The Galaxy-Halo Connection

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OUTLINE

LECTURE 1

- A primer on Structure Formation
- The Halo Model
- Halo Occupation Modeling
  - Halo Occupation Distribution (HOD)
  - Conditional Luminosity Function (CLF)
  - Subhalo Abundance Matching (SHAM)

LECTURE 2

- Empirical Constraints
  - Galaxy clustering
  - Galaxy-Galaxy lensing
  - Satellite kinematics
- Galaxy-Halo Connection
  - Stellar Mass-Halo Mass Relation (SHMR)
  - Scatter in SHMR
  - Satellite Galaxies
- Cosmological Constraints
  - The S_8 tension
- Issues & Concerns
  - Artificial Subhalo Disruption & Orphans
  - Baryonic Effects
  - Assembly Bias

PRELIMINARIES

- This `review' is far from complete

  I sincerely apologize to those whose work I am unable to cite
  I sincerely apologize to those whose work I misrepresent

- This `review' is biased both in terms of views and scope

  I will occasionally express my personal views and opinions.
  Please feel free to disagree in silence or to engage in a discussion.

  I will mainly highlight topics that I personally find particularly exciting

- This `review' is more historical than up-to-date

  Basically, I have a hard time keeping up with the exponentially growing body of literature on this topic

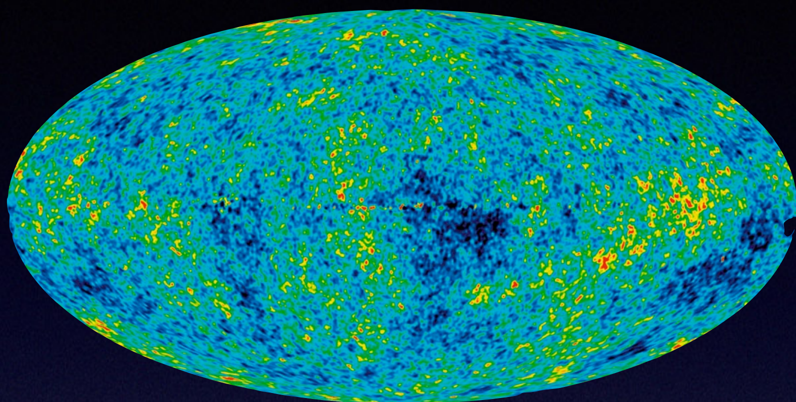
- This `review' hopefully will spawn interest in this field

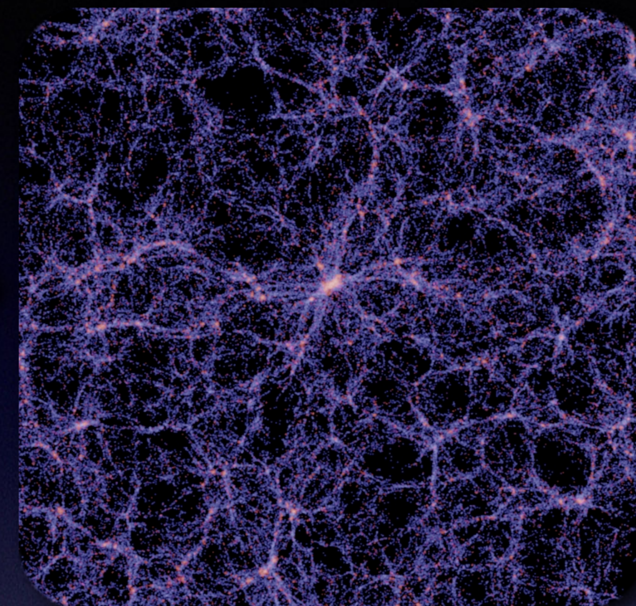
  Like any field in astrophysics/cosmology, we need new, bright minds to make progress..

- Do NOT hesitate to ask questions. Feel free to interpret any time!!!!

Structure Formation
...a primer...

Structure Formation





Initial Conditions

Linear Growth

Non-Linear Collapse

Inflation

Linear Perturbation Theory

N-body Simulations

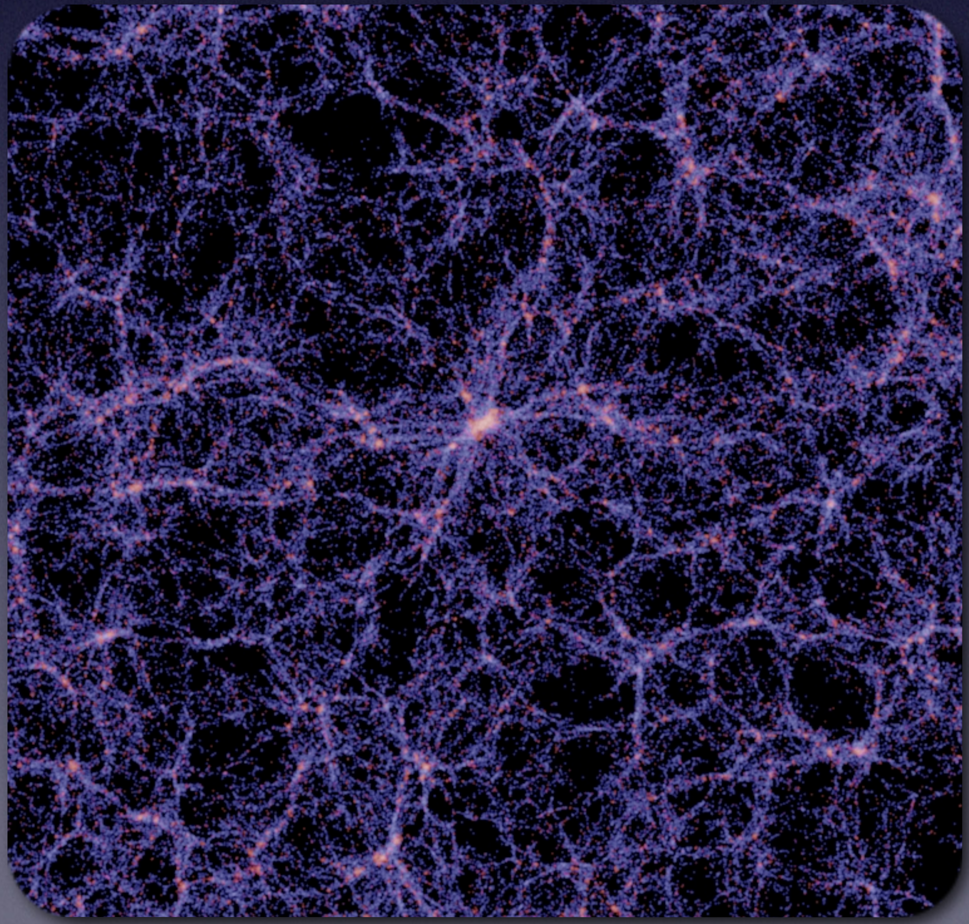
The Density Field

Let $\rho(\vec{x})$ be the density distribution of matter at location $\vec{x}$

It is useful to define the corresponding overdensity field

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

Note: $\delta(\vec{x})$ is believed to be the outcome of some random process in the early Universe (i.e., quantum fluctuations in inflaton)



Let $P(\delta)$ describe the probability that a random location in the Universe has an overdensity δ

First Moment

$$\langle\delta\rangle = \int \delta\, \mathcal{P}(\delta)\, \mathrm{d}\delta = \int \delta(\vec{x})\, \mathrm{d}^3\vec{x} = 0$$

ergodic principle: ensemble average = spatial average

Second Moment

$$\langle\delta^2\rangle = \int \delta^2 \mathcal{P}(\delta)\, \mathrm{d}\delta = \sigma^2$$

variance of density field

The Two-Point Correlation Function

But what about $\mathcal{P}(\delta_1, \delta_2)$ where $\delta_1 = \delta(\vec{x}_1)$ and $\delta_2 = \delta(\vec{x}_2)$ with $\vec{x}_2 = \vec{x}_1 + \vec{r}_{12}$

If δ_1 and δ_2 are independent, then $\mathcal{P}(\delta_1, \delta_2) = \mathcal{P}(\delta_1)\,\mathcal{P}(\delta_2)$ and $\langle \delta_1 \delta_2 \rangle = \langle \delta_1 \rangle \langle \delta_2 \rangle = 0$

However, because of gravity, δ_1 and δ_2 are correlated; we define the

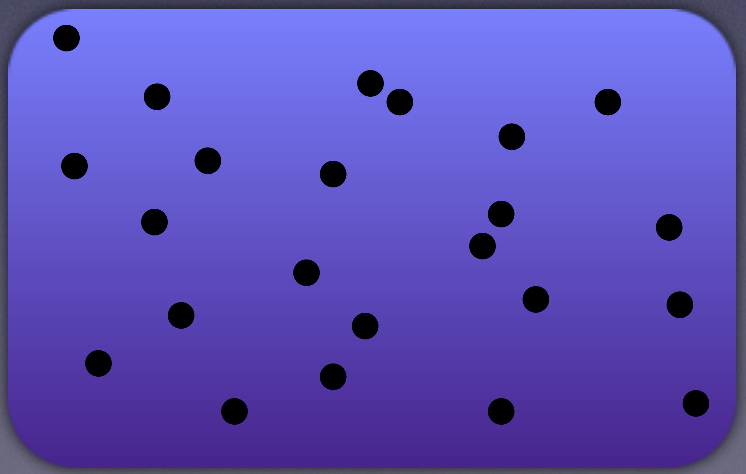
two-point correlation function $\xi(r_{12}) = \langle \delta_1 \delta_2 \rangle \qquad r_{12} = |\vec{x}_1 - \vec{x}_2|$

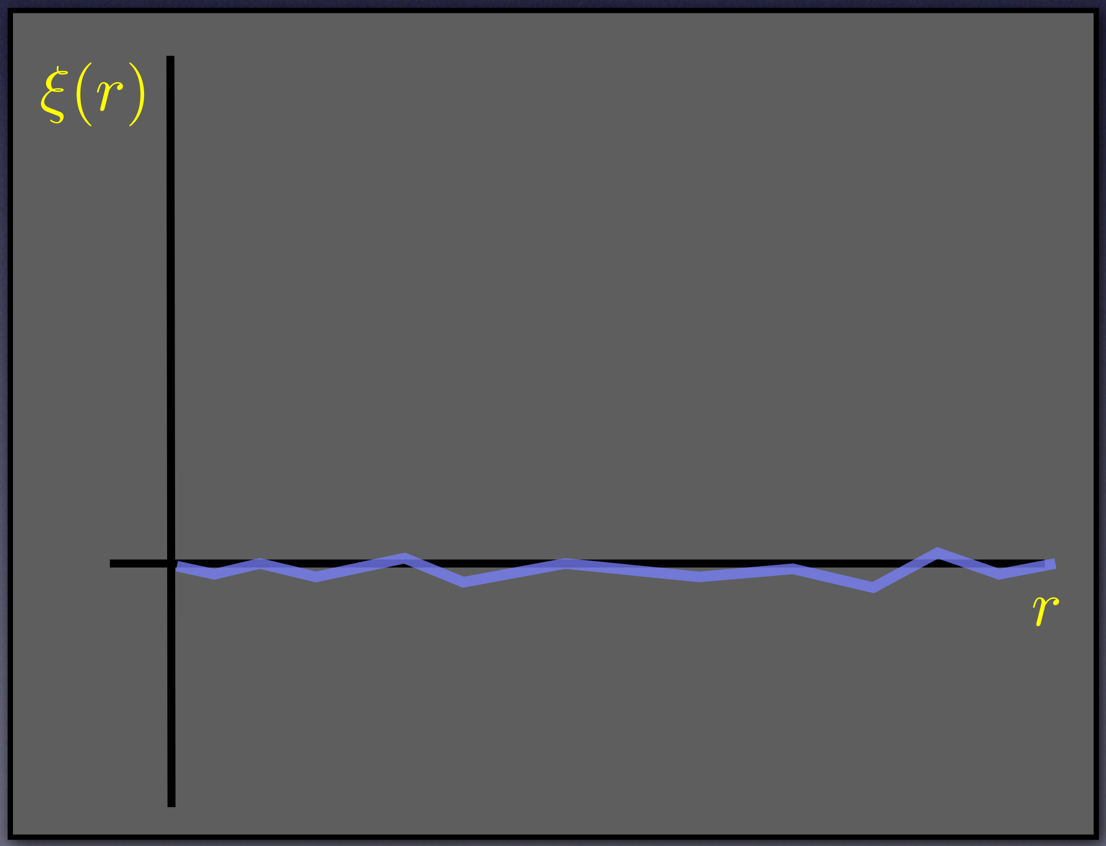
Note: $\xi(0) = \sigma$

for discrete points:

$$1 + \xi(r) = \frac{n_{\rm pair}(r \pm {\rm d}r)}{n_{\rm random}(r \pm {\rm d}r)}$$

Poisson distribution





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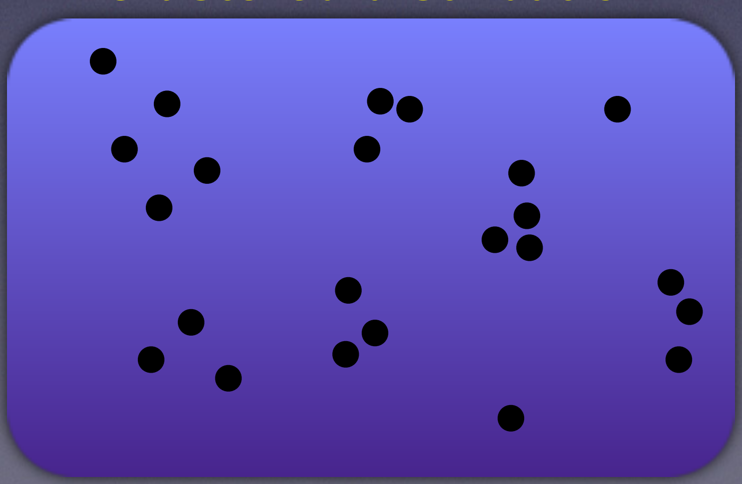
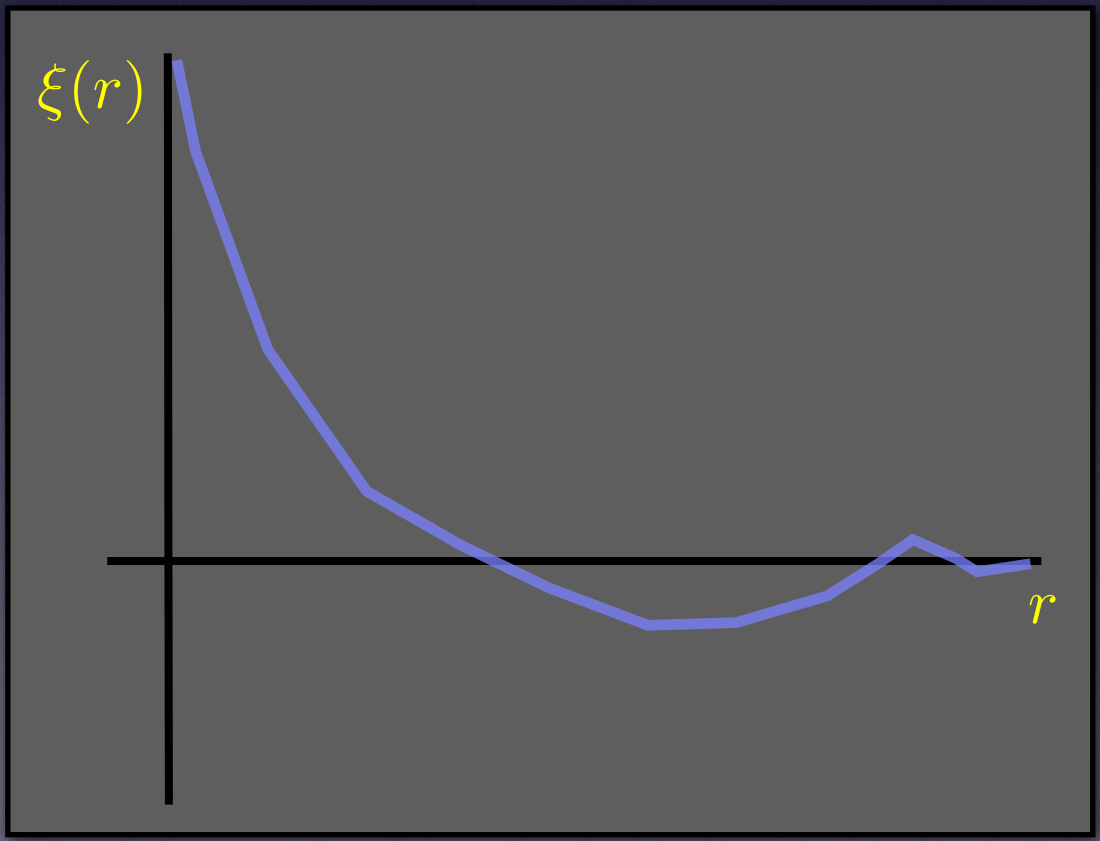
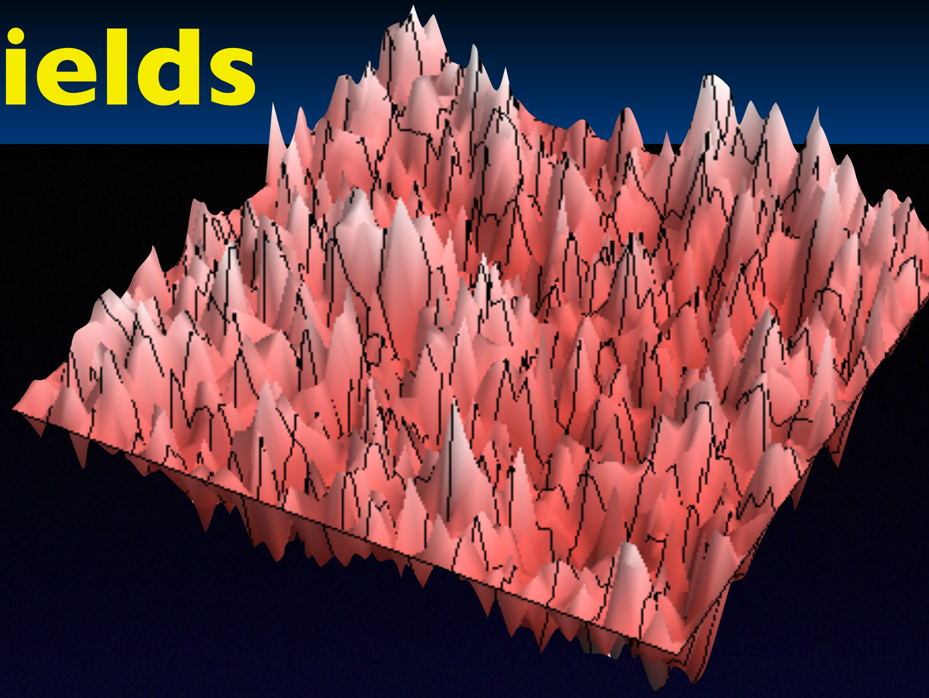
Clustered distribution





Gaussian Random Fields



How many moments do we need to completely specify the matter distribution?

