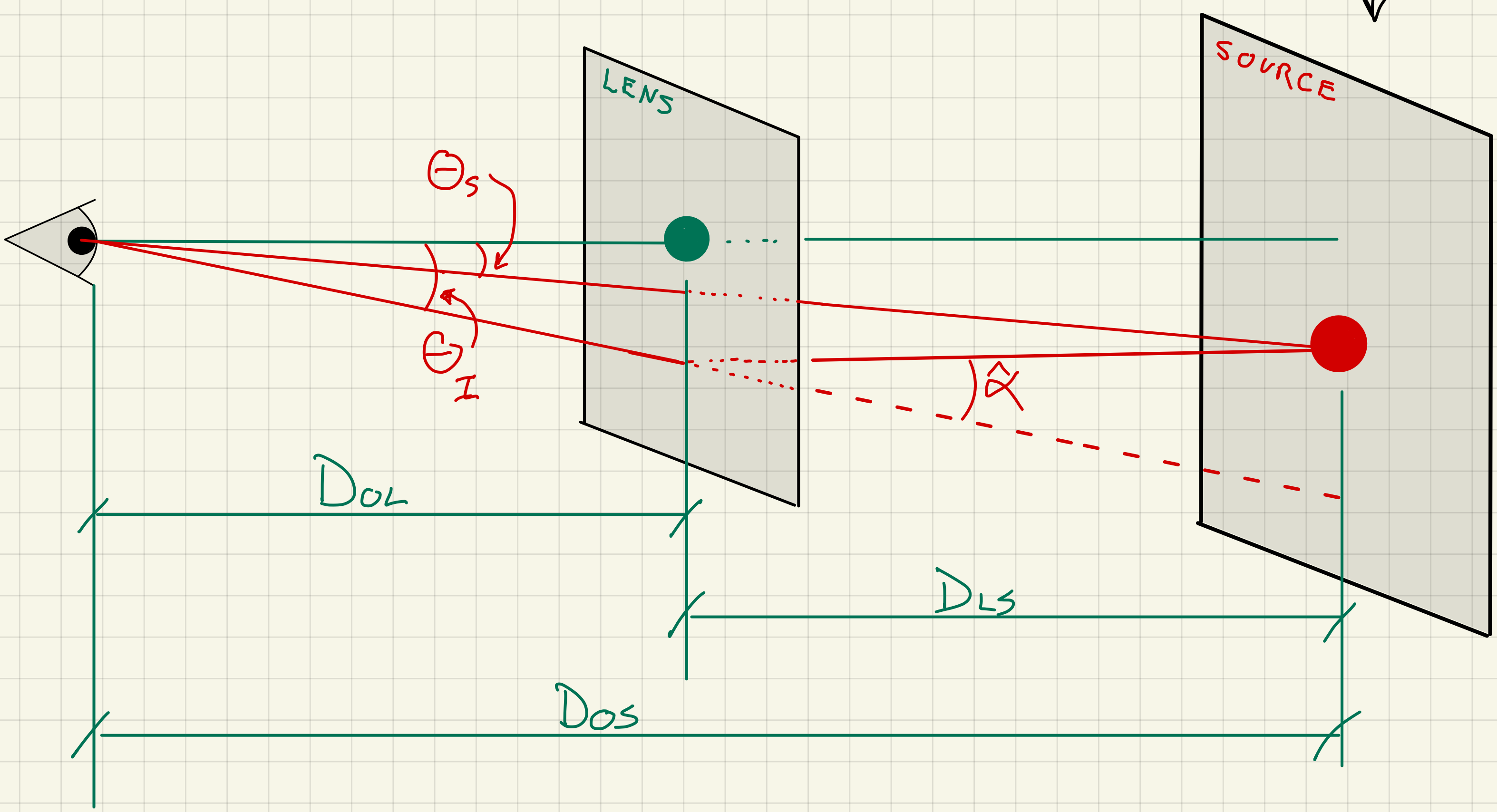
Weak Gravitational Lensing

The only things you really need to know from GR:

- Friedmann equations
- Deflection of light:

$$\boxed{\hat{\alpha} = \frac{4GM}{bc^2}}$$





$$\Theta_I \cdot D_{OS} = \Theta_S \cdot D_{OS} + \hat{\alpha} \cdot D_{LS} \Longrightarrow \underline{\Theta_S = \Theta_I - \hat{\alpha}\frac{D_{LS}}{D_{OS}} = \Theta_I - \frac{4GM}{c^2}\frac{D_{LS}}{\Theta_I D_{OL} D_{OS}}}$$

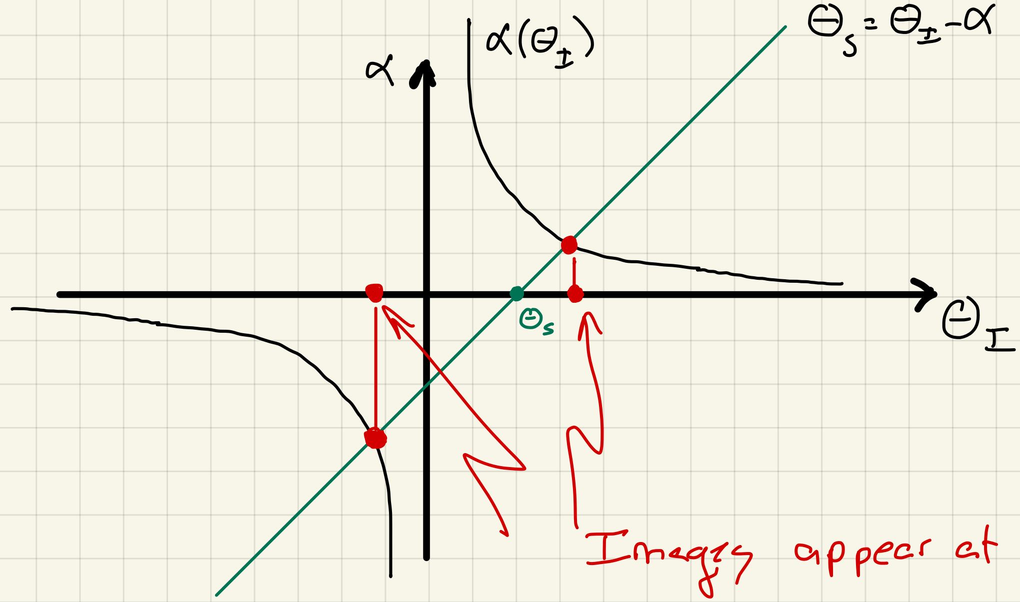
Circularly symmetric lenses

- An annular ring of mass causes no deflection interior to it
- Acts as a point mass for rays exterior to it.

$$\Rightarrow \qquad \Theta_s = \Theta_I - \underbrace{\frac{4G}{c^2} \cdot \frac{D_{LS}}{D_{OS} D_{OL}} \cdot \frac{M(<\Theta_I)}{\Theta_I}}_{\equiv\, \alpha(\Theta_I)}$$

$\leftarrow$ mass interior to Θ_I.

Graphical solution:

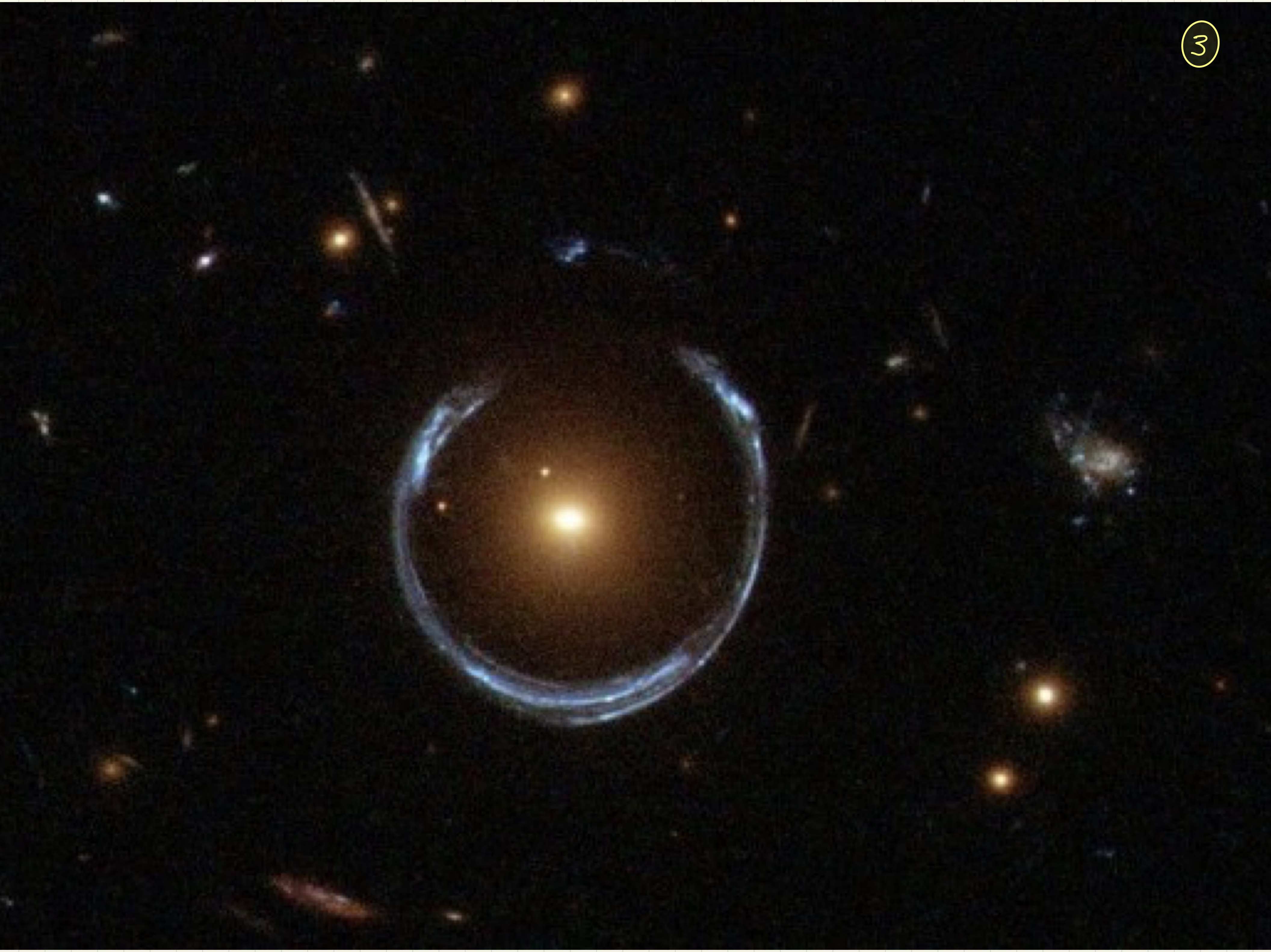


Images appear at these radii

Einstein ring: If $\Theta_s = 0$ (observer - source - lens alignment)

And $\Theta_I = \alpha(\Theta_I)$, then a ring image is formed around source.

e.g. point mass $\quad 0 = \Theta_E - \dfrac{4GMD_{LS}}{D_{OS} D_{OL} c^2} \cdot \dfrac{1}{\Theta_E} \;\Rightarrow\; \Theta_E = \sqrt{\dfrac{4GMD_{LS}}{D_{OS} D_{OL} c^2}}$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Example: singular isothermal sphere (SIS)

What 3d spherically-symmetric mass distribution $\rho(r)$ would host a galaxy with a flat rotation curve $v(r) = v_{circ}$?