In principle infinitely many......

However, initial density distribution is believed to be a Gaussian random field...

A random field $\delta(\vec{x})$ is said to be Gaussian if the distribution of the field values at an *arbitrary* set of N points is an N-variate Gaussian:

$$\mathcal{P}(\delta_1, \delta_2, ..., \delta_N) = \frac{\exp(-Q)}{[(2\pi)^N \det(\mathcal{C})]^{1/2}}$$

$$Q \equiv \frac{1}{2} \sum_{i,j} \delta_i \, (\mathcal{C}^{-1})_{ij} \delta_j$$

$$\mathcal{C}_{ij} = \langle \delta_i \delta_j \rangle = \xi(r_{12})$$

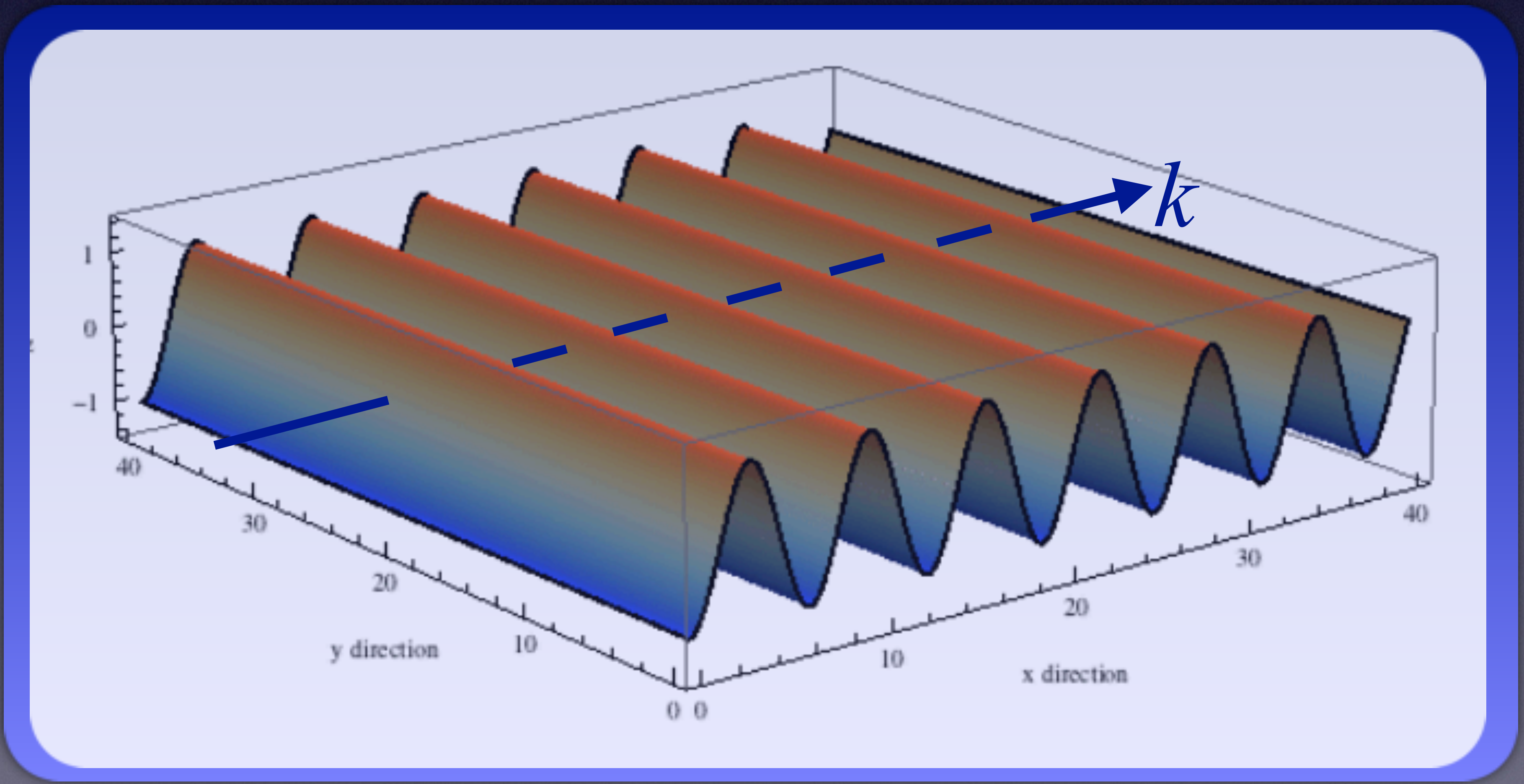
A Gaussian random field is completely specified by its second moment, the two-point correlation function $\xi(r)$!!!!

The Power Spectrum

Often it is very useful to describe the matter field in Fourier space:

$$\delta(\vec{x}) = \sum_k \delta_{\vec{k}}\, e^{+i\vec{k}\cdot\vec{x}} \qquad \delta_{\vec{k}} = \frac{1}{V}\int \delta(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}\, \mathrm{d}^3\vec{x}$$

Note: the perturbed density field can be written as a sum of plane waves of different wave numbers k (called `modes')



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The Fourier transform (FT) of the two-point correlation function is called the power spectrum and is given by

$$\begin{aligned} P(\vec{k}) &\equiv V\langle|\delta_{\vec{k}}|^2\rangle \\ &= \int \xi(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}\, \mathrm{d}^3\vec{x} \\ &= 4\pi \int \xi(r)\, \frac{\sin kr}{kr}\, r^2\, \mathrm{d}r \end{aligned}$$

Note: P(k) has units of volume!

A Gaussian random field is completely specified by either the two-point correlation function $\xi(r)$, or, equivalently, the power spectrum $P(k)$

Structure Formation in the Linear Regime

As long as $|\delta| \ll 1$, we can use linear perturbation theory to describe the evolution of the density field:

$$\frac{\mathrm{d}^2\delta_{\vec{k}}}{\mathrm{d}t^2} + 2\frac{\dot{a}}{a}\frac{\mathrm{d}\delta_{\vec{k}}}{\mathrm{d}t} = \left[4\pi G\bar{\rho} - \frac{k^2 c_{\rm s}^2}{a^2}\right]\delta_{\vec{k}} - \frac{2}{3}\frac{\bar{T}}{a^2}k^2\,S_{\vec{k}}$$

Hubble drag — gravity — pressure

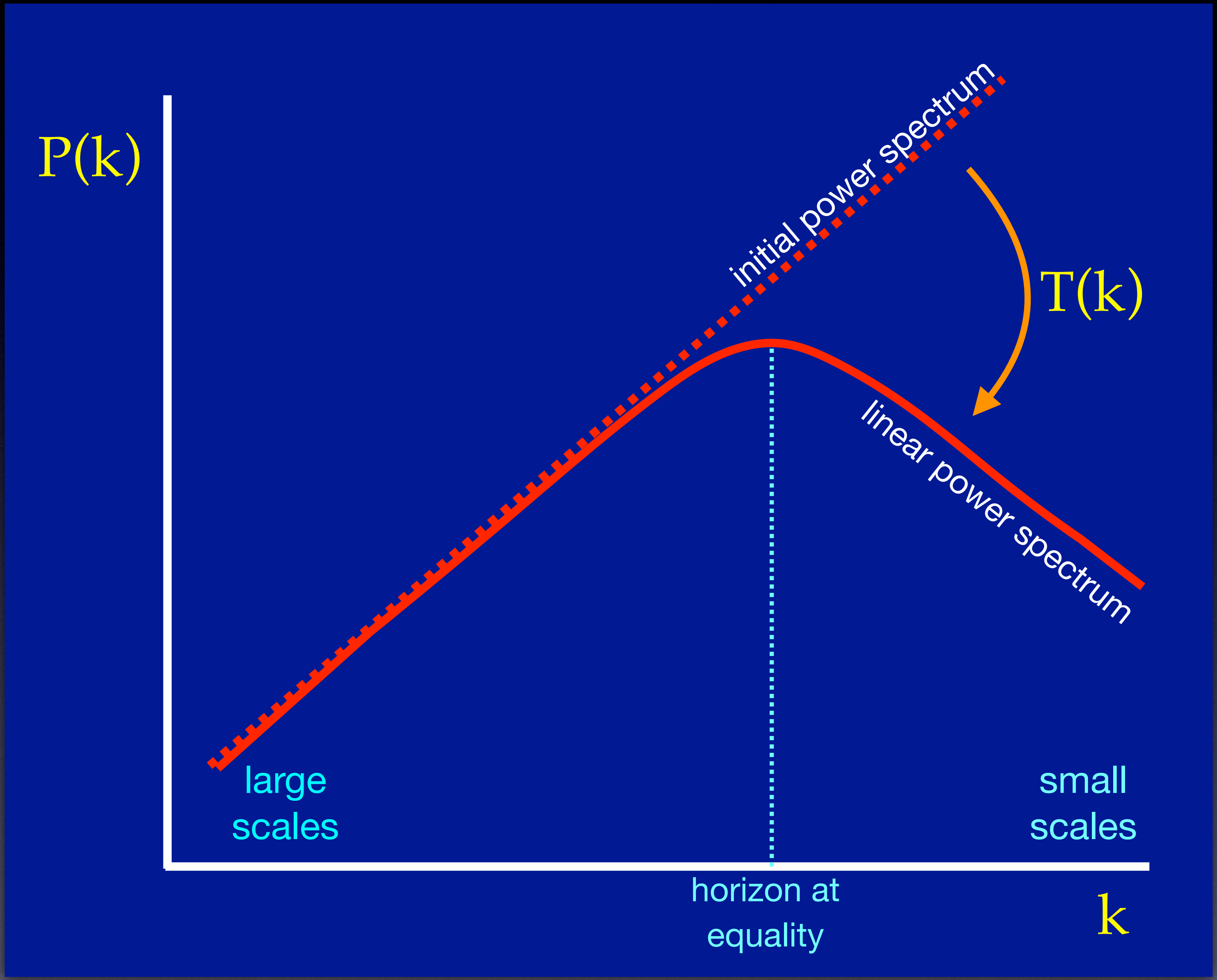
Note that each mode, $\delta_{\vec{k}}(t)$, evolves independently (sign of linearity)!!

In the linear regime, the power spectrum evolves as $$P(k,t) = P_{\rm i}(k)\,T^2(k)\,D^2(t)$$

- $P_{\rm i}(k)$ is the initial power spectrum (i.e., shortly after creation of perturbations)
- $T(k)$ is called the transfer function (depends on nature of dark matter)
- $D(t)$ is the linear growth rate (cosmology dependent)

The Transfer Function

The transfer function describes what happens to the perturbations prior to decoupling…



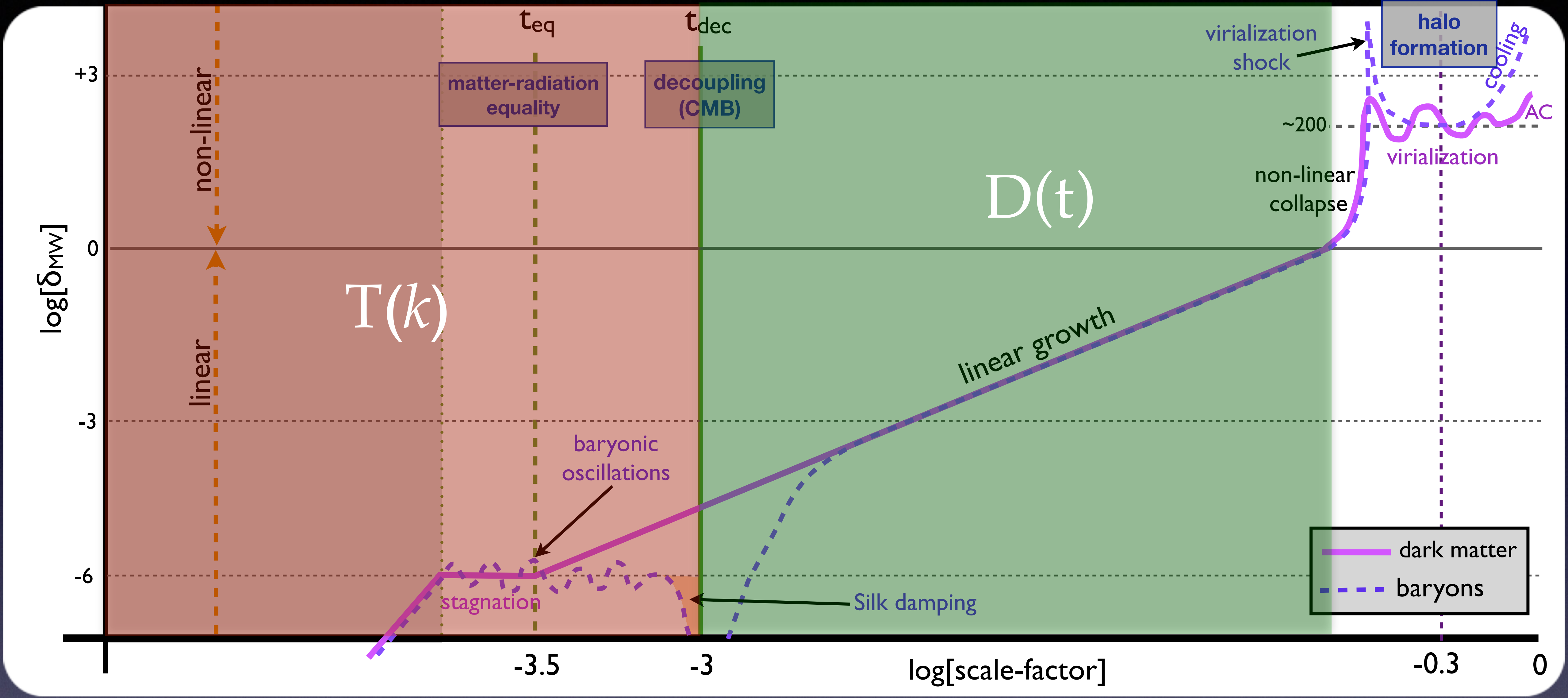
Main effect: stagnation (Meszaros effect) = retarded growth due to Hubble drag

also free streaming (dark matter)

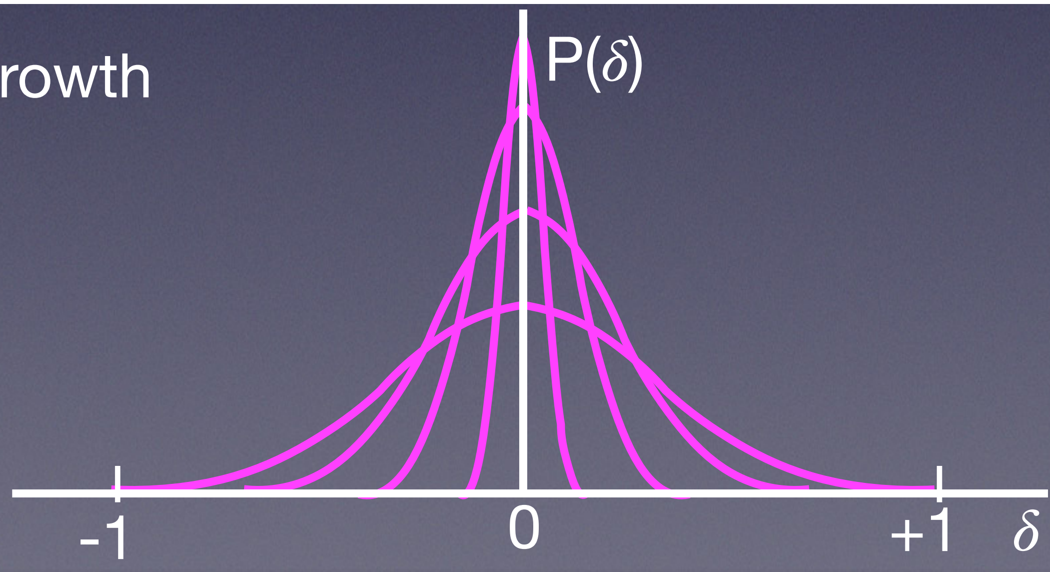
acoustic oscillations (baryons only)

for details, see Mo, vdB & White 2010

Structure Formation in a Nutshell



During linear growth



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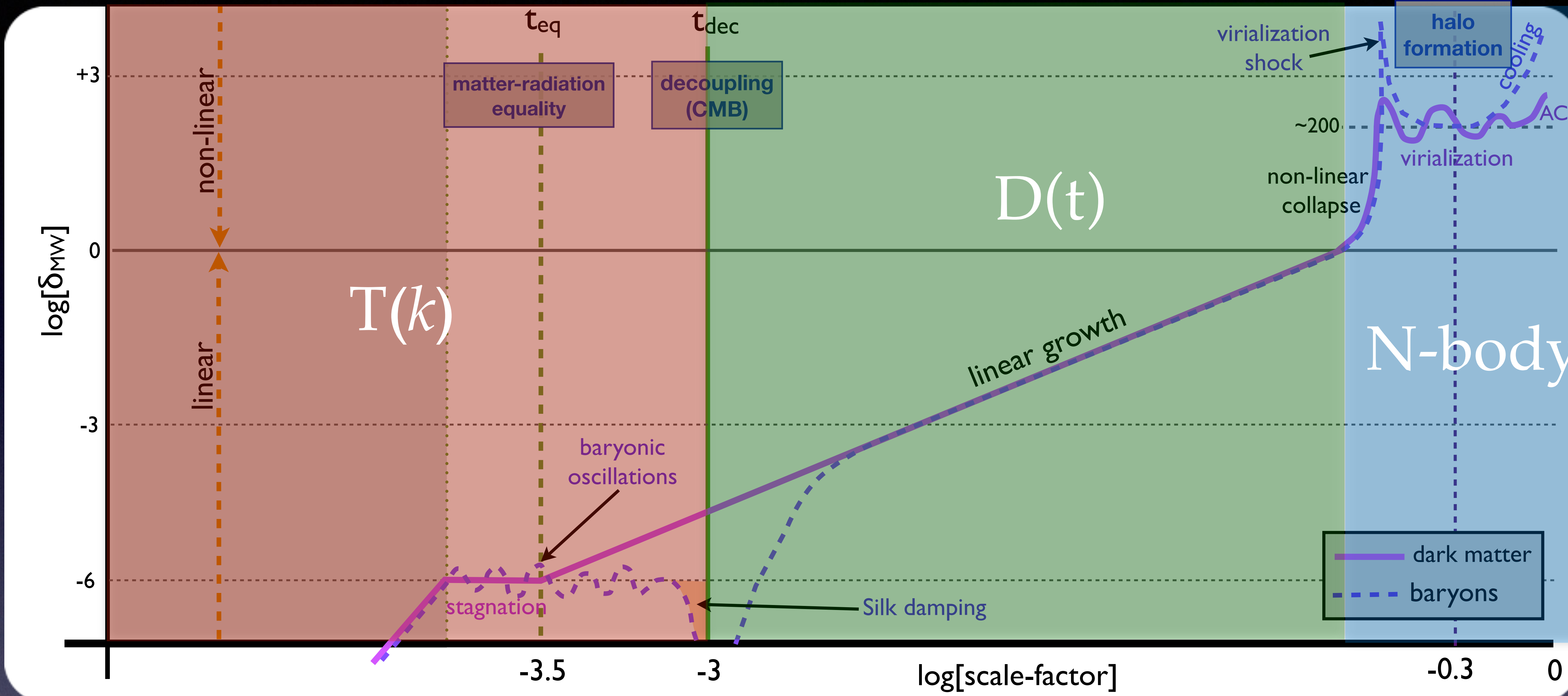
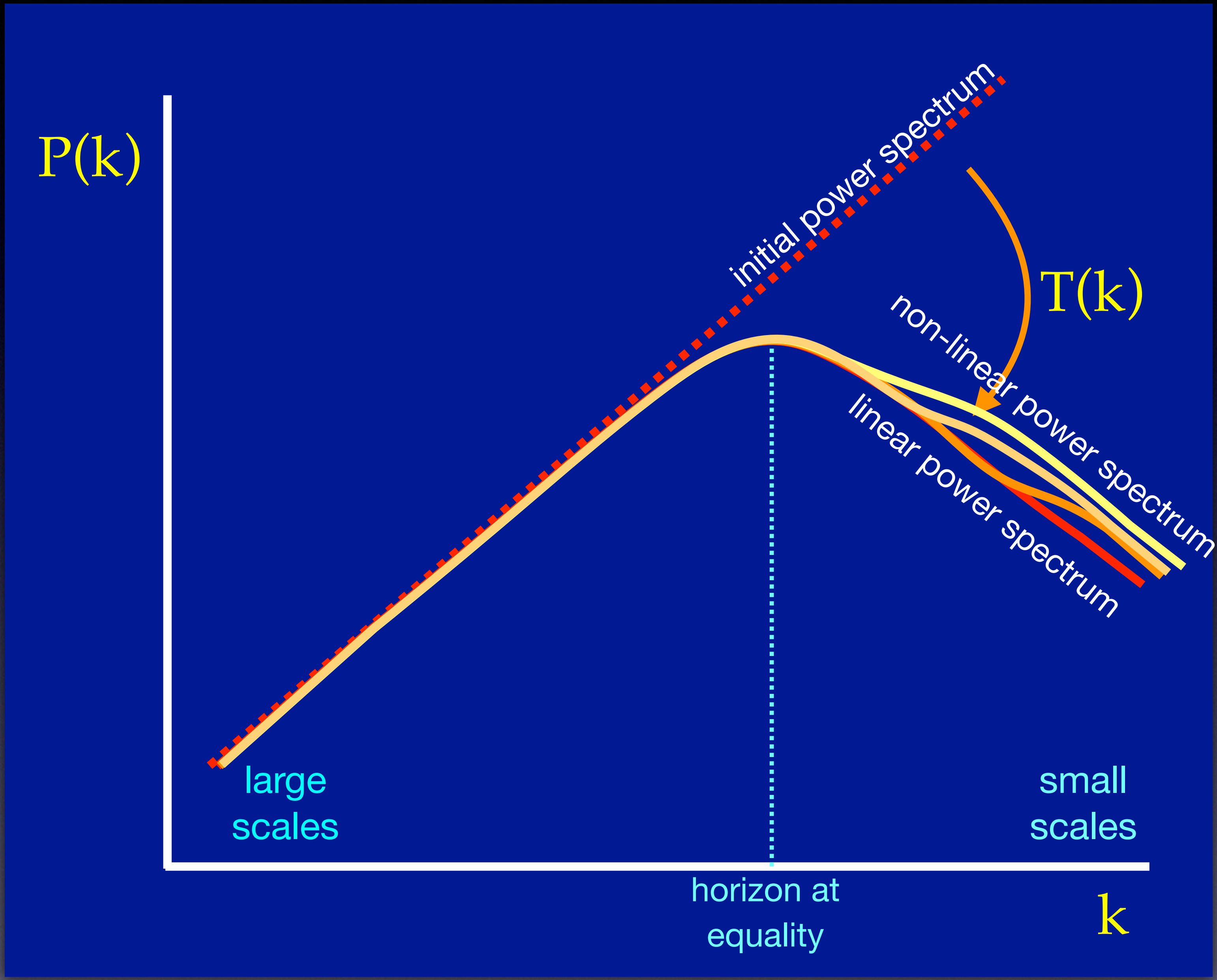


Once density field becomes non-linear:

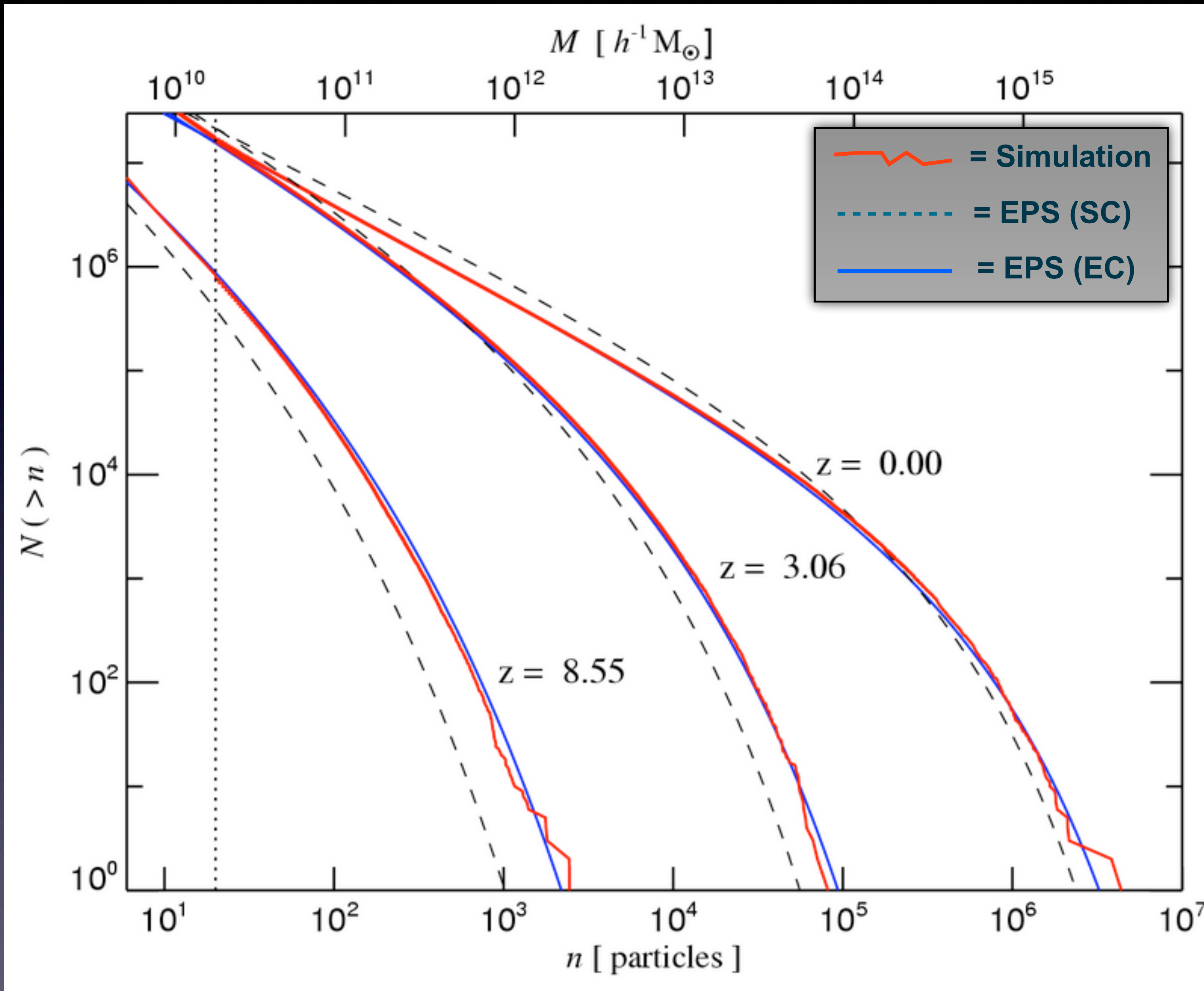
- linear perturbation theory no longer valid (use simulations instead)
- mode-coupling
- non-Gaussianities develop (higher-order correlation functions needed)
- non-linear collapse —> halo formation

The Non-Linear Matter Power Spectrum



Since the non-linear matter power spectrum describes structure growth in the non-linear regime, we typically need to resort to N-body simulations for its computation…

Halo Mass Function



Source: Millenium Simulation; Springel V., 2004 (MPA research highlight)

The halo mass function, n(M), expresses the number of halos of mass M per comoving volume

The halo mass function can be obtained using N-body simulations, or from the Press-Schechter formalism

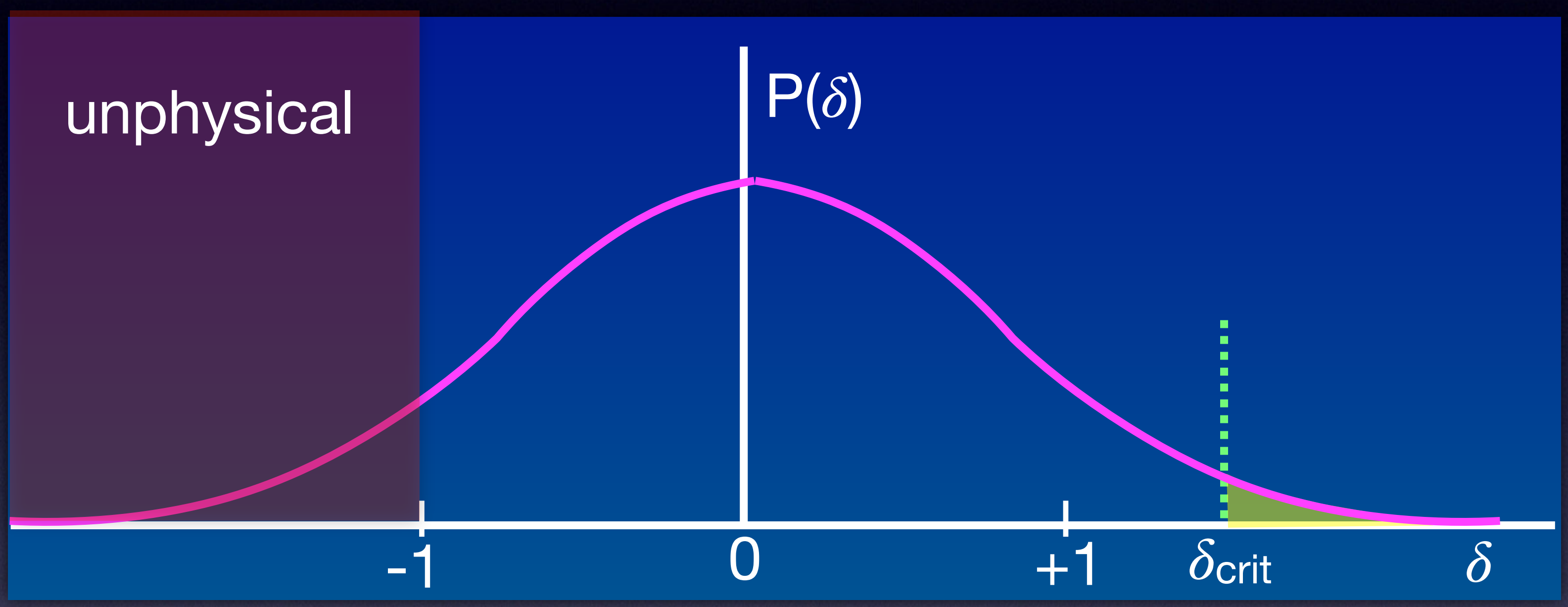
SC = spherical collapse
EC = ellipsoidal collapse

Press & Schechter 1974
Bond et al. 1991
Sheth, Mo & Tormen 2000

Press-Schechter (PS) formalism: compute n(M) from statistics of Gaussian density field.