Newton says $$\frac{v_{circ}^2}{r} = \frac{GM(<r)}{r^2}$$

$$\Rightarrow \quad GM(<r) = v_{circ}^2 \, r$$

$$\Rightarrow \frac{d}{dr} GM(<r) = 4\pi G r^2 \rho(r) = v_{circ}^2$$

$$\Rightarrow \quad \rho(r) = \frac{v_{circ}^2}{4\pi G} \cdot \frac{1}{r^2}$$

The projected surface density is

$$\Sigma(b) = \int_{-\infty}^{\infty} dz \cdot \rho\left(\sqrt{b^2+z^2}\right) = \frac{v_{circ}^2}{4\pi G} \cdot \int_{-\infty}^{\infty} \frac{dz}{b^2+z^2} = \frac{v_{circ}^2}{4\pi G b} \cdot \tan^{-1} u \Big|_{-\infty}^{\infty}$$

$$= \frac{v_{circ}^2}{4Gb} = \frac{\sigma^2}{2Gb}$$ ← velocity dispersion for isotropic v's.

$$\hat{\alpha}_{SIS}(b) = \frac{4G}{c^2 b} \cdot \int_{r=0}^{b} 2\pi r \, dr \, \Sigma(r) = \frac{8\pi G}{c^2 b} \cdot \int_0^b r \, dr \cdot \frac{\sigma^2}{2Gr} = 4\pi \frac{\sigma^2}{c^2}$$

The deflection angle of an SIS is independent of radius!

The critical density -

Our Einstein radius satisfies

$$\theta = \theta_E - \frac{4G}{c^2} \cdot \frac{D_{LS}}{D_{OL} D_{OS}} \frac{M(<\theta_E)}{\theta_E}$$

$$\Rightarrow \quad \underline{1} = \frac{4G}{c^2} \cdot \frac{D_{LS}}{D_{OL} D_{OS}} \frac{M(<\theta_I)}{\theta_E^2} = \frac{4\pi G}{c^2} \cdot \frac{D_{OL} D_{LS}}{D_{OS}} \cdot \frac{M(<b)}{\pi b^2}$$

$$= \bar{\Sigma}(<\theta_E) / \Sigma_{crit} \,, \qquad \boxed{\Sigma_{crit} \equiv \frac{c^2}{4\pi G} \cdot \frac{D_{OS}}{D_{OL} D_{LS}}}$$

mean projected physical surface density within θ_E

Every Einstein ring encloses averaged surface density $\bar{\Sigma}(<\theta_E) = \Sigma_{crit}$!

For cosmological distances $D \sim c/H_0$,

$$\Sigma_{crit} \sim \frac{cH_0}{4\pi G} \approx \frac{3 \times 10^8 \, m/s \times (4.5 \times 10^{17} s)^{-1}}{4\pi \times 6.67 \times 10^{-11} \, kg^{-1} m^3 s^{-2}} = 0.8 \, \frac{kg}{m^2}$$

$$= 0.08 \, g/cm^2$$

* We always know the mass inside an Einstein ring!

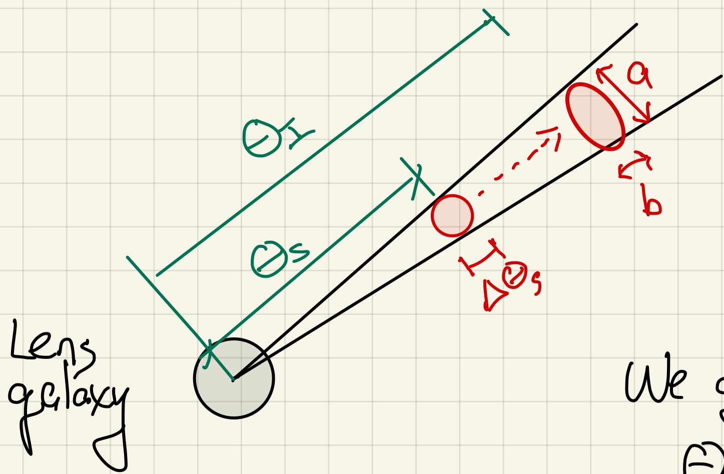
  ... BUT ...
* Only systems with $\Sigma > \Sigma_{crit}$ will cause "strong lensing" like multiple imaging
* Less than 0.1% of cosmological lines of sight have enough density for SL ☹

Sub-critical circular lensing

- If your source is not close enough to the lens center to be multiply imaged, how can you tell it's lensed??

We can see Θ_I but have no clue of Θ_S without removing the lens!

- But the SHAPE of the source is also changed by lensing!



Consider a small circular source of diameter $\Delta\Theta_S$.

It must fit in the same wedge after it's lensed.

Its major axis will be $a = \frac{\Theta_I}{\Theta_S} \cdot \Delta\Theta_S = \frac{\Theta_I}{\Theta_I - \alpha} \cdot \Delta\Theta_S$

We get its minor axis by differentiating the lens eq'n:

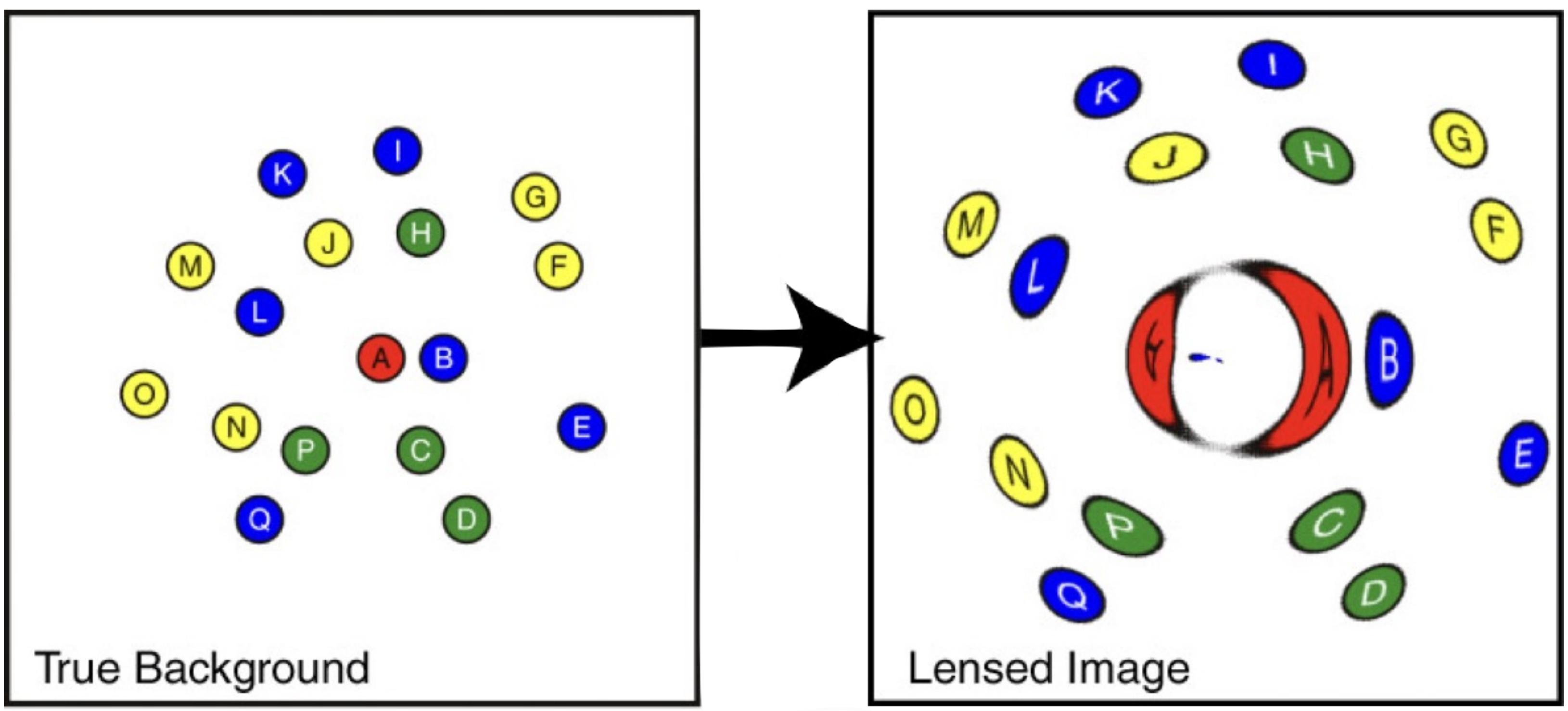
$$\Theta_S = \Theta_I - \alpha(\Theta_I)$$

$$\Delta\Theta_S = \Delta\Theta_I - \left.\frac{d\alpha}{d\Theta}\right|_{\Theta_I} \Delta\Theta_I \Rightarrow \Delta\Theta_I = b = \frac{\Delta\Theta_S}{1 - d\alpha/d\Theta_I}$$

The galaxy image is now elliptical with tangential elongation described by

$$\gamma_t \equiv \frac{a-b}{a+b} \simeq \frac{1}{2} \cdot \left(\frac{\alpha}{\Theta_I} - \frac{d\alpha}{d\Theta_I}\right) \quad (\text{in limit } \alpha \ll \Theta_S) \quad \text{SHEAR}$$

... and it is larger and brighter by $\quad 1 + \mu = \frac{ab}{\Delta\Theta_S^2} \Rightarrow \mu = \frac{\alpha}{\Theta_I} + \frac{d\alpha}{d\Theta_I}$. MAGNIFICATION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Weak lensing "aperture mass" formula.