Extended Press-Schechter (EPS): uses excursion set formalism to compute n(M) and $P(M_1,z_1|M_2,z_2)$

Halo Mass Function



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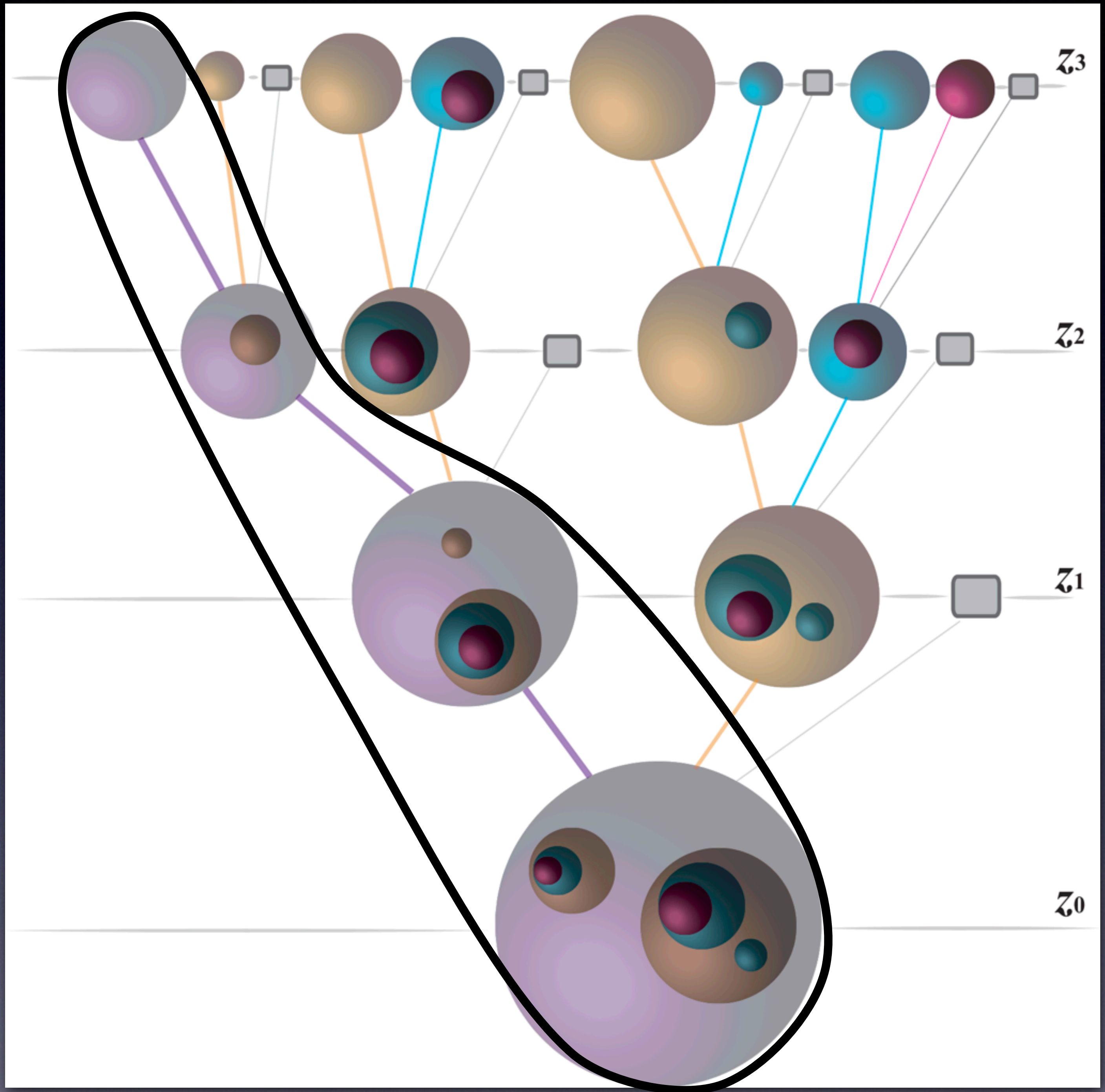
The Anatomy of a Halo Merger Tree

Hierarchical formation gives rise to hierarchy of substructure



EPS

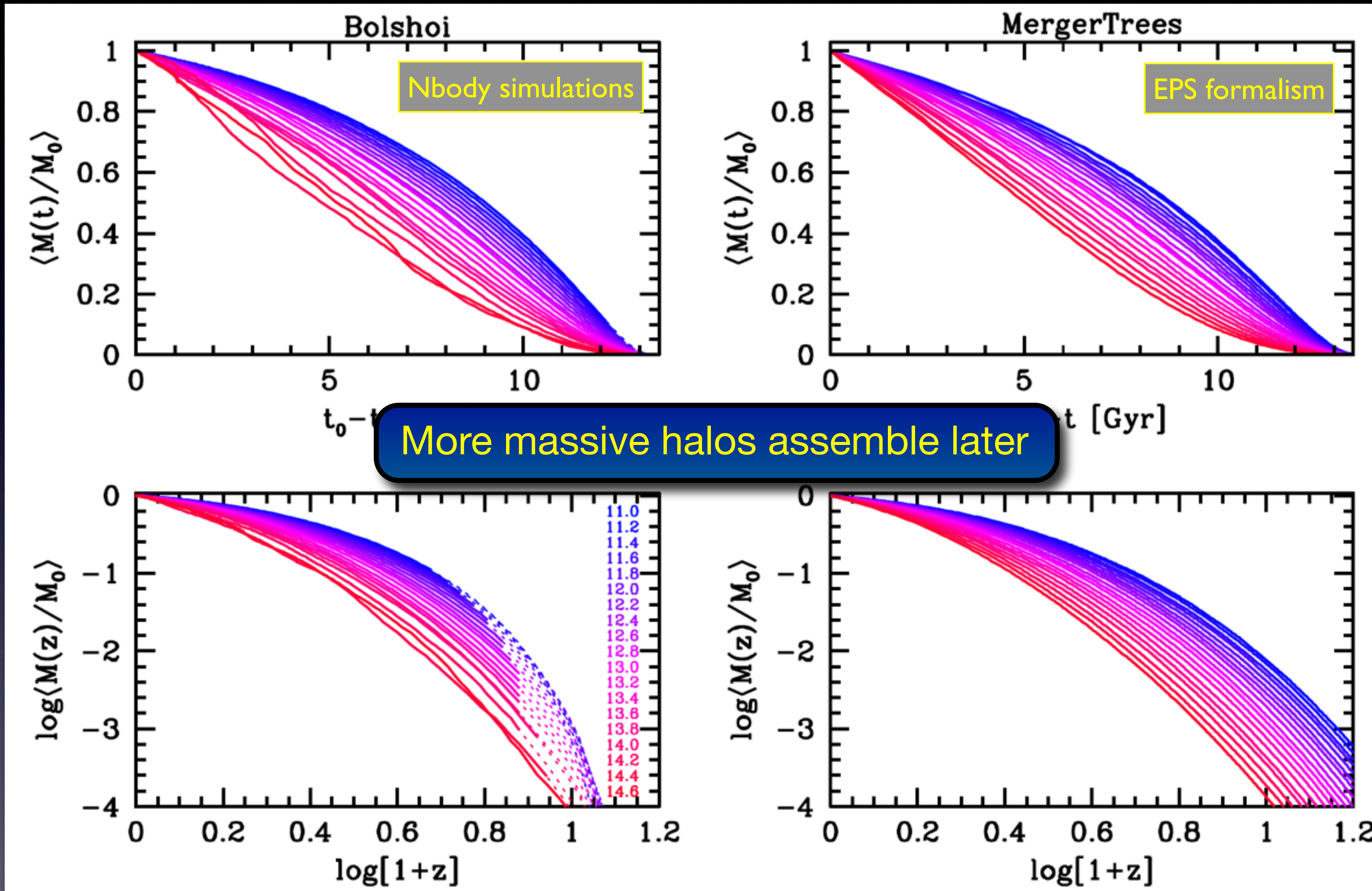
$P(M_3,z_3|M_2,z_2)$

$P(M_2,z_2|M_1,z_1)$

$P(M_1,z_1|M_0,z_0)$

Main Progenitor History = Mass Assembly History = Mass Accretion History = MAH

Mass Assembly Histories



More massive halos assemble later

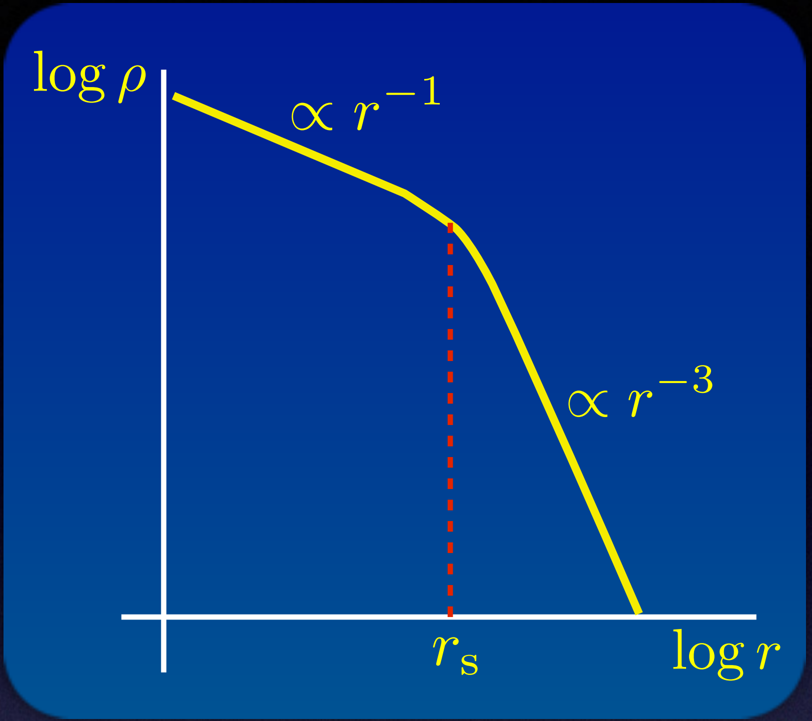
Source: van den Bosch 2014

EPS merger trees in excellent agreement with N-body simulations

The NFW Profile

The NFW profile is given by

$$\rho(r) = \rho_{\rm crit} \frac{\delta_{\rm char}}{(r/r_{\rm s})\,(1+r/r_{\rm s})^2}$$



It is completely characterized by the mass $M_{\rm vir}$ and the concentration parameter $c = r_{\rm vir}/r_{\rm s}$, which is related to the characteristic overdensity according to:

$$\delta_{\rm char} = \frac{\Delta_{\rm vir}\,\Omega_{\rm m}}{3} \frac{c^3}{f(c)}$$

where $f(x) = \ln(1+x) - x/(1+x)$

Navarro, Frenk & White 1996
Navarro, Frenk & White 1997

The circular velocity of an NFW profile is

$$V_{\rm c}(r) = V_{\rm vir}\sqrt{\frac{f(cx)}{x\,f(c)}}$$

which has a maximum $V_{\rm max} \simeq 0.465 V_{\rm vir}\sqrt{c/f(c)}$ at $r_{\rm max} \simeq 2.163 r_{\rm s}$

$$V_{\rm max} = V_{\rm max}(M_{\rm vir}, c)$$

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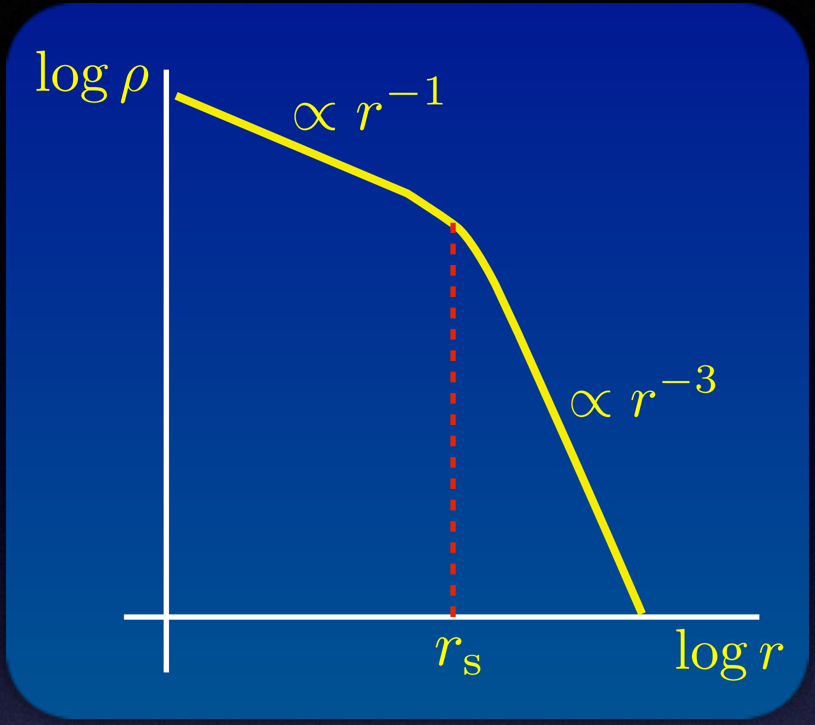

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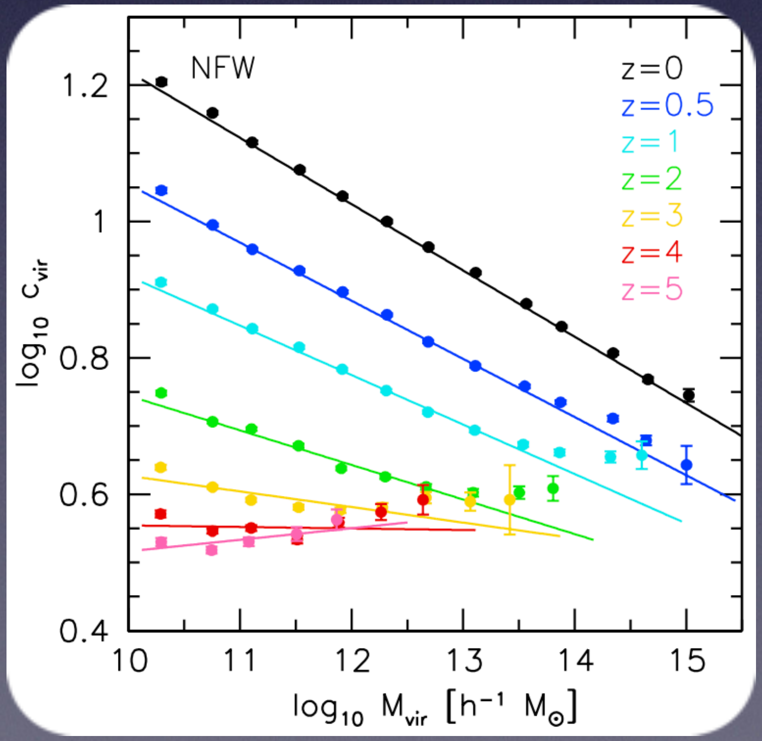


Source: Dutton & Maccio 2014

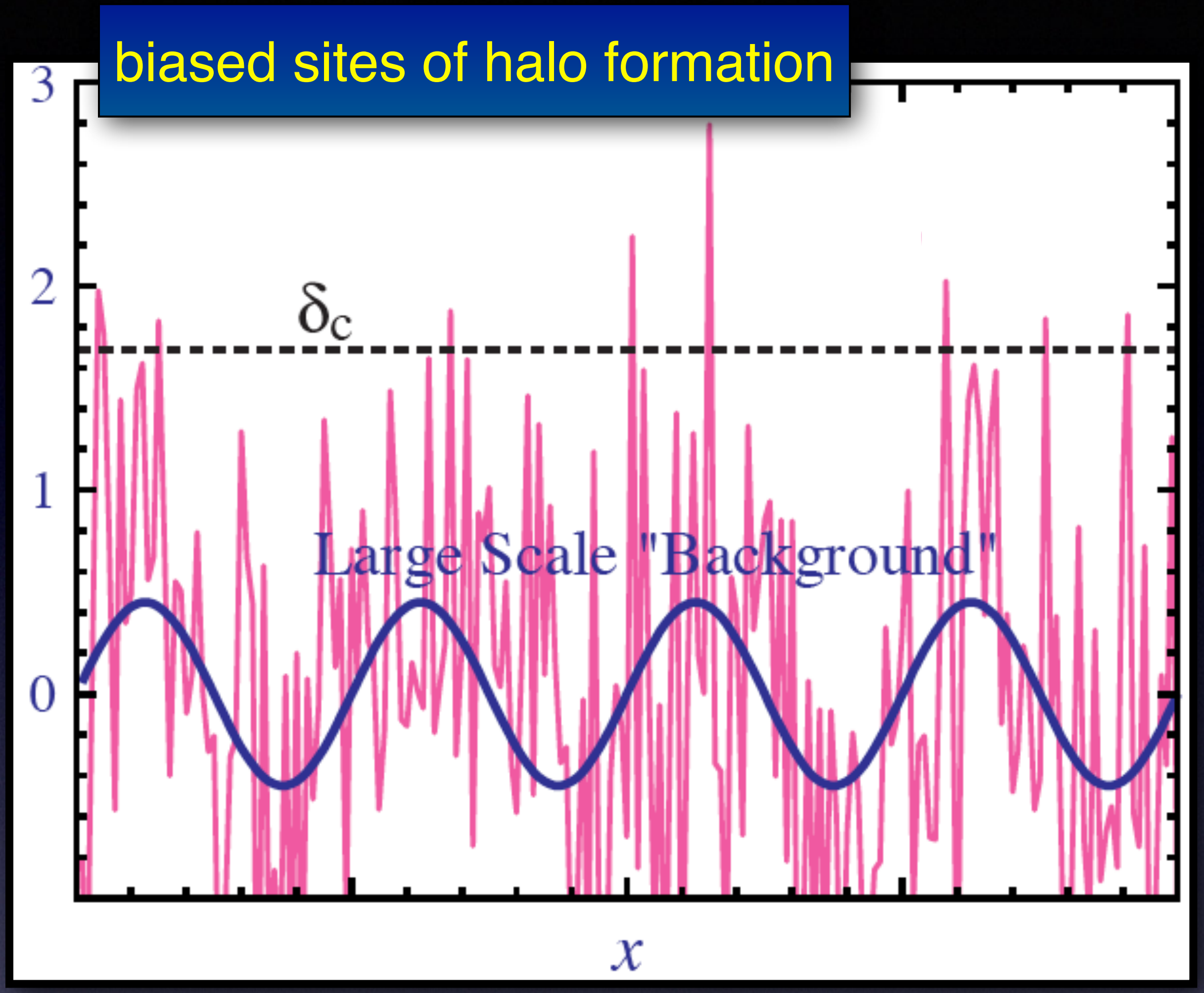
Typically, less massive halos are more concentrated

Navarro, Frenk & White 1997; Bullock et al. 2001
Eke et al . 2001; Maccio et al. 2008

This is a consequence of less massive halos forming earlier, when Universe is denser

Navarro, Frenk & White 1997; Wechsler et al. 2002;
Zhao et al. 2009; Correa et al. 2015

Halo Bias



- Dark matter halos form from over-densities with $\delta > \delta_{crit} \simeq 1.686$
- Halos are a biased tracer of mass distribution, moduated by large-scale modes
- Snowfall occurs at high altitudes (and in Michigan)
- Snow is a biased tracer of land-mass, modulated by mountain ranges

The Mass Dependence of Halo Bias

Halo bias function, $b(M)$, expresses how halos of mass M are clustered compared to dark matter particles:

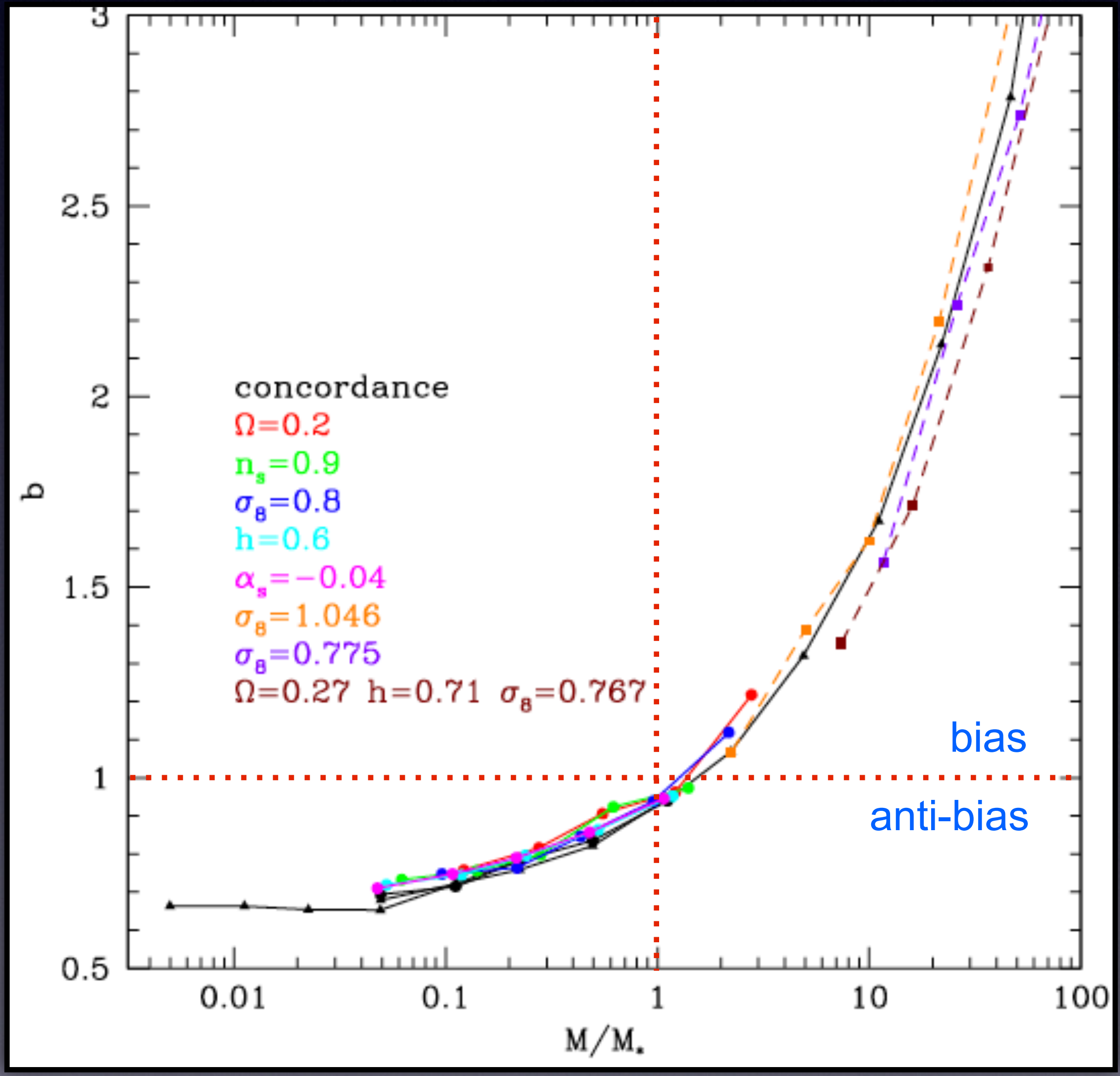
$$\xi_{\rm hh}(r|M) = \langle \delta_{\rm h}(\vec{x})\delta_{\rm h}(\vec{x}+\vec{r})\rangle = b^2(M)\,\langle \delta_{\rm m}(\vec{x})\delta_{\rm m}(\vec{x}+\vec{r})\rangle = b^2(M)\,\xi_{\rm mm}(r\,|\,M)$$

we see that $b(M) = \langle \xi_{\rm hh}/\xi_{\rm mm}\rangle^{1/2}$ where $\xi_{\rm mm}(r)$ is the two-point correlation function of the dark matter particles, and $\langle\cdot\rangle$ indicates an averaging over large (linear) radii)

Both simulations and EPS theory show that $b(M)$ increases with increasing halo mass:

Cole & Kaiser 1989; Mo & White 1996, Tinker et al. 2010

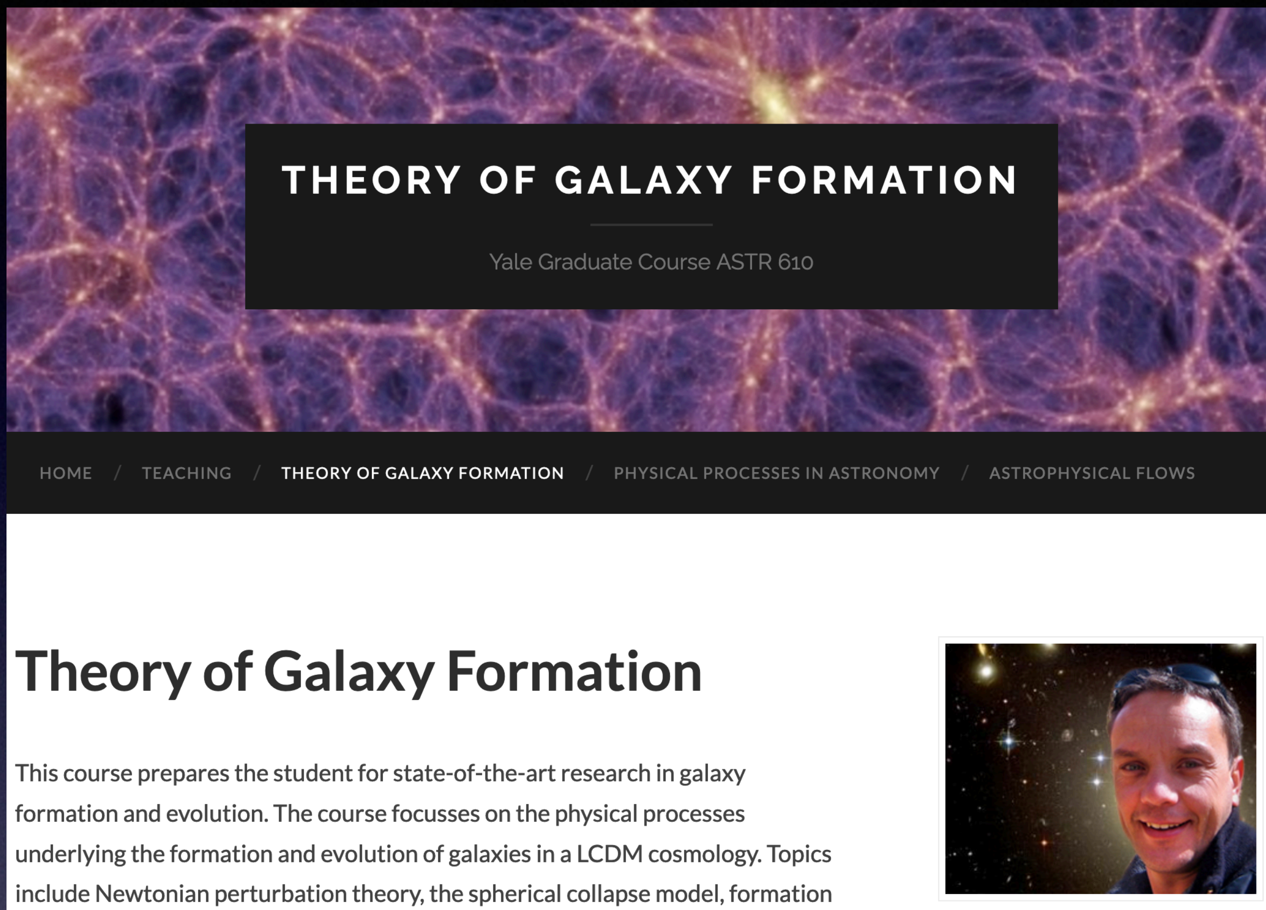
More massive haloes, are more strongly clustered.

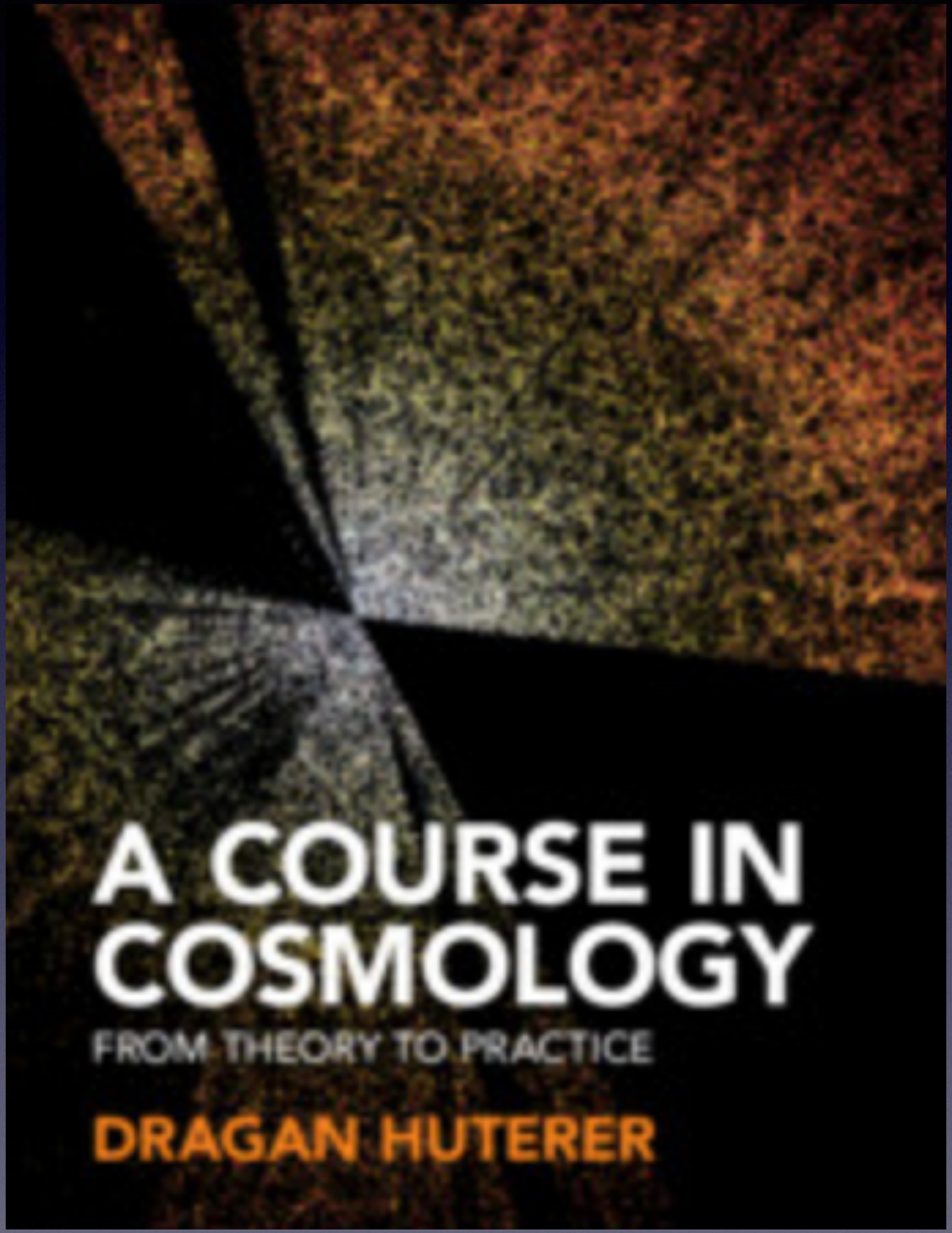


Source: Seljak & Warren, 2004, 355, 129

For a detailed review, see Desjacques, Jeong & Schmidt 2018

For More Details...





See https://campuspress.yale.edu/astro610/ for video lectures and detailed lecture notes of my ASTR 610 graduate course on The Theory of Galaxy Formation

The Galaxy-Halo Connection

The Galaxy - Halo Connection

Key Premises

- Galaxies form and reside in halos (including subhalos)
- There exist some halo property(ies) that are tightly correlated with the properties of the galaxies they host

The Galaxy - Halo Connection

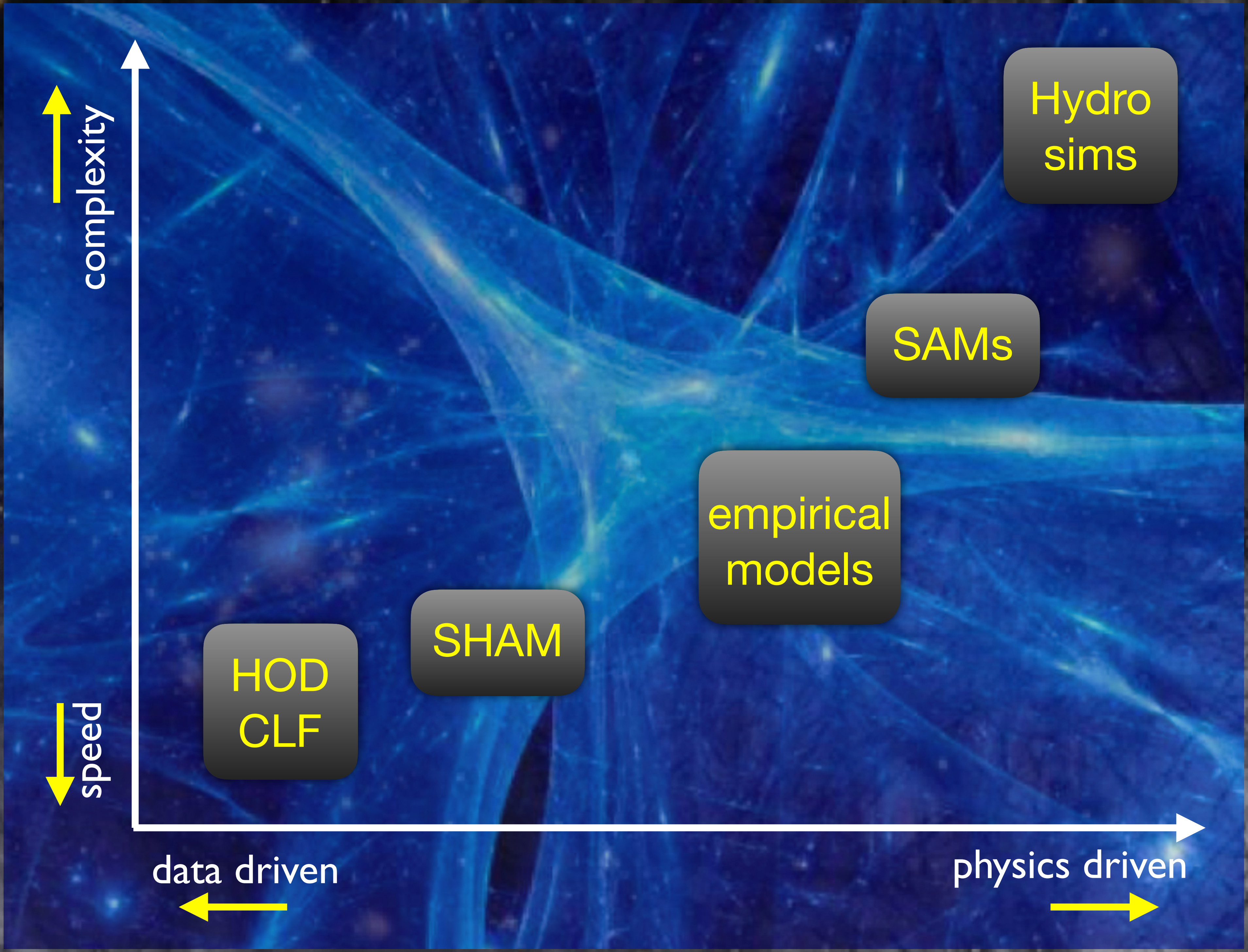
GOAL: constrain the galaxy-dark matter connection $P(\mathcal{G}|\mathcal{H})$

galaxy properties $\mathscr{G} = (G_1, G_2, \ldots, G_K)$
halo properties $\mathscr{H} = (H_1, H_2, \ldots, H_N)$

Why do we care?