Remember
$$\alpha(\theta_I) = \frac{4G}{c^2} \cdot \frac{D_{LS}}{D_{OS} D_{OL}} \cdot \frac{M(<\theta_I)}{\theta_I}$$

$$\Rightarrow \frac{\alpha}{\theta_I} = \frac{4\pi G}{c^2} \frac{D_{LS} D_{OL}}{D_{OS}} \cdot \frac{M(<\theta_I)}{\pi(\theta_I D_{OL})^2} = \bar{\Sigma}(\theta_I) / \Sigma_{crit}$$

$$\frac{d\alpha}{d\theta_I} = \frac{4G}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \left(-\frac{M(<\theta_I)}{\theta_I^2} + \frac{1}{\theta_I} \frac{dM(<\theta_I)}{d\theta_I} \right)$$

$$= \frac{4\pi G}{c^2} \frac{D_{LS} D_{OL}}{D_{OS}} \left(-\frac{M(<\theta_I)}{\pi(D_{OL}\theta_I)^2} + \frac{2\cancel{\pi} \cdot (\cancel{D_{OL}} \cancel{\theta_I}) \cdot \Sigma(D_{OL}\theta_I) \cdot \cancel{D_{OL}}}{\cancel{\pi} \cancel{D_{OL}^2} \cancel{\theta_I}} \right)$$

$$= \frac{1}{\Sigma_{crit}} \cdot \left(-\bar{\Sigma}(<\theta_I) + 2\Sigma(\theta_I) \right)$$

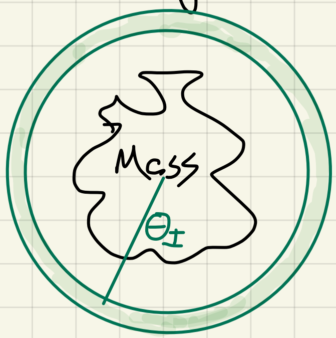
We then get

$$\boxed{\begin{array}{lll} \text{SHEAR:} & \gamma_t = \dfrac{\bar{\Sigma}(<\theta_I) - \Sigma(\theta_I)}{\Sigma_{crit}} & = \bar{\kappa}(<\theta_I) - \kappa(\theta_I) \\ \text{MAGNIFICATION:} & \mu = 2 \cdot \Sigma(\theta_I) / \Sigma_{crit} & = 2\kappa(\theta_I) \end{array}}$$

$\kappa \equiv \Sigma / \Sigma_{crit}$ is called convergence

Remarkably, these formulae work for arbitrary mass distributions!

... if we average around a circle

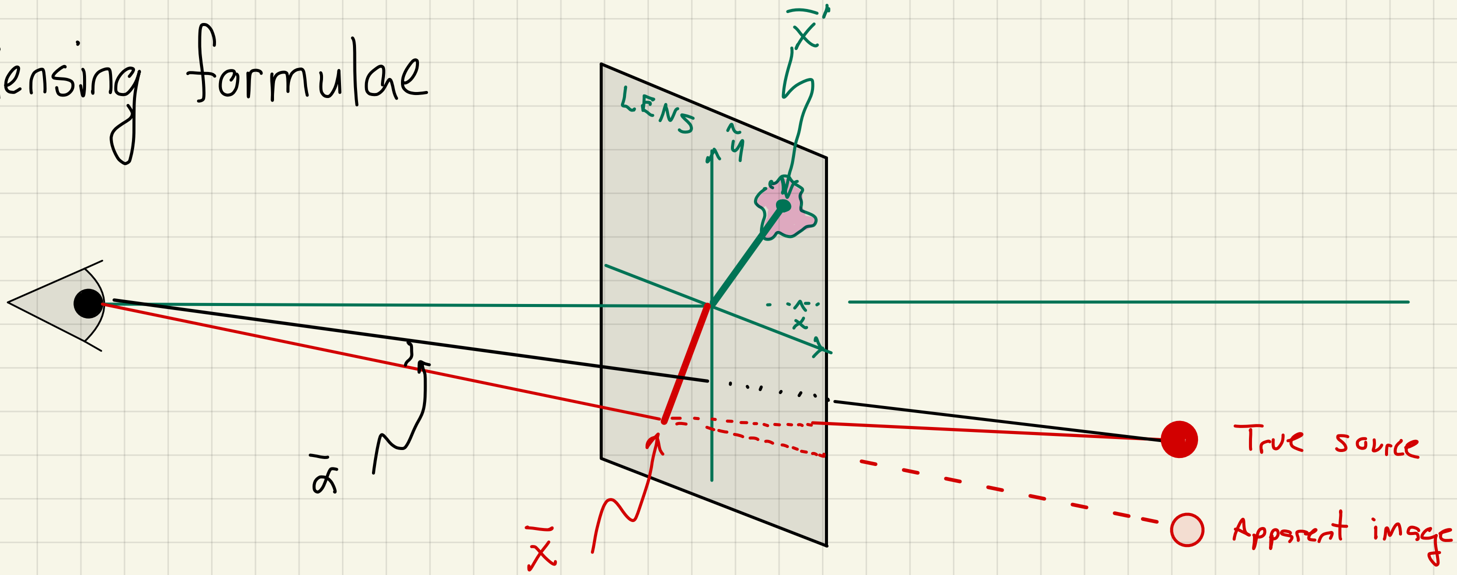


$$\langle \gamma_t \rangle_{\theta_I} = \bar{\kappa}(<\theta_I) - \langle \kappa(\theta_I) \rangle$$

$$\langle \mu \rangle = 2 \langle \kappa(\theta_I) \rangle$$

We can use these formulae to measure the total mass profiles of a single or collection of objects!

General 2d lensing formulae



With $\bar{x}$ = 2d location on lens plane of observed pos'n. $\left(\bar{\Theta}_I = \bar{x}/D_{OL}\right)$

$\bar{x}'$ = position of mass on lens plane $\left(\bar{\Theta}_L = \bar{x}'/D_{OL}\right)$

$\Sigma(\bar{x}')$ = surface mass distribution of lens

$\bar{\alpha}$ = apparent 2d deflection angle

$$\bar{\alpha}(\bar{x}) = \frac{4G}{c^2}\frac{D_{LS}}{D_{OS}} \int d^2\bar{x}' \, \frac{-\Sigma(\bar{x}')}{|\bar{x}-\bar{x}'|^2}(\bar{x}-\bar{x}')$$

$$= -\frac{4G}{c^2}\frac{D_{LS}}{D_{OS}} \bar{\nabla}_x \left[\int d^2x' \, \Sigma(\bar{x}') \ln|\bar{x}-\bar{x}'| \right]$$

The apparent deflection $\bar{\alpha}$ is the gradient of the lensing potential ψ

$$\bar{\alpha}(\bar{\Theta}_I) = \bar{\nabla}_{\Theta_I} \psi(\Theta_I) \quad , \quad \psi(\Theta_I) = \frac{1}{\pi}\int d^2\bar{\Theta}_L \, \underbrace{\frac{\Sigma(\bar{\Theta}_L)}{\Sigma_{crit}}}_{\kappa(\bar{\Theta}_L)} \ln|\bar{\Theta}_I - \bar{\Theta}_L|$$

Convergence, shear

Consequences of $\bar{\alpha} = \bar{\nabla}\psi$:

- the deflection field is curl-free: $\bar{\nabla}\times\bar{\alpha} = \frac{\partial}{\partial x}\alpha_y - \frac{\partial}{\partial y}\alpha_x = 0$

- The lensing potential & deflection are defined by a Poisson-like equation

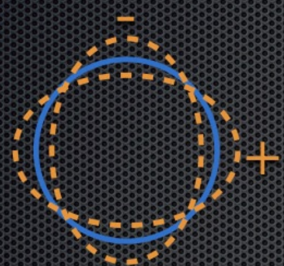
$$\nabla^2\psi = \bar{\nabla}\cdot\bar{\alpha} = \nabla_\theta^2\left[\frac{1}{\pi}\cdot\int d^2\bar{\theta}'\,\kappa(\bar{\theta}')\ln|\bar{\theta}-\bar{\theta}'|\right]$$
$$= \frac{1}{\pi}\int d^2\bar{\theta}'\,\kappa(\bar{\theta}')\cdot 2\pi\delta^2(\bar{\theta}-\bar{\theta}')$$
$$= 2\kappa(\bar{\theta}) = 2\Sigma(\bar{\theta})/\Sigma_{crit}$$

- Since $\bar{\theta}_s = \bar{\theta}_I - \alpha(\bar{\theta}_I)$, the Jacobian of the lensing map $\bar{\theta}_I \to \bar{\theta}_s$ can be written as

$$A = \frac{d\bar{\theta}_s}{d\bar{\theta}_I} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$2\kappa = \mathrm{Tr}(A) - 2 = -(\partial_x^2\psi + \partial_y^2\psi)$$
$$\gamma_1 = -(\partial_x^2\psi - \partial_y^2\psi)$$
$$\gamma_2 = -2\partial_x\partial_y\psi$$

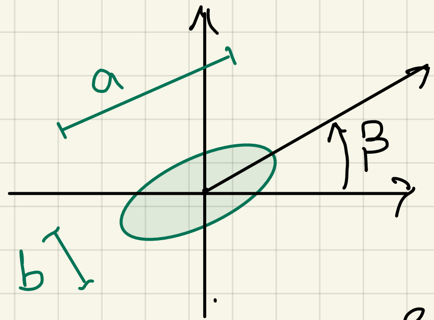
WL observables

		Model fitting	Moments
Magnification: $\bar{\Theta}_s = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} \bar{\Theta}_I$ κ		$f \to (1+2\kappa)f$ $r_e \to (1+\kappa)r_e$	$M_f = \int dx\, dy\, I(x,y) \to M_f(1+2\kappa)$ $M_r = \int dx\, dy\, I(x,y)\left(x^2+y^2\right) \to M_r(1+\kappa)$
Shear: $\bar{\Theta}_s = \begin{pmatrix} 1-\gamma_1 & 0 \\ 0 & 1+\gamma_1 \end{pmatrix} \bar{\Theta}_I$ γ_+ or γ_1		$e_+ = e\cos 2\theta \to e_+ + 2\gamma_+$	$M_+ = \int dx\, dy\, I(x,y)\left(x^2-y^2\right) \to M_+ + 2M_f\gamma_+$
$\bar{\Theta}_s = \begin{pmatrix} 1 & -\gamma_2 \\ -\gamma_2 & 1 \end{pmatrix} \bar{\Theta}_I$ $\gamma_\times$ or γ_2		$e_\times = e\sin 2\theta \to = e_\times 2\gamma_\times$	$M_\times = \int dx\, dy\, I(x,y)\left(2xy\right) \to M_\times + 2M_f\gamma_\times$

x

Shear is a "spin 2" quantity:

If the source is circular $(e=0)$, then an image will be elliptical.



$a, b =$ major/minor axes

$\beta =$ position angle of major

$$e \equiv \frac{a^2-b^2}{a^2+b^2}$$

$$e_1 \text{ (or } e_+) \equiv e \cdot \cos 2\beta$$

$$e_2 \text{ (or } e_\times) \equiv e \cdot \sin 2\beta$$

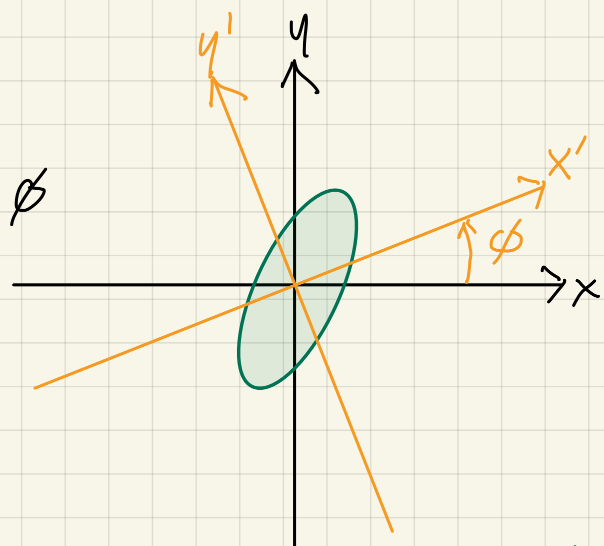
Lensing shear turns $e=0$ to $(e_1, e_2) = (2\gamma_1, 2\gamma_2)$

Coordinate rotations:

If a galaxy has (e_1, e_2) in coordinates (x,y)

...and we have new coords (x', y') rotated ccw by ϕ

... then the shape (e_1', e_2') in new coords is



$$\begin{pmatrix} e_1' \\ e_2' \end{pmatrix} = \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ -\sin 2\phi & \cos 2\phi \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

- or -

$$e_1' + i e_2' = (e_1 + i e_2) \cdot \exp(-2i\phi)$$

... same for γ_1, γ_2

[We need this eq. when figuring out the γ_t from the γ_1 and γ_2 of sources]

Shear and magnification are the second derivatives of lensing potential!