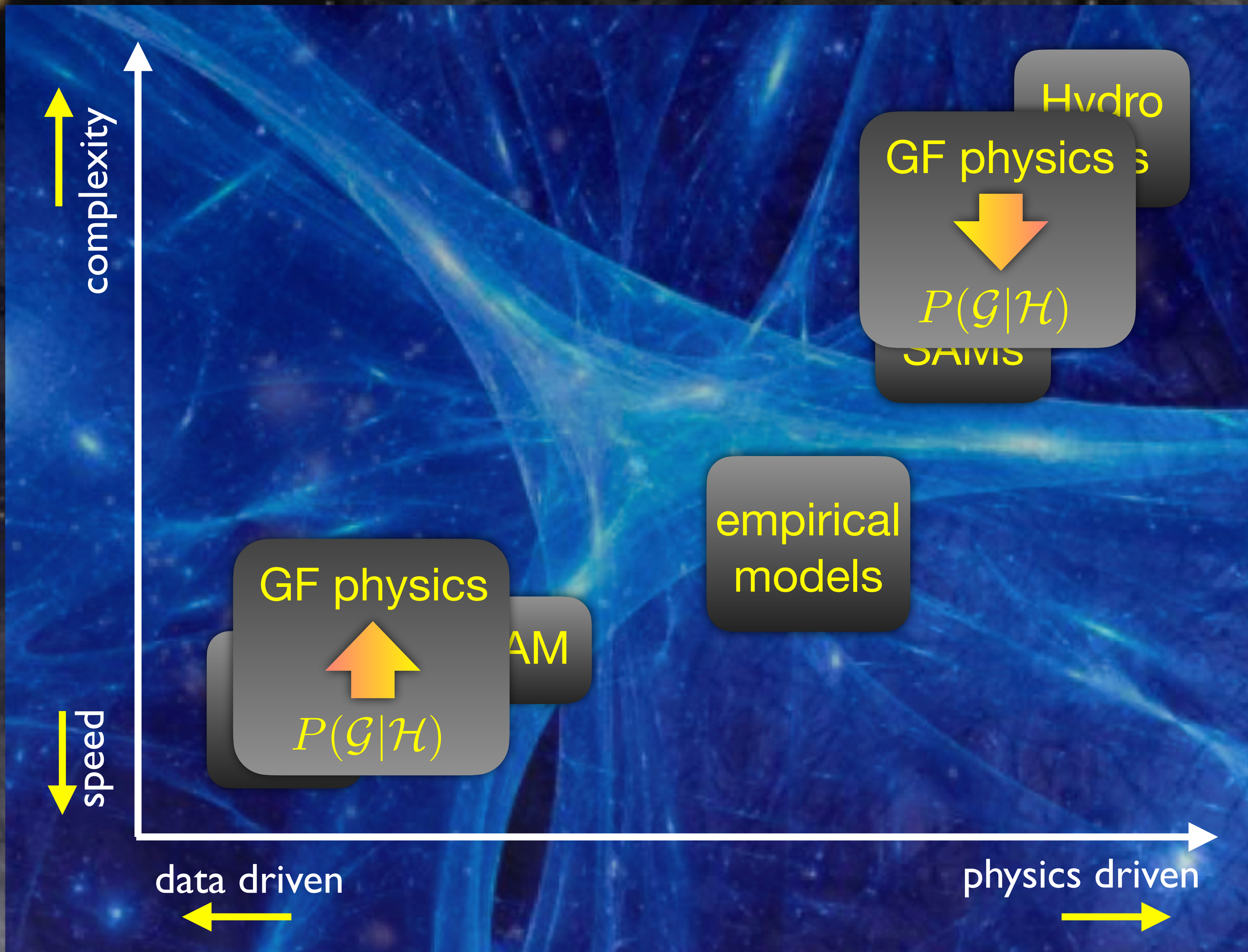
- Galaxy-Halo connection characterizes the effective outcome of galaxy formation
- Galaxy-Halo connection links what we can see (galaxies) to what governs the dynamics of the Universe (dark matter)
- Galaxy-Halo connection is required whenever one uses galaxies to constrain cosmology

Halo Occupation Modeling



inspired by Wechsler & Tinker 2018 and a KITP talk by Sownak Bosek

Halo Occupation Modeling



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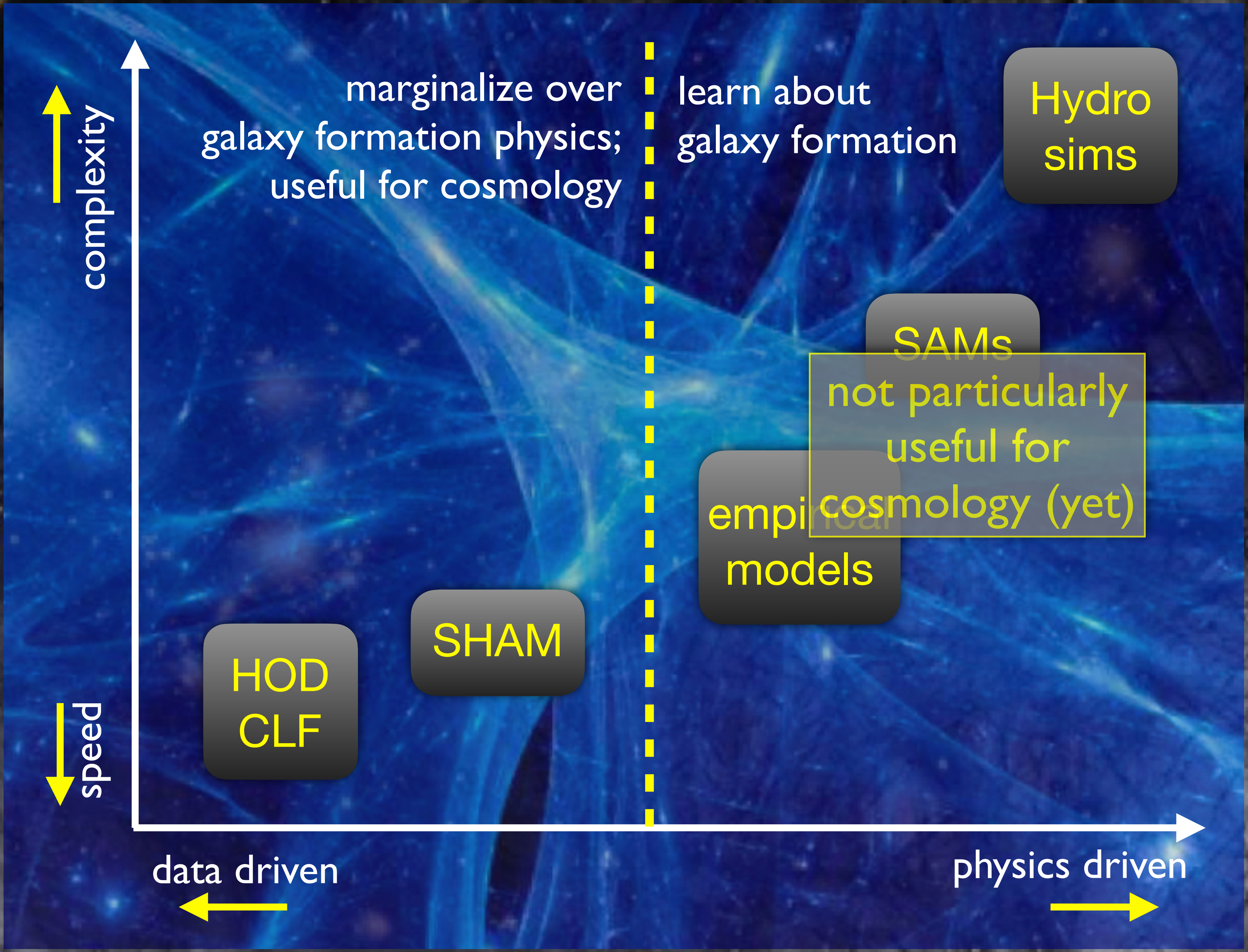
Halo Occupation Modeling



Halo Occupation Modeling

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galaxy properties $\mathcal{G} = (G_1, G_2, \ldots, G_K)$

halo properties $\mathcal{H} = (H_1, H_2, \ldots, H_N)$

Galaxy Surveys

clustering LSS

gravitational lensing

satellite kinematics

$P(\mathcal{G} \,|\, \mathcal{H})$

Cosmology

Galaxy Formation

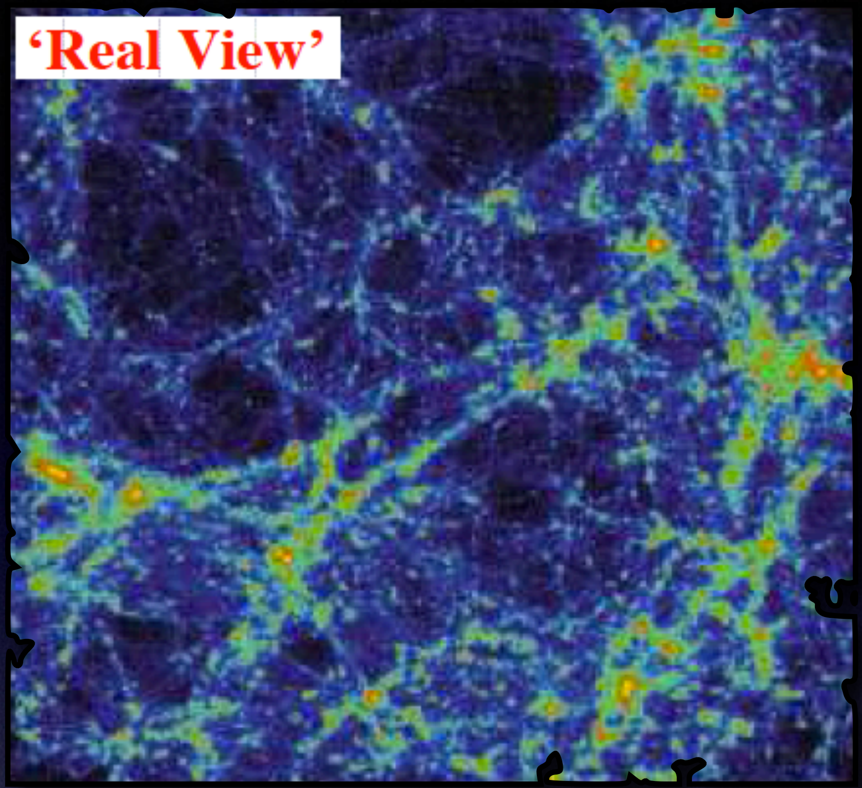
Galaxy Properties: luminosity, stellar ... gy, metallicity,...

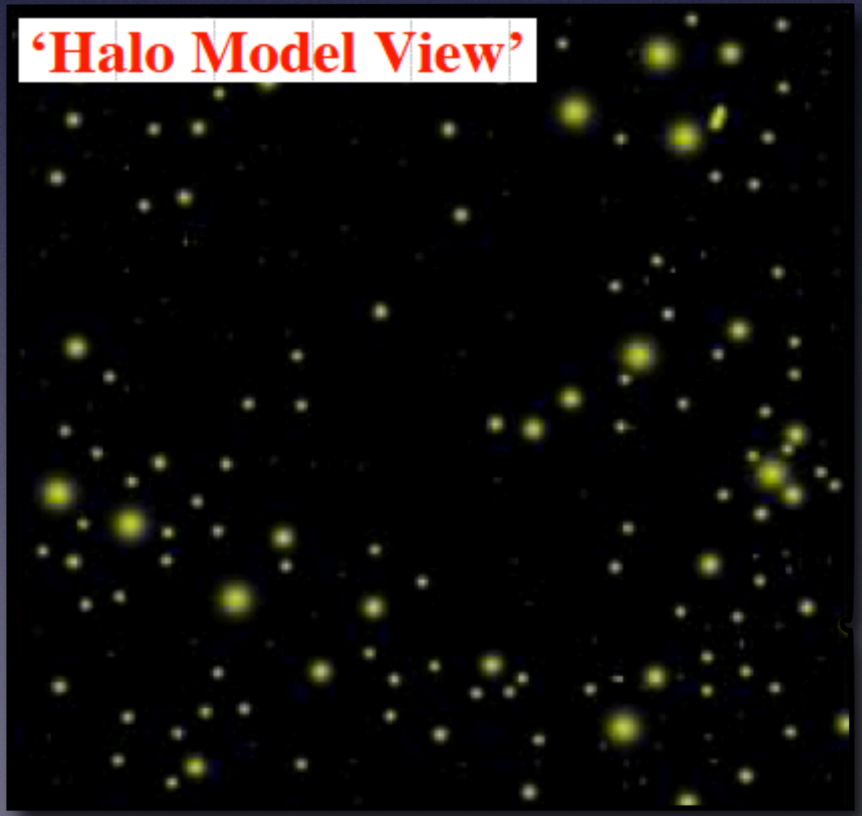
Halo Properties: mass, max. circul ... formation time,...

$$P(\mathcal{G} \,|\, \mathcal{H}) \rightarrow P(L \,|\, M)$$

The Halo Model

The Halo Model





Halo Model: an analytical model that describes dark matter density distribution in terms of halo building blocks.

Ansatz: *all* dark matter is partitioned over haloes.

As highlighted in the introduction; we know how to compute

- number density of halos $n(M,z)$
- density profiles of halos $\rho(r\,|\,M,z)$
- clustering of halos $b(M,z)$
- linear power spectrum $P^{\rm LIN}(k,z)$

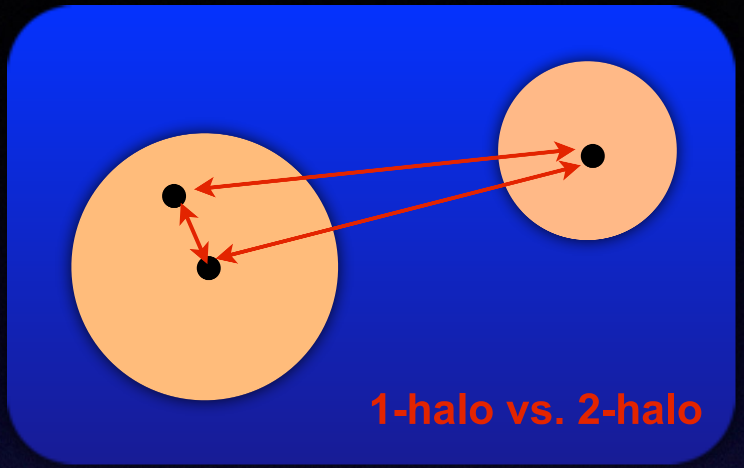
These can be combined to compute the non-linear power spectrum, $P^{\rm NL}(k,z)$, without having to resort to N-body simulations…

Neyman & Scot 1952; Ma & Fry 2000; Seljak 2000; Scoccimarro et al. 2001

The Halo Model

Recall: $P(k)$ is the Fourier Transform of $\xi(r)$

$\xi(r)$ describes number of pairs of particles in excess of that of a random distribution



the two particles of a pair either reside in the same halo (1-halo term) or in two separate halos (2-halo term)

Throughout we assume that all dark matter haloes are spherical, and have a density distribution that only depends on halo mass:

$$\rho(r|M) = M\, u(r|M)$$

Here $u(r|M)$ is the normalized density profile:

$$\int \mathrm{d}^3\vec{x}\, u(\vec{x}|M) = 1$$

Its Fourier Transform is

$$\tilde{u}(\vec{k}|M) = \int u(\vec{x}|M) e^{-i\vec{k}\cdot\vec{x}}\, \mathrm{d}^3\vec{x} = 4\pi \int_0^\infty u(r|M) \frac{\sin kr}{kr}\, r^2\, \mathrm{d}r$$

The Halo Model

After some algebra

$$P(k) = P^{1\mathrm{h}}(k) + P^{2\mathrm{h}}(k)$$

$$P^{1\mathrm{h}}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M\, M^2\, n(M)\, |\tilde{u}(k|M)|^2$$

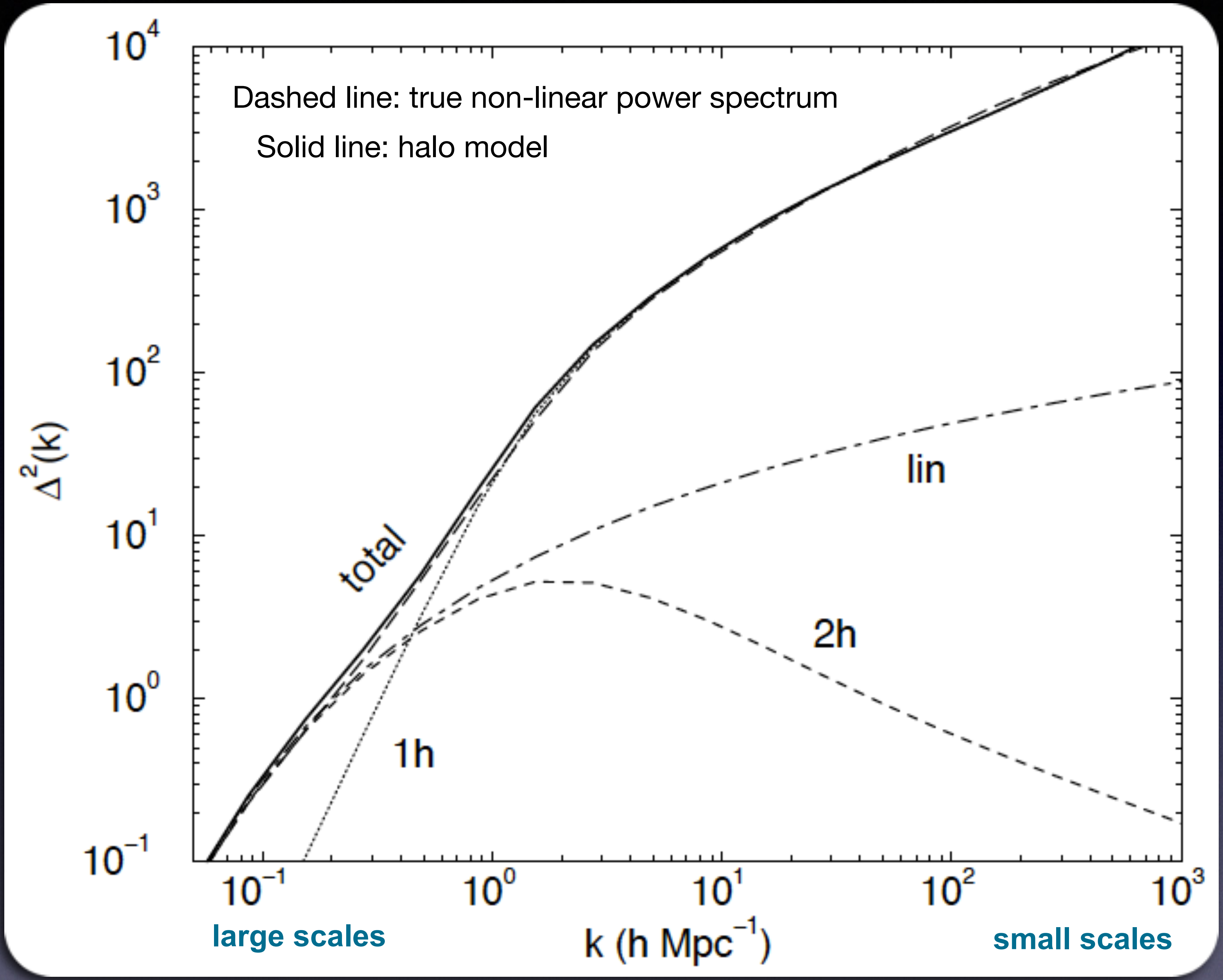
$$P^{2\mathrm{h}}(k) = P^{\mathrm{lin}}(k) \left[\frac{1}{\bar{\rho}} \int \mathrm{d}M\, M\, b(M)\, n(M)\, \tilde{u}(k|M)\right]^2$$

The (non-linear) two-point correlation function of the matter field, $\xi_{\mathrm{mm}}(r)$, is obtained by Fourier Transforming this (non-linear) power spectrum $P(k)$

For a detailed derivation, see

- Extra Lecture Notes
- Lecture 13 of my ASTR 610 course [https://campuspress.yale.edu/astro610/]
- van den Bosch et al. 2013, MNRAS, 430, 725
- Cooray & Sheth, 2002, Phys. Rep. 372, 1 [Halo Model review paper]

The Halo Model in Fourier Space



Source: Cooray & Sheth 2002

$$\Delta^2(k) = \frac{1}{2\pi^2} k^3 P(k)$$

Dimensionless power spectrum

Halo Occupation Modelling

The Galaxy Power Spectrum

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M\, M^2\, n(M)\, |\tilde{u}(k|M)|^2$$

$$P^{2h}(k) = P^{\rm lin}(k) \left[\frac{1}{\bar{\rho}} \int \mathrm{d}M\, M\, b(M)\, n(M)\, \tilde{u}(k|M)\right]^2$$

The above equations describe the halo model predictions for the matter power spectrum

The same formalism can also be used to compute the galaxy power spectrum:

simply replace:

$$\frac{M}{\bar{\rho}} \rightarrow \frac{\langle N \rangle_M}{\bar{n}_{\rm g}}$$

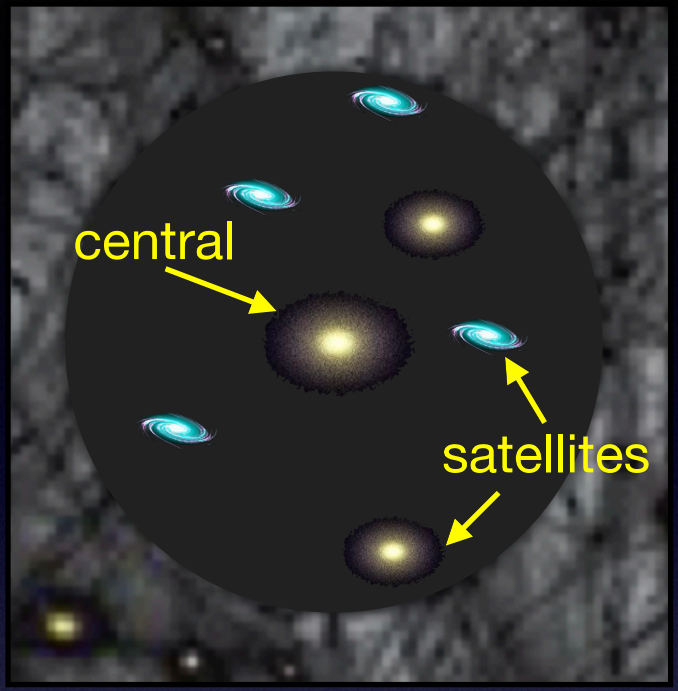
$$\frac{M^2}{\bar{\rho}^2} \rightarrow \frac{\langle N(N-1) \rangle_M}{\bar{n}_{\rm g}^2}$$

$$\tilde{u}(k|M) \rightarrow \tilde{u}_{\rm g}(k|M)$$

$\langle N \rangle_M$ describes average number of galaxies that reside in a halo of mass M
$\bar{n}_{\rm g}$ is the average number density of those galaxies.
$u_{\rm g}(r|M)$ is the normalized, radial distribution of galaxies in haloes of mass M.

Halo Occupation Statistics

It is important to treat central and satellite galaxies separately.



Centrals: those galaxies that reside at the center of their dark matter (host) halo

Satellites: those galaxies that reside at center of a sub-halo, and are orbitting inside a larger host halo.

Central Galaxies

$$\langle N_c \rangle_M = \sum_{N_c=0}^{1} N_c\, P(N_c|M) = P(N_c|M)$$

$$u_c(r|M) = \delta^D(r)$$

Satellite Galaxies

$$\langle N_s \rangle_M = \sum_{N_s=0}^{\infty} N_s\, P(N_s|M)$$

$$\langle N_s^2 \rangle_M = \sum_{N_s=0}^{\infty} N_s^2\, P(N_s|M)$$

$$u_s(r|M) = \mathrm{TBD}$$

Halo Occupation Statistics

Central Galaxies

$$\langle N_{\rm c}\rangle_M = \sum_{N_{\rm c}=0}^{1} N_{\rm c}\, P(N_{\rm c}\,|\,M) = P(N_{\rm c}\,|\,M)$$

$$u_{\rm c}(r|M) = \delta^{\rm D}(r)$$

Satellite Galaxies

$$\langle N_{\rm s}\rangle_M = \sum_{N_{\rm s}=0}^{\infty} N_{\rm s}\, P(N_{\rm s}|M)$$

$$\langle N_{\rm s}^2\rangle_M = \sum_{N_{\rm s}=0}^{\infty} N_{\rm s}^2\, P(N_{\rm s}|M)$$

$$u_{\rm s}(r|M) = {\rm TBD}$$

Calculating galaxy-galaxy correlation functions requires following halo occupation statistic ingredients:

Halo occupation distribution for centrals	$P(N_{\rm c}\|M)$
Halo occupation distribution for satellites	$P(N_{\rm s}\|M)$
Radial number density profile of satellites	$u_{\rm s}(r\|M)$

In principle, one also requires $P(N_{\rm c}, N_{\rm s}|M)$, but it is common to assume that occupation statistics of centrals and satellites are independent, i.e., $P(N_{\rm c}, N_{\rm s}|M) = P(N_{\rm c}|M) \times P(N_{\rm s}|M)$

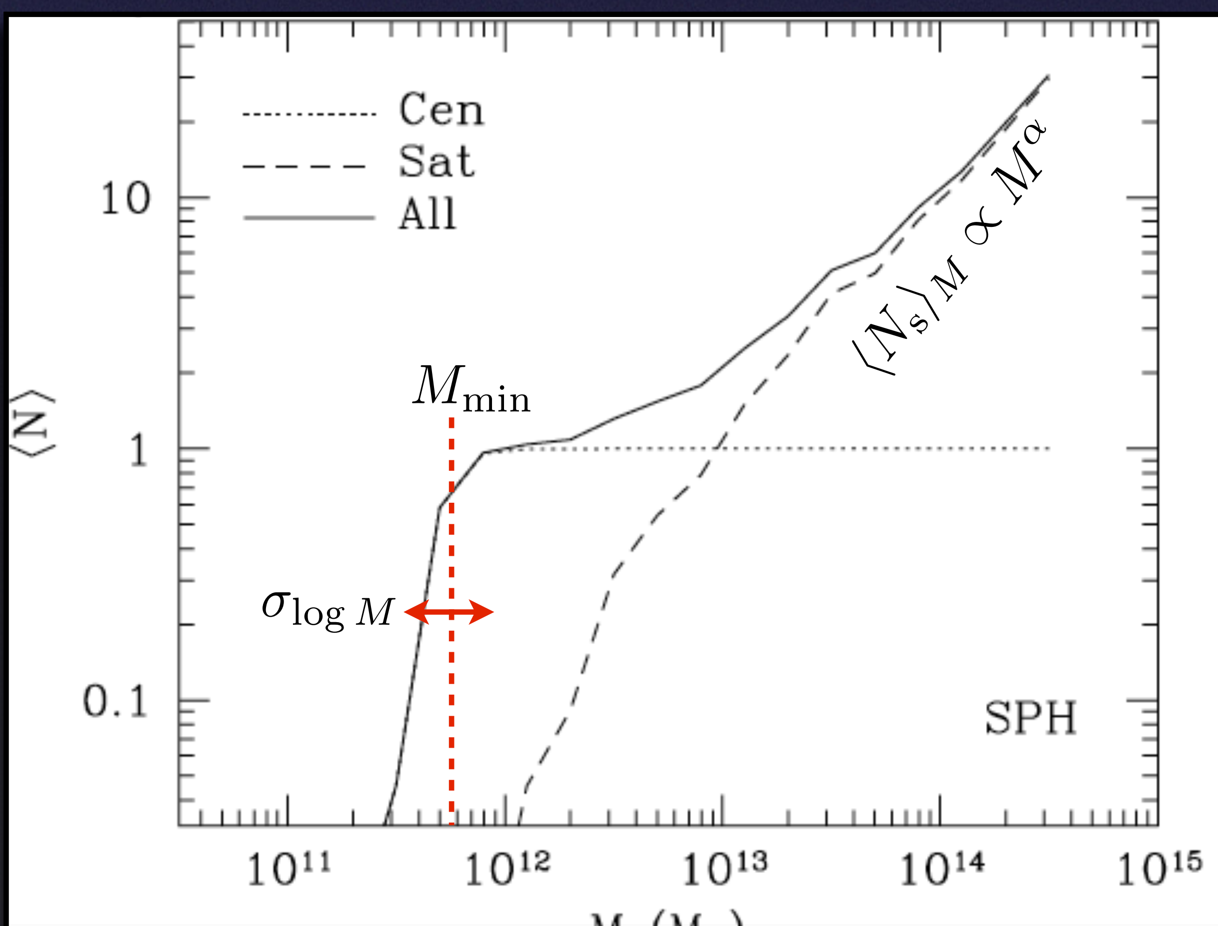
Halo Occupation Distribution (HOD)

Consider a luminosity threshold sample; all galaxies brighter than some threshold luminosity. The halo occupation statistics for such a sample are typically parameterized as follows:

$$\langle N_{\rm c}\rangle_M = \frac{1}{2}\left[1 + {\rm erf}\left(\frac{\log M - \log M_{\rm min}}{\sigma_{\log M}}\right)\right]$$

$$\langle N_{\rm s}\rangle_M = \begin{cases} \left(\frac{M}{M_1}\right)^{\alpha} & \text{if } M > M_{\rm cut} \\ 0 & \text{if } M < M_{\rm cut} \end{cases}$$

- $M_{\rm min}$ = characteristic minimum mass of haloes that host centrals above luminosity threshold
- $\sigma_{\log M}$ = characteristic transition width due to scatter in L-M relation of centrals
- α = slope of satellite occupation numbers
- M_1 = normalization of satellite occupation numbers
- $M_{\rm cut}$ = cut-off mass below which you have zero satellites above luminosity threshold



Source: Zheng et al., 2005

This popular HOD model requires only 5 parameters to characterize occupation statistics for a luminosity threshold sample.

This model is (partially) motivated by the occupation statistics in hydro simulations

Conditional Luminosity Function (CLF)

An alternative parameterization, which has the advantage that it describes the occupation statistics for any luminosity sample (not only threshold samples), is the conditional luminosity function $\Phi(L|M)$

The CLF describes the average number of galaxies of luminosity L that reside in a dark matter halo of mass M.

$$\Phi(L) = \int_0^\infty \Phi(L|M)\, n(M)\, \mathrm{d}M$$

CLF is the direct link between the halo mass function and the galaxy luminosity function.