With the Born approximation we can calculate the lensing caused by a 3d mass distribution $\rho(\bar{\Theta}, \chi)$ viewed along χ axis

(χ is comoving distance to redshift z)

- We can add up the potential / deflections / shear / mag along the l.o.s., remembering $\Sigma = a\rho d$

$$\kappa = \frac{\Sigma}{\Sigma_{crit}} \Rightarrow \quad \kappa(\bar{\Theta}) = \int_0^{\chi_s} d\chi_L \frac{\rho(\bar{\Theta}, \chi_L)}{\Sigma_{crit}} = \int_0^{\chi_s} \frac{4\pi G}{c^2 D_{os}} D_{OL} D_{LS} \cdot \rho(\bar{\Theta}, \chi_L)\, d\chi_L\, a$$

Let's convert this into an integral over redshift z, with $a = (1+z)^{-1}$.

$$H(z) = \frac{1}{a} \cdot \frac{da}{dt} \Rightarrow dt = \frac{1}{aH} \cdot d(1+z)^{-1} = \frac{1}{aH}(1+z)^{-2}\, dz = \frac{a}{H}\, dz$$

The distance $\chi(z)$ has $d\chi = \frac{c \cdot dt}{a} = \frac{c\, dz}{H}$

* a constant mass density causes no deflection!

... with overdensity $\delta(\bar{\Theta}, z) = \rho(\bar{\Theta}, z) / \bar{\rho}(z) - 1 \Rightarrow \rho(\bar{\Theta}, z) = \delta(\bar{\Theta}, z) \cdot \Omega_m \cdot \rho_{crit} \cdot (1+z)^3$

In a flat universe, $D_{os} = \chi_s a_s$, $D_{OL} = \chi_L a_L$, $D_{LS} = (\chi_s - \chi_L) a_s$

$$\kappa(\bar{\Theta}) = \frac{\cancel{4\pi G}\, \Omega_m}{2c} \cdot \frac{3H_0^2}{\cancel{8\pi G}} \cdot \int_0^{z_s} dz_L \frac{\chi_L(\chi_s - \chi_L)}{\chi_s} \cdot \delta(\bar{\Theta}, z) \cdot \frac{(1+z)^2}{H} \cdot a_L$$

$$= \int_0^{z_s} dz_L\, \delta(\bar{\Theta}, z_L) \cdot \underbrace{\frac{3H_0^2 \Omega_m}{2c} \cdot \frac{\chi_L(\chi_s - \chi_L)}{\chi_s} \frac{1}{Ha}}_{\text{"lensing kernel"}}$$

Wonderful weak lensing math

$$\kappa(\bar\theta) = \int_0^{z_s} dz_L\, \delta(\bar\theta, z_L) \cdot W(z_s, z_L) \,, \quad W = \frac{3H_0^2 \Omega_m}{2c} \cdot \frac{\chi_L(\chi_s - \chi_L)}{\chi_s} \frac{1}{H a_L}$$

$$\nabla^2 \psi = 2\kappa$$

$$\bar\alpha = \bar\nabla \psi \,, \qquad \gamma_1 = -(\partial_x^2 - \partial_y^2)\psi \qquad \gamma_2 = -(2\partial_x \partial_y)\psi$$

$$\bar\nabla \times \bar\alpha = 0$$

- Knowing any one of κ, $\bar\alpha$, (γ_1, γ_2) or ψ we can get the others!

Easiest to see in Fourier space: if $\psi(\bar\theta) = \int d^2\ell\, \tilde\psi(\bar\ell)\, e^{i\bar\ell\cdot\bar\theta}$

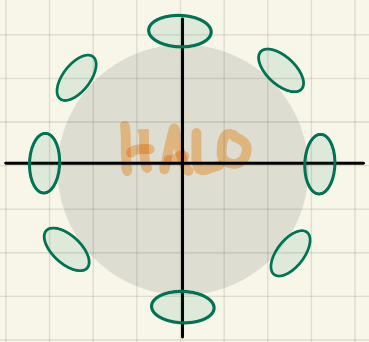
then $\tilde\kappa(\bar\ell) = -(\ell_x^2 + \ell_y^2)\,\tilde\psi(\bar\ell)$, $\tilde\gamma_1 = -(\ell_x^2 - \ell_y^2)\,\tilde\psi(\bar\ell)$, $\tilde\gamma_2(\bar\ell) = -2\ell_x \ell_y \tilde\psi(\bar\ell)$

Measure γ pattern, get a mass map!

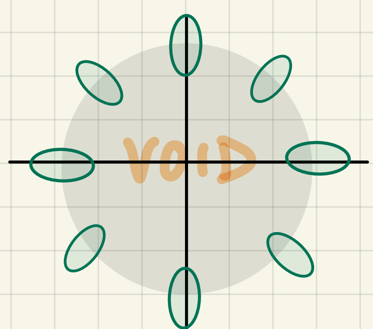
- γ_1, γ_2 have a consistency relation because they are different 2nd derivs of the <u>same ψ</u>

$$2\partial_x \partial_y \gamma_1 = (\partial_x^2 - \partial_y^2)\gamma_2$$

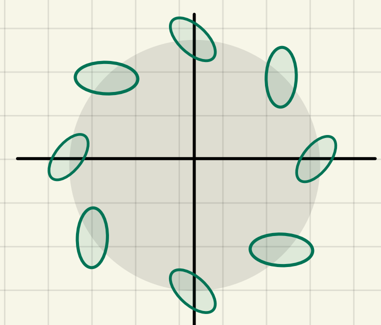
$\Rightarrow$ The shear field is pure "E mode", its "B mode" must be zero.



This can happen
$\langle \gamma_t \rangle > 0$

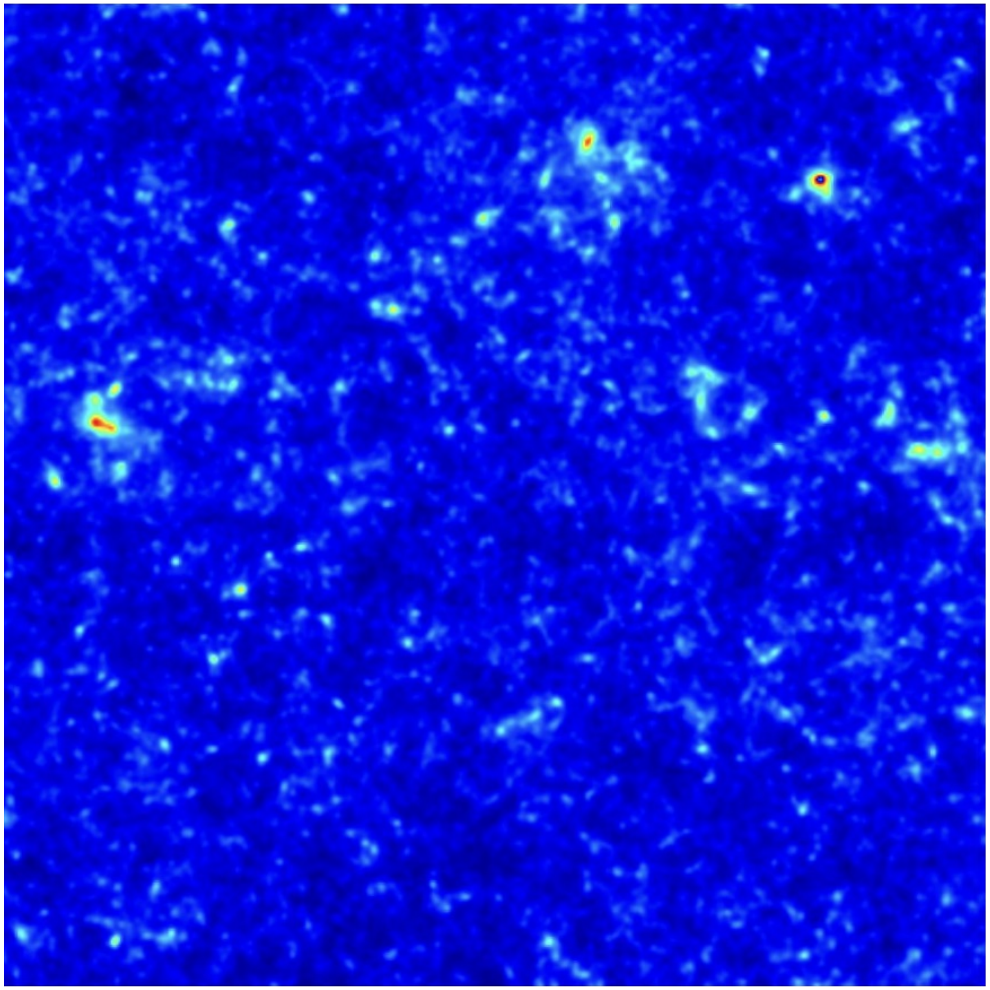


$\langle \gamma_t \rangle < 0$

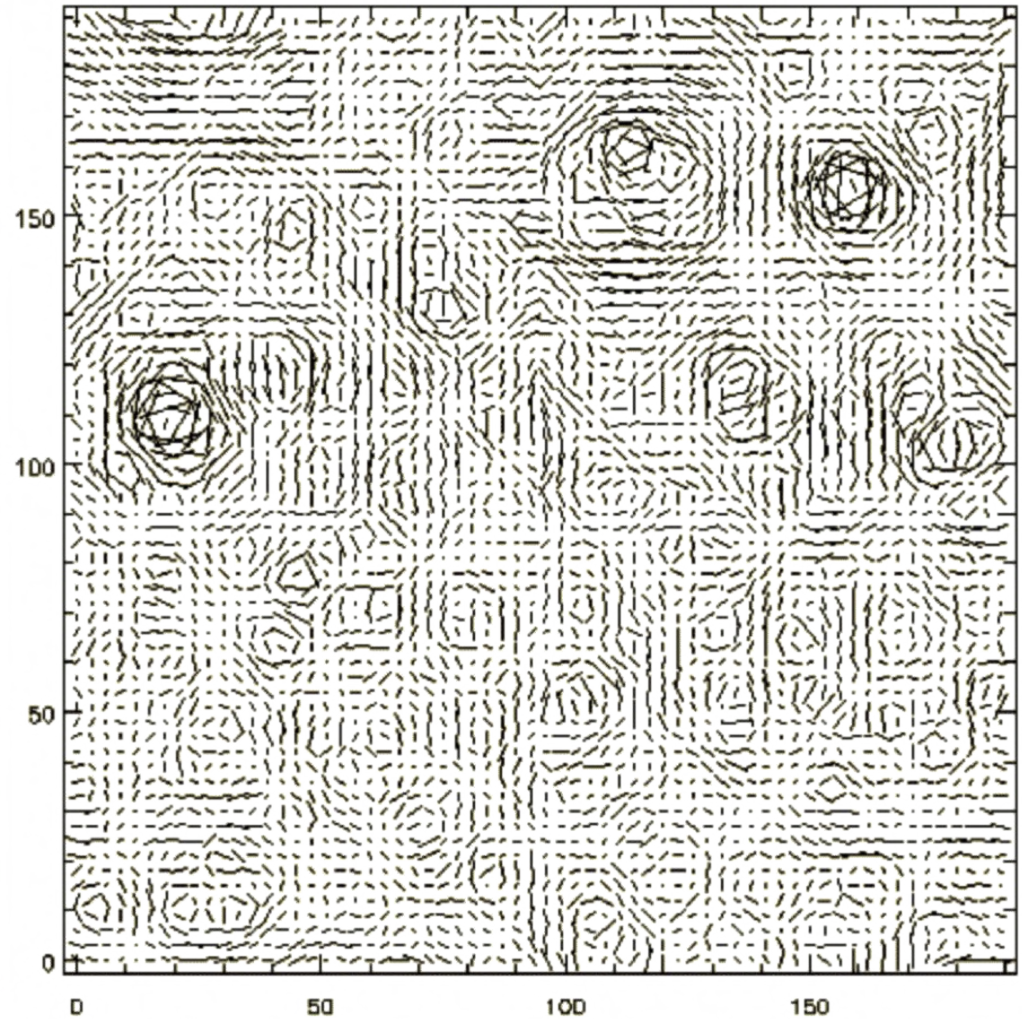


This cannot happen - $\langle \gamma_x \rangle = 0$ required!

(B. Jain)



Projected mass map

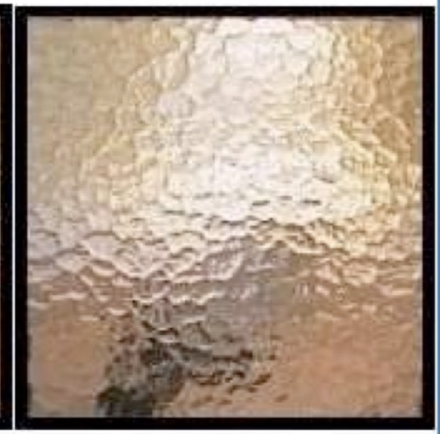


Gravitational shear map

Clear Glass
Obscurity Level 0

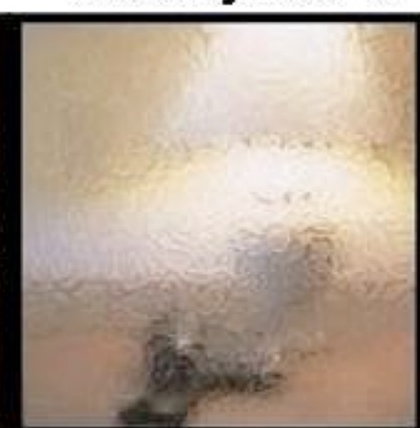
Cotswold
Obscurity Level 5

Everglade
Obscurity Level 5

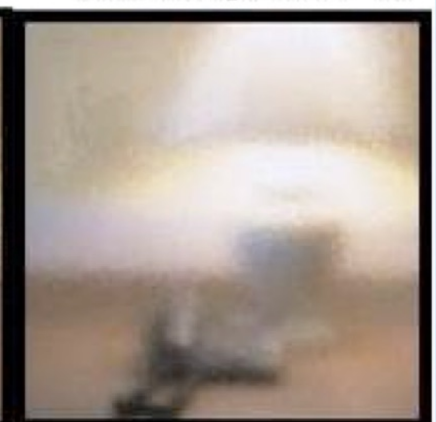


Artic
Obscurity Level 4

Bamboo
Obscurity Level 4

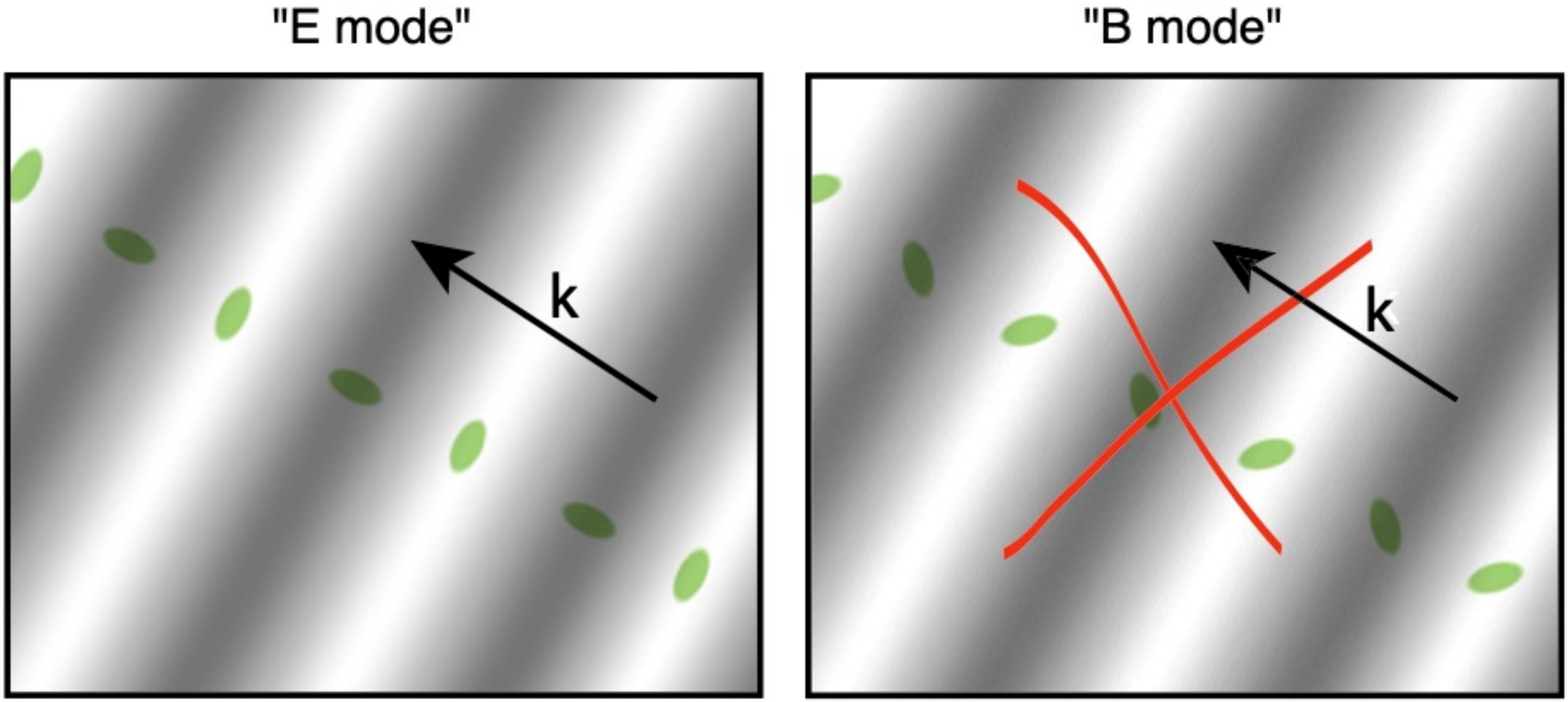


Florielle
Obscurity Level 4



Stippolyte
Obscurity Level 4

,



Foreground mass sinusoid produces ellipticity pattern at the same k-vector

Lensing cannot produce ellipticity pattern at 45 degrees to k-vector

Perpendicular/along the wave vector

2-point functions of lensing

Using our $\kappa(\vec{\theta}) = \int_0^{z_s} dz_L \, \delta(\vec{\theta}, z_L) \, \bar{W}(z_L, z_s)$

the Limber approximation tells us that the power spectrum of WL (shear or mag) will be

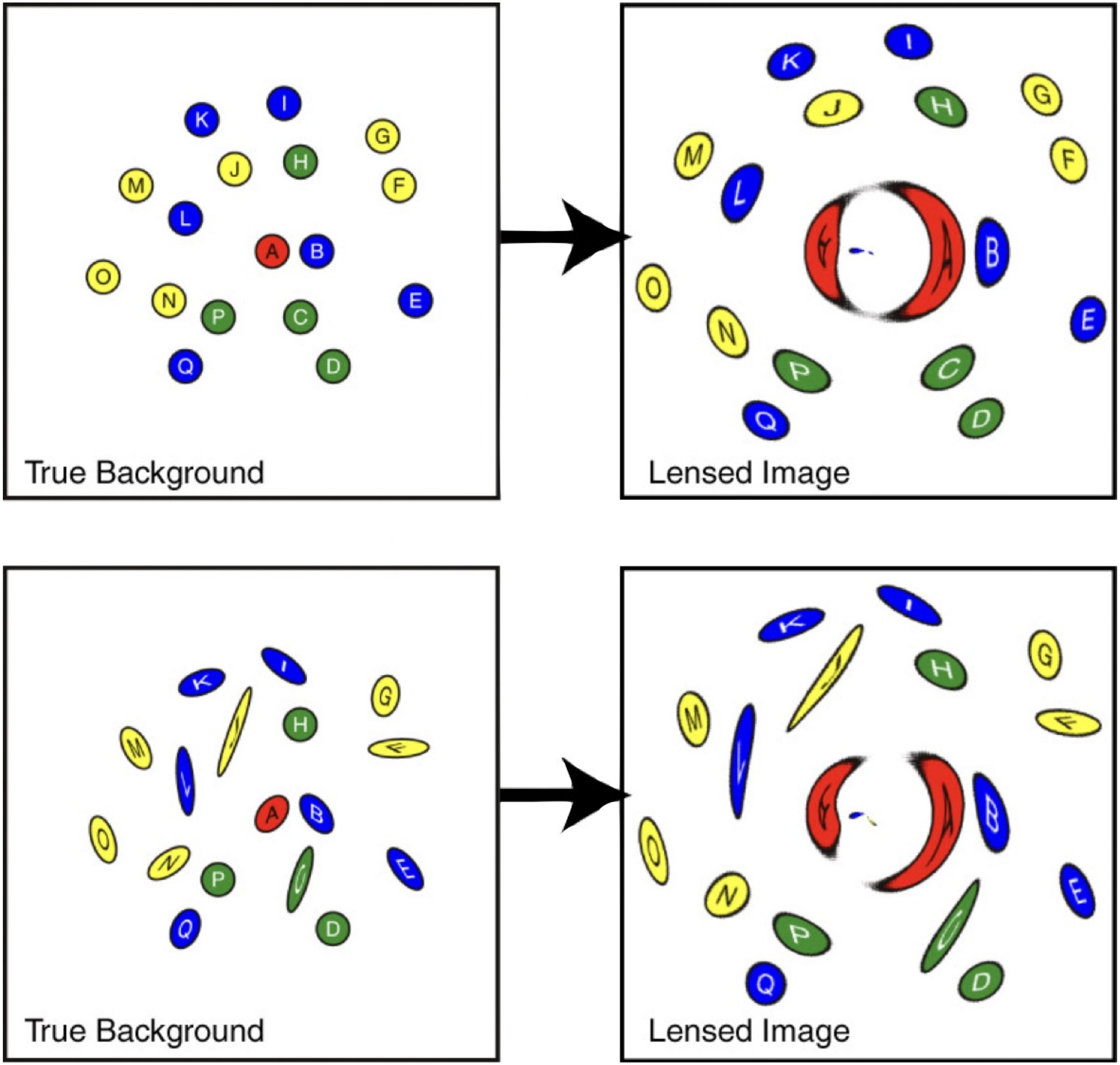
$$P_\kappa(\ell) = \int_0^{z_s} dz_L \cdot P_\delta(k = \ell/\chi_L, z_L) \cdot \bar{W}^2(z_L, z_s) \cdot \frac{H(z_L)}{\chi_L^2}$$

measure this

compare to this cosmological theory

Equivalently - measure 2-point correlations ξ of the shear

Because shear has 2 components, there are multiple possible ξ's.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True Background
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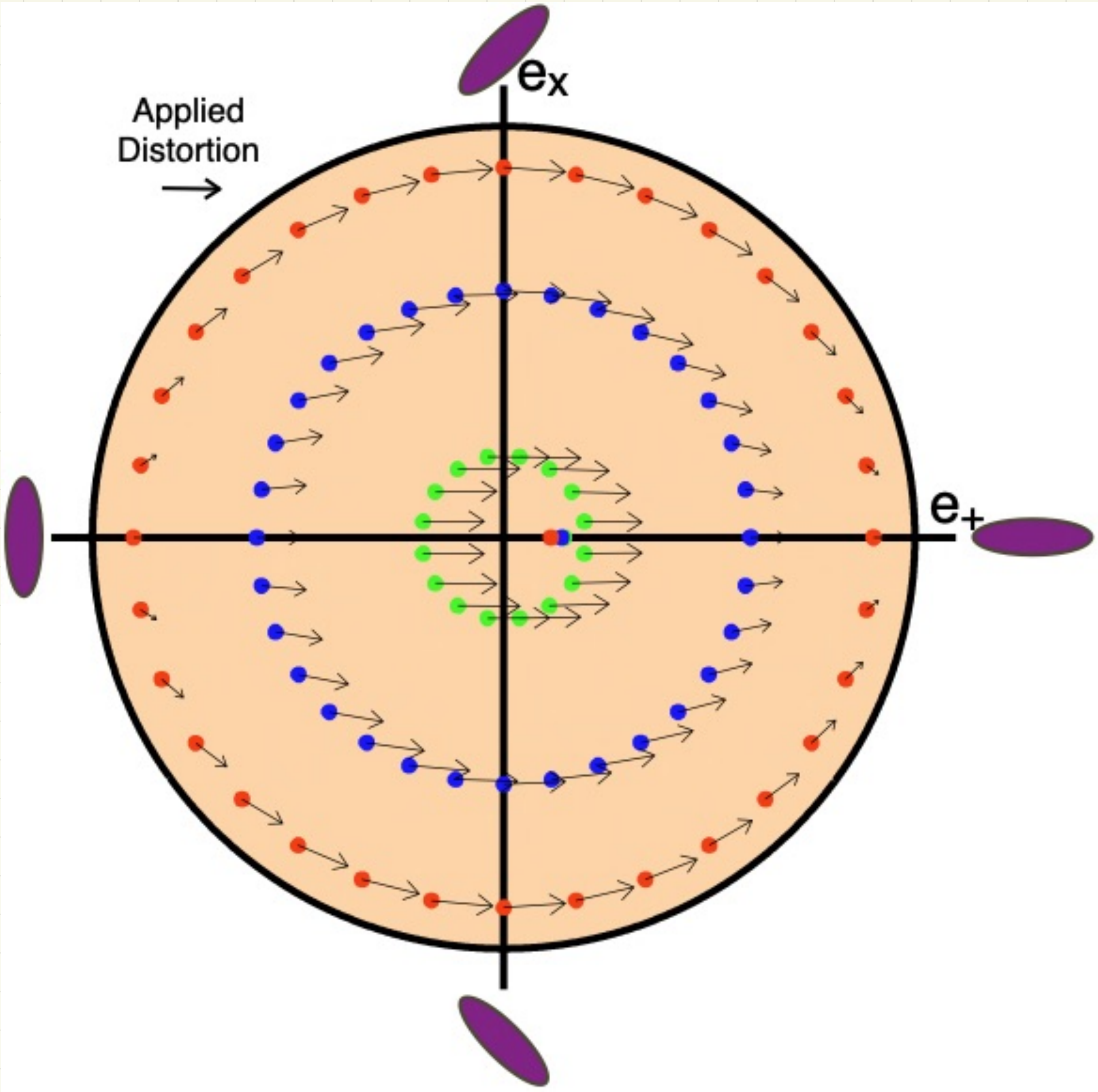
SHAPE NOISE



Galaxies are not intrinsically circular!

Intrinsic shape $\bar{e}_I = (e_{I,1}, e_{I,2})$

$$|e_I| = \frac{a^2-b^2}{a^2+b^2}$$

is altered by applied shear $\bar{\gamma}$ roughly (not exactly) as

$$\bar{e}_{obs} = \bar{e}_I + 2\bar{\gamma}$$

... so the average shape of N galaxies is

$$\langle \bar{e}_{obs} \rangle = \langle \bar{e}_I \rangle + 2\bar{\gamma} = 2\bar{\gamma}$$

avg intrinsic shape is zero in an isotropic universe!

If $Var(e_{I,1}) = Var(e_{I,2}) = \sigma_e^2$, then we estimate

shape noise!

$$\boxed{\hat{\gamma}_1 = \frac{\langle e_{obs,1} \rangle}{2} \pm \frac{\sigma_e}{2\sqrt{N}}}$$

we like big N!

- Typically $\sigma_e = 0.3 - 0.4$
- RMS shear on $z \approx 1$ line of sight: 0.02
- If we want to measure shear (and mass) power to 1% accuracy we need $\sigma_\gamma \approx \frac{0.02}{100} \lesssim \frac{\sigma_e}{2\sqrt{N}}$
- $\Rightarrow \quad N \gtrsim \left(\frac{\sigma_e}{2\sigma_\gamma}\right)^2 = \left(\frac{0.4}{2 \times 0.0002}\right)^2 = 10^6$

... and this is optimistic for several reasons.

Weak lensing is a numbers game!

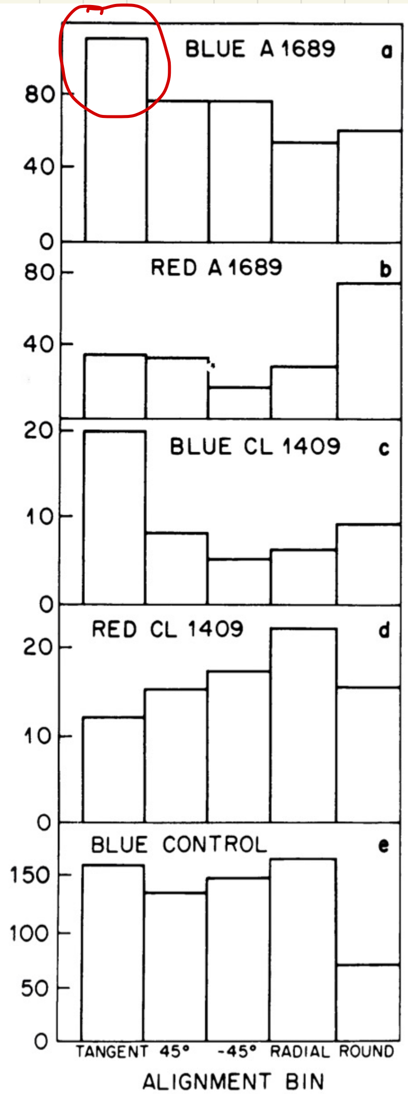


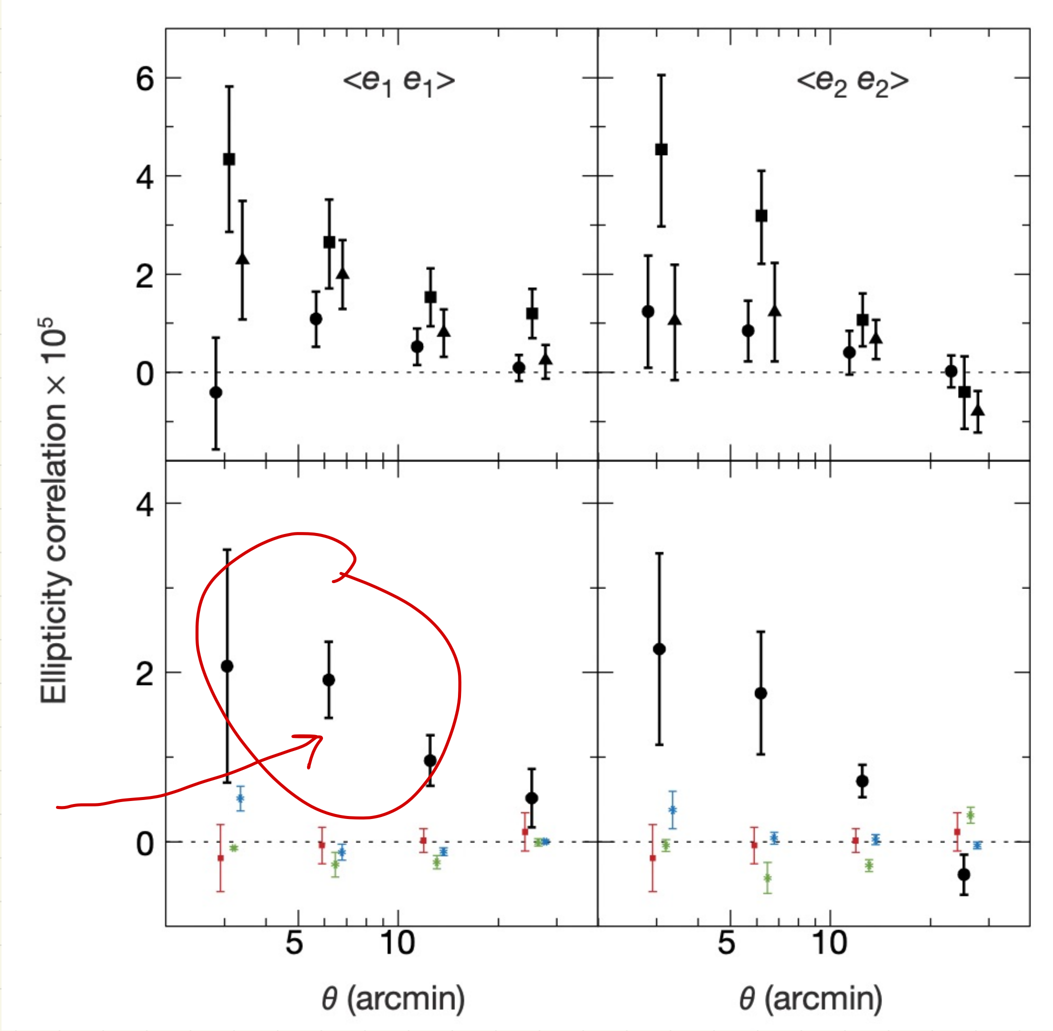
FIG. 5.—Histograms of faint galaxy major axis alignments relative to the vector to the cluster center are binned (for ellipticities above 0.2) into four orientations: tangent (90°), radial (0°), and ±45°. Only galaxies of 22–26 B_J mag are included. An excess number of blue galaxies are aligned orthogonal to the radius of the cluster center (tangent bin), due to the lens distortion. Blue (background) and red (cluster) galaxy alignments for the A1689 field are shown in Figs. 5*a* and 5*b*, and for CL 1409+52 in Figs. 5*c* and 5*d*. Figure 5*e* shows the faint blue galaxy alignment histogram for the sum of 11 similar high-latitude comparison fields with no foreground clusters.

Tyson, Valdes, & Wenk 1990

Excess tangent alignment around massive clusters

$\boxed{\gtrsim 300}$ galaxies in (blue) background

Single $\lesssim 1$ Mpix CCD



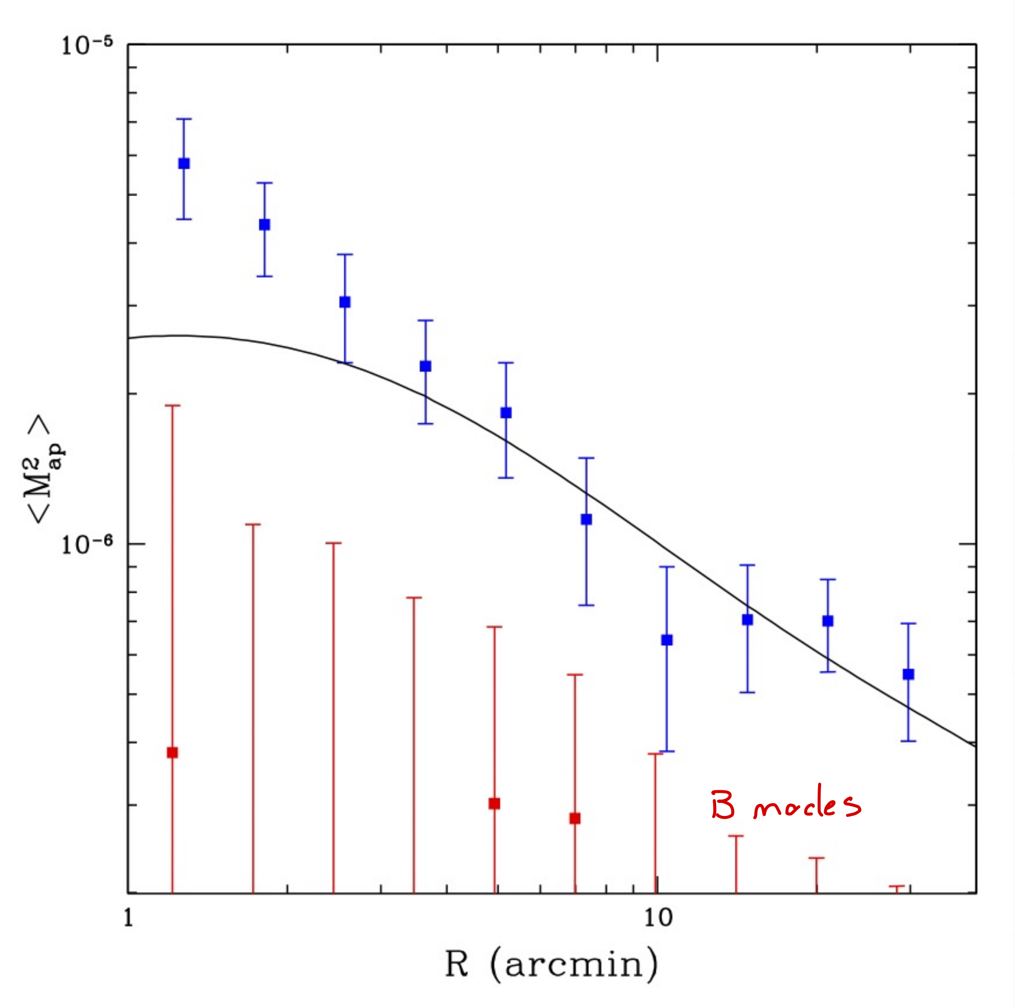
Wittman et al 2000

First detection (w/2 others) of "cosmic shear" correlations in random fields

$\boxed{10^5 \text{ galaxies}}$,

1.5 deg^2

16 Mpix camera



Jarvis et al 2006

$\boxed{2 \times 10^6 \text{ galaxies}}$

75 deg^2

16 Mpix → 64 Mpix

Kilbinger et al 2013, CFHTLens

$\boxed{4 \times 10^6 \text{ galaxies}}$

154 deg^2

340 Mpix camera

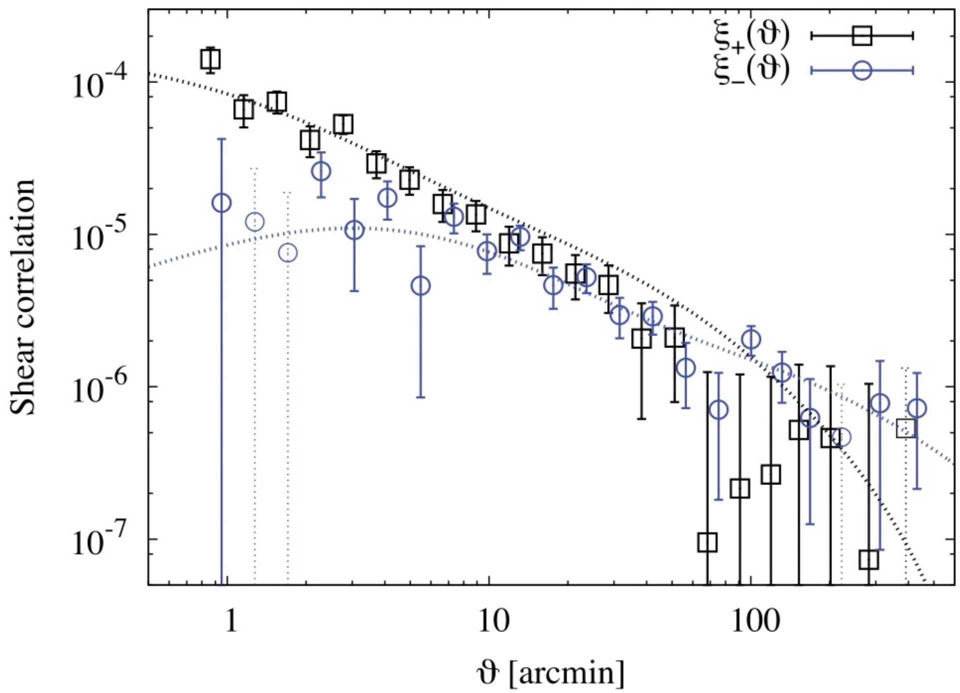


Figure 6. The measured shear correlation functions ξ_+ (black squares) and ξ_- (blue circles), combined from all four Wide patches. The error bars correspond to the total covariance diagonal. Negative values are shown as thin points with dotted error bars. The lines are the theoretical prediction using the *WMAP*7 best-fitting cosmology and the non-linear model described in Section 4.3. The data points and error bars are listed in Table B1.

Amon et al 2022

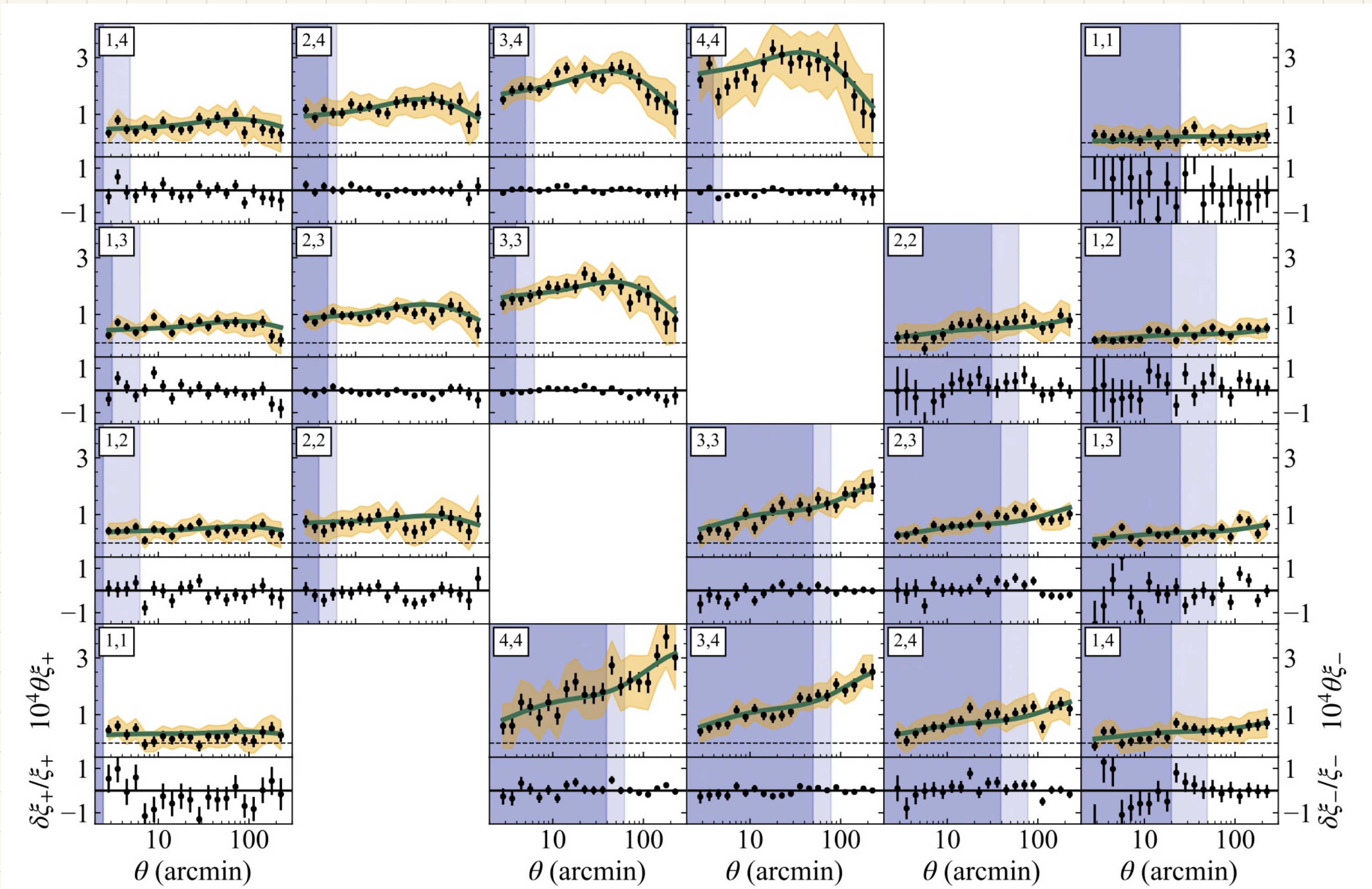
Dark Energy Survey Year 3

$\boxed{10^8 \text{ galaxies}}$

4200 deg^2

500 Mpix camera

Shear-shear correlations among 4 redshift bins.



See also:

- Kilo-Degree Survey
- Hyper Suprime Cam Survey

All working on final analyses.

- LSST: ≈ 10^9 galaxies!

≈ 18,000 sq deg

2 Gpix camera

- Euclid (ESO)
- Roman (NASA)

space-based surveys.

Measuring Weak lensing

For a truly elliptical-shaped galaxy, we have clear definitions of a & b for a chosen isophote

$$e = \frac{a^2 - b^2}{a^2 + b^2}, \quad \begin{matrix} e_1 \\ e_2 \end{matrix} = e \cdot \begin{matrix} \cos 2\beta \\ \sin 2\beta \end{matrix}$$

① ... and we know $\langle e_1 \rangle = \langle e_2 \rangle = 0$ in absence of lensing

② ... and we know exactly how WL $\gamma_1, \gamma_2, \kappa$ will affect e_1, e_2, and $r^2 = a^2 + b^2$.

PROBLEM #1

For a not-elliptical galaxy, how would we define e_1, e_2, r^2 such that ① and ② hold?

Here's a solution: for galaxy with brightness distribution $I(x,y)$, define

$$M_x \equiv \int d^2x \, (x - x_0) \, I(x,y)$$

$$M_y \equiv \int d^2x \, (y - y_0) \, I(x,y)$$

$$\left.\begin{matrix} M_{xx} \\ M_{xy} \\ M_{yy} \end{matrix}\right\} \equiv \int d^2x \left\{ \begin{matrix} (x-x_0)^2 \\ (x-x_0)(y-y_0) \\ (y-y_0)^2 \end{matrix} \right\} I(x,y)$$

$$\text{flux} = \int d^2x \, I(x,y)$$

0th/1st/2nd CENTRAL MOMENTS

(1) Find x_0, y_0 such that $M_x = M_y = 0$

(2) Define

$$e_1 = \frac{M_{xx} - M_{yy}}{M_{xx} + M_{yy}}$$

$$e_2 = \frac{2 M_{xy}}{M_{xx} + M_{yy}}$$

$$r^2 = \frac{M_{xx} + M_{yy}}{\text{flux}}$$

These have same ① & ② properties as true ellipses!

To see why:

Recall $A = \frac{d\bar{\theta}_s}{d\bar{\theta}_I} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$

- set $x_0, y_0 = 0$ for both lensed & unlensed

$$\begin{pmatrix} x_I \\ y_I \end{pmatrix} = A \cdot \begin{pmatrix} x_s \\ y_s \end{pmatrix}$$ across galaxy

$$I_{obs}(x_I, y_I) = I_{true}(x_s, y_s)$$

$$\begin{pmatrix} M_{xx} & M_{xy} \\ M_{xy} & M_{yy} \end{pmatrix}_{obs} = \int d^2x_I \, I_{obs}(\bar{x}_I) \begin{pmatrix} x_I^2 & x_I y_I \\ x_I y_I & y_I^2 \end{pmatrix} = \int d^2x_I \, I_{obs}(\bar{x}_I) \, \bar{x}_I \bar{x}_I^T$$

$$= \int |A| \, d^2x_s \cdot I_{true}(\bar{x}_s) \cdot (A\bar{x}_s)(A\bar{x}_s)^T$$

$$= (1-\kappa^2-\gamma^2) \cdot \int d^2x_s \, I_{true}(\bar{x}_s) \cdot A \left(\bar{x}_s \bar{x}_s^T\right) A^T$$

$$= (1-\kappa^2-\gamma^2) \cdot A \begin{pmatrix} M_{xx} & M_{xy} \\ M_{xy} & M_{yy} \end{pmatrix}_{true} \cdot A^T \longrightarrow$$ linear transformations of moments!

PROBLEM #2 : We have to observe a blurred version of the galaxy, which alters the size and shape!

Solution: need to know the point spread function (PSF) very accurately and remove its effect on e's. This is simple for our 2nd moments.

The observed image is the convolution of the true (lensed) sky image by the PSF:

$$I_{obs}(x,y) = [I_{sky} * PSF](x,y) = \int d^2\bar{x}'\, I_{sky}(\bar{x}',\bar{y}') \cdot PSF(\bar{x}-\bar{x}')$$

$$M_{xx}^{obs} = \int d^2x\, I_{obs}(\bar{x})\, x^2 = \int d^2x\, d^2x'\; x^2 I_{sky}(\bar{x}',\bar{y}') \cdot PSF(\bar{x}-\bar{x}')$$

define $x'' = x - x'$

$x^2 = (x' + x'')^2$

$$= \int d^2x' \int d^2x'' \left((x')^2 + (x'')^2 + 2x'x''\right) I_{sky}(\bar{x}')\, PSF(\bar{x}'')$$

$$= \int d^2\bar{x}'\, (x')^2 I_{sky}(x') \cdot \int d^2x''\, PSF(x'') + \int d^2\bar{x}'\, I_{sky}(x') \cdot \int d^2x''\, PSF(x'')(x'')^2$$

$$= M_{xx}^{sky} + flux \cdot M_{xx}^{PSF}$$

(since $\int d^2x\, PSF(x) = 1$)

$$\frac{M_{xx}^{sky}}{flux} = \frac{M_{xx}^{obs}}{flux} - M_{xx}^{PSF}$$

- we just subtract away the PSF moments!

PROBLEM #3: Every image of the sky has noise in every pixel.

$\Rightarrow$ moment measurements are noisy.

$\Rightarrow$ e_1, e_2, r^2 measures are noisy and biased

Indeed our moments acquire infinite noise because $\int d^2x$ goes to $\pm\infty$!

Solution: use weighted moments

some function with finite area

$$M_{xx} \equiv \int d^2\bar{x}\, I(\bar{x})\, x^2 \cdot W(\bar{x})$$

... alas, including W breaks the nice relations btwn e_1, e_2 and γ_1, γ_2 and breaks the property that PSF simply adds to moments.

It took almost 25 years after Tyson, Valdes & Wenk's 1990 paper to develop a method that measure γ to 1 part per thousand accuracy from galaxy images in presence of PSF and noise!

see Bayesian Fourier Domain (BFD) methods (Bernstein et al 2016)

Metacalibration, Metadetection (Huff, Sheldon, Mandelbaum, Becker... papers 2020, 2017)

"FPFS" (Li & Mandelbaum 2023)

MORE PROBLEMS & SOLUTIONS

- The detector gives us a pixelized version of the image
- Detectors are not strictly linear recorders of $I(\bar{x})$
- The PSF is a function of λ but detectors mix photons over range of λ into the image
- "Selection biases" exist – WL can make galaxies disappear from the sample!
- Blending – nearby galaxies can overlap. How do we know if this has happened? How do we reallocate the photons to the individual galaxies?
- Redshifts – we need to know the z_s to make accurate theory predictions of γ. But it is infeasible to measure absorption/emission lines' λ's for 10^8 galaxies.

Photometric redshifts – estimate z_s to low precision but very high accuracy using its broadband colors + knowledge of galaxies.
(A big topic of its own!)

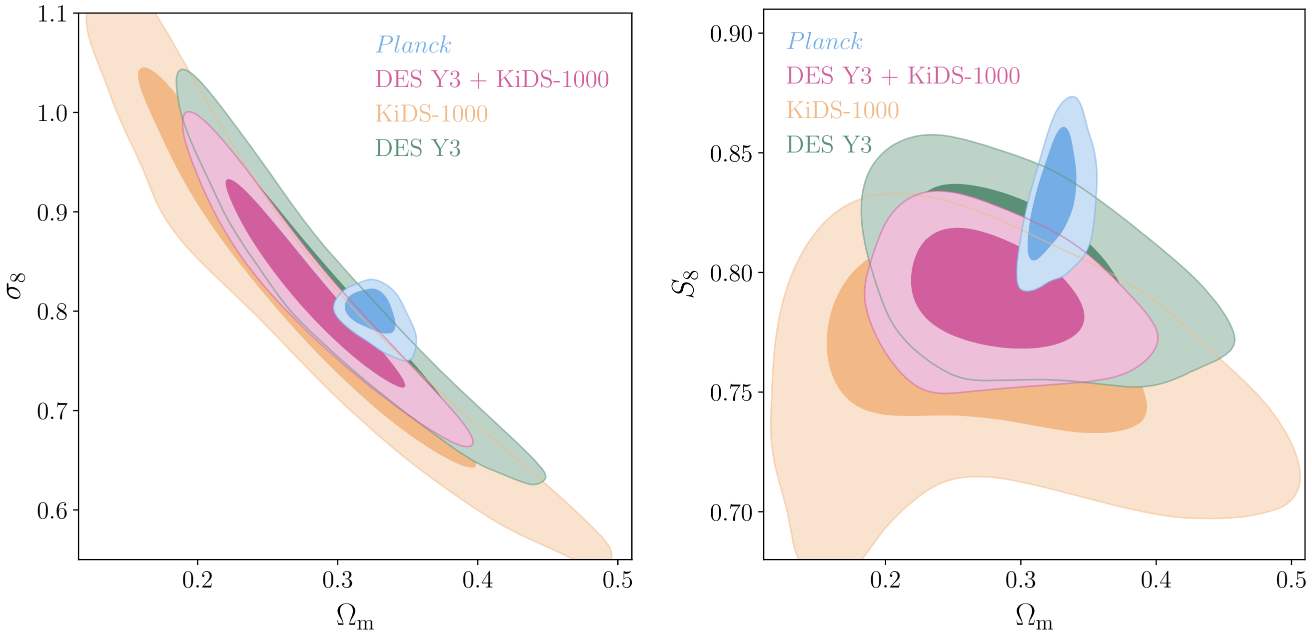


Figure 1. Cosmological constraints on the clustering amplitude, σ_8, (left) and S_8 (right) with the matter density, Ω_m in flat-ΛCDM . The marginalised posterior contours (inner 68% and outer 95% credible intervals) are shown for the DES Y3 + KiDS-1000 Hybrid analysis in pink and Planck Collaboration (2020) CMB (TT,TE,EE+lowE) in blue. The yellow contours represent the Hybrid analysis of KiDS-1000 only and the green, of DES Y3 only.