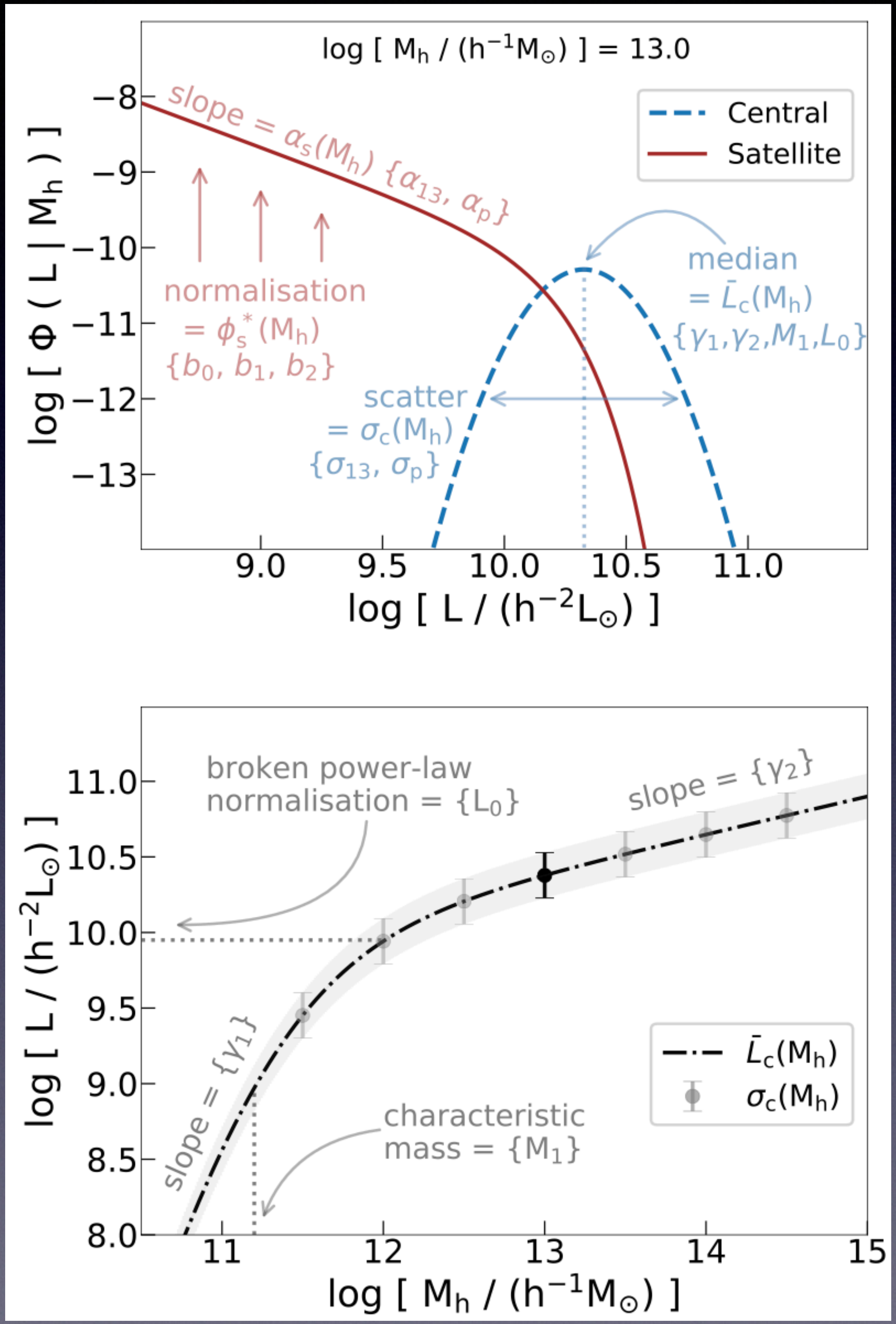
$$\langle L \rangle_M = \int_0^\infty \Phi(L|M)\, L\, \mathrm{d}L$$

CLF describes link between luminosity and mass

$$\langle N_\mathrm{x} \rangle_M = \int_{L_1}^{L_2} \Phi_\mathrm{x}(L|M)\, \mathrm{d}L$$

CLF describes first moments of halo occupation statistics of any luminosity sample

The Conditional Luminosity Function



We split the CLF in a central and a satellite term:

$$\Phi(L|M) = \Phi_c(L|M) + \Phi_s(L|M)$$

For centrals we adopt a log-normal distribution:

$$\Phi_c(L|M)\mathrm{d}L = \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left[-\left(\frac{\ln(L/L_c)}{\sqrt{2}\sigma_c}\right)^2\right] \frac{\mathrm{d}L}{L}$$

For satellites we adopt a Schechter function:

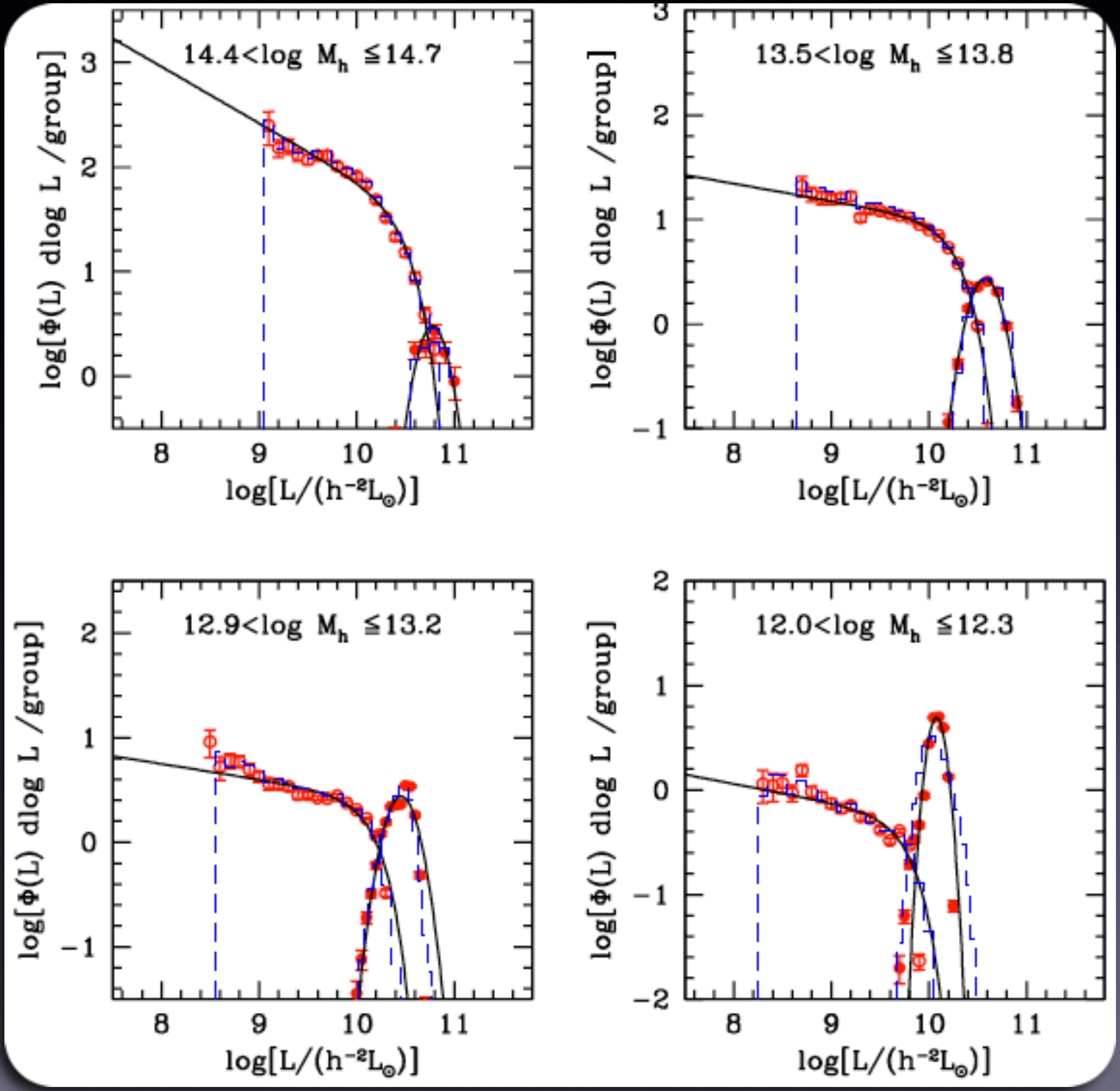
$$\Phi_s(L|M)\mathrm{d}L = \frac{\phi_s}{L_s}\left(\frac{L}{L_s}\right)^{\alpha_s} \exp\left[-(L/L_s)^2\right]\mathrm{d}L$$

Note: $\{L_c, L_s, \sigma_c, \phi_s, \alpha_s\}$ all depend on halo mass

Characterized by $\mathcal{O}(10)$ free parameters, to be constrained by data

The Conditional Luminosity Function

The functional form for the CLF is supported by data from galaxy group catalogues



Source: Yang, Mo & van den Bosch, 2008

Halo Occupation Statistics

In addition to the HOD/CLF, one also needs to specify:

- The second moment of the satellite occupation distribution:

$$\langle N_{\rm s}(N_{\rm s}-1)\rangle_M = \sum_{N_{\rm s}=0}^{\infty} N_{\rm s}(N_{\rm s}-1)\, P(N_{\rm s}|M) \equiv \beta(M)\, \langle N_{\rm s}\rangle^2$$

where we have introduced the function $\beta(M)$

If the occupation statistics of satellite galaxies follow Poisson statistics, i.e.,

$$P(N_{\rm s}|M) = \frac{\lambda^{N_{\rm s}}\, e^{-\lambda}}{N_{\rm s}!} \quad \textbf{with} \quad \lambda = \langle N_{\rm s}\rangle_M$$

then $\beta(M) = 1$. Distributions with $\beta > 1$ ($\beta < 1$) are broader (narrower) than Poisson.

The second moment of the halo occupation statistics is completely described by $\beta(M)$

Halo Occupation Statistics

In addition to the HOD/CLF, one also needs to specify:

- The second moment of the satellite occupation distribution:

$$\langle N_{\rm s}(N_{\rm s}-1)\rangle_M = \sum_{N_{\rm s}=0}^{\infty} N_{\rm s}(N_{\rm s}-1)\, P(N_{\rm s}|M) \equiv \beta(M)\,\langle N_{\rm s}\rangle^2$$

where we have introduced the function $\beta(M)$

- The radial number density profile of satellite galaxies

$$n_{\rm sat}(r|M) \propto \left(\frac{r}{\mathcal{R} r_{\rm s}}\right)^{\gamma} \left[1 + \frac{r}{\mathcal{R} r_{\rm s}}\right]^{\gamma-3}$$

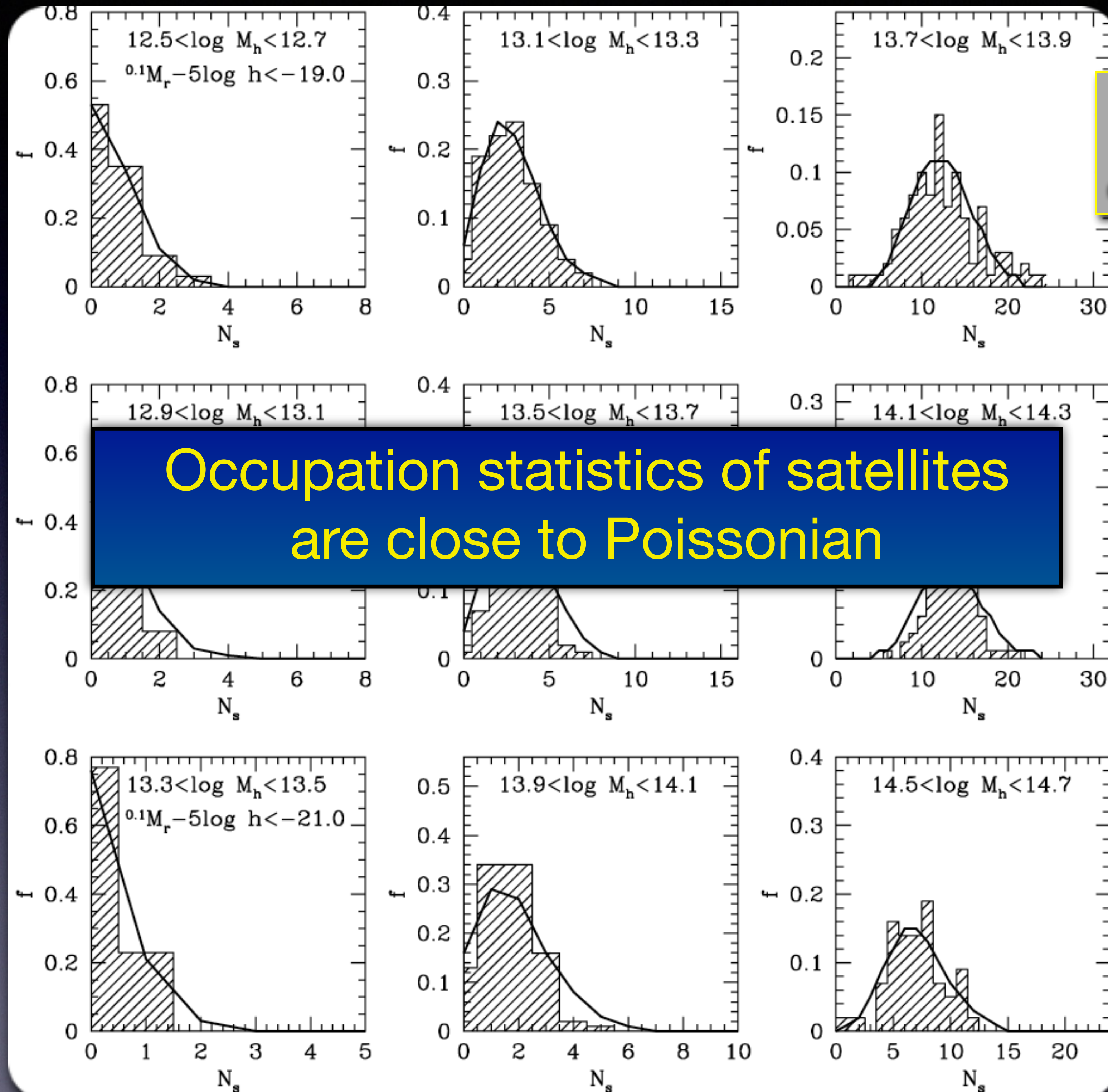
This is a `generalized NFW profile'

Majority of studies assume that

$\beta(M) = 1$ i.e., satellites obey Poisson statistics

$\gamma = \mathcal{R} = 1$ i.e., satellites are unbiased tracer of halo mass distribution

Halo Occupation Statistics



Solid lines are the Poisson distribution corresponding to <Ns>

Occupation statistics of satellites are close to Poissonian

Source: Yang, Mo & van den Bosch 2008

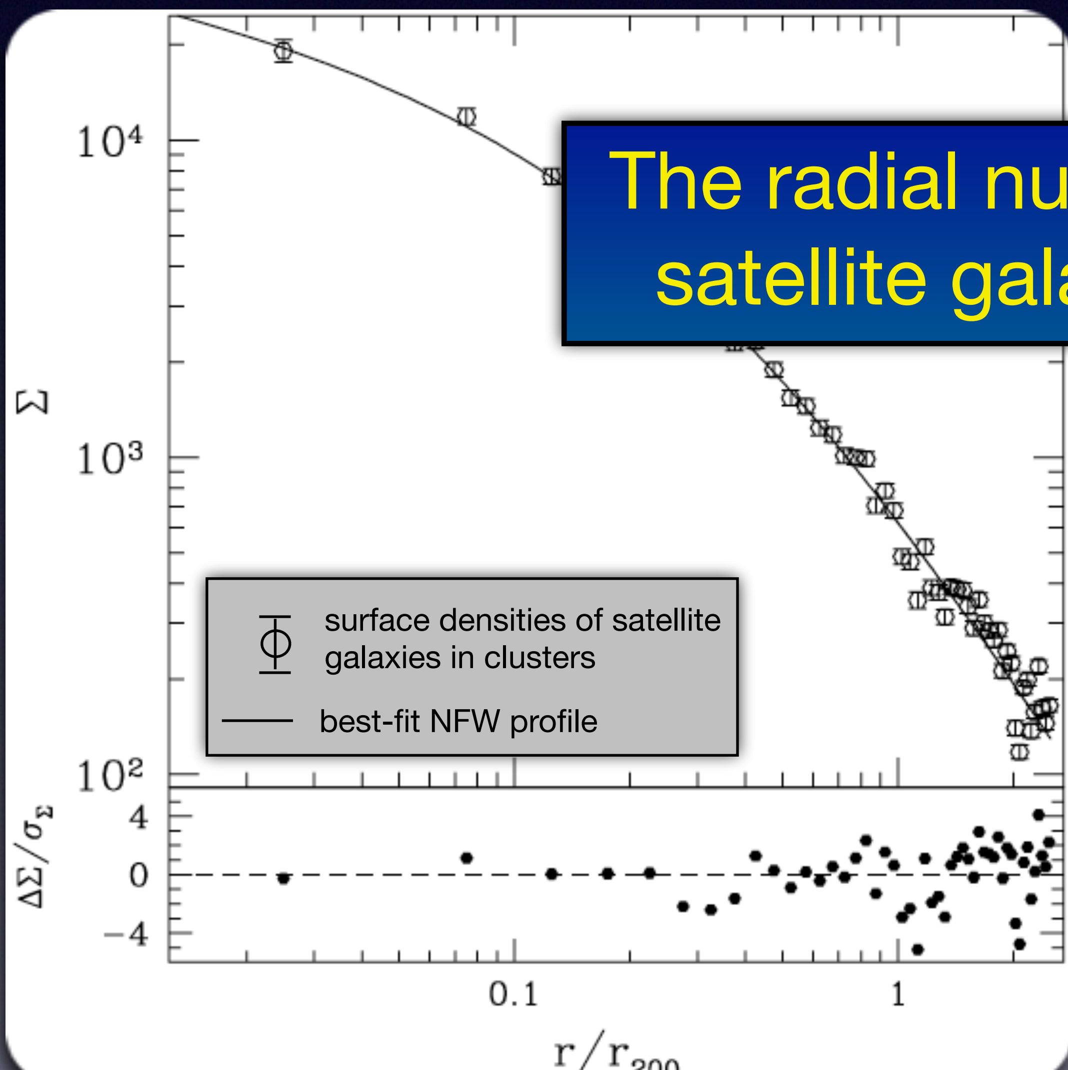
Radial Number Density Profile of Satellites

The radial number density profile of satellites is typically modelled as a \`generalized NFW profile':

$$n_{\rm sat}(r|M) \propto \left(\frac{r}{\mathcal{R} r_{\rm s}}\right)^{\gamma} \left[1 + \frac{r}{\mathcal{R} r_{\rm s}}\right]^{\gamma - 3}$$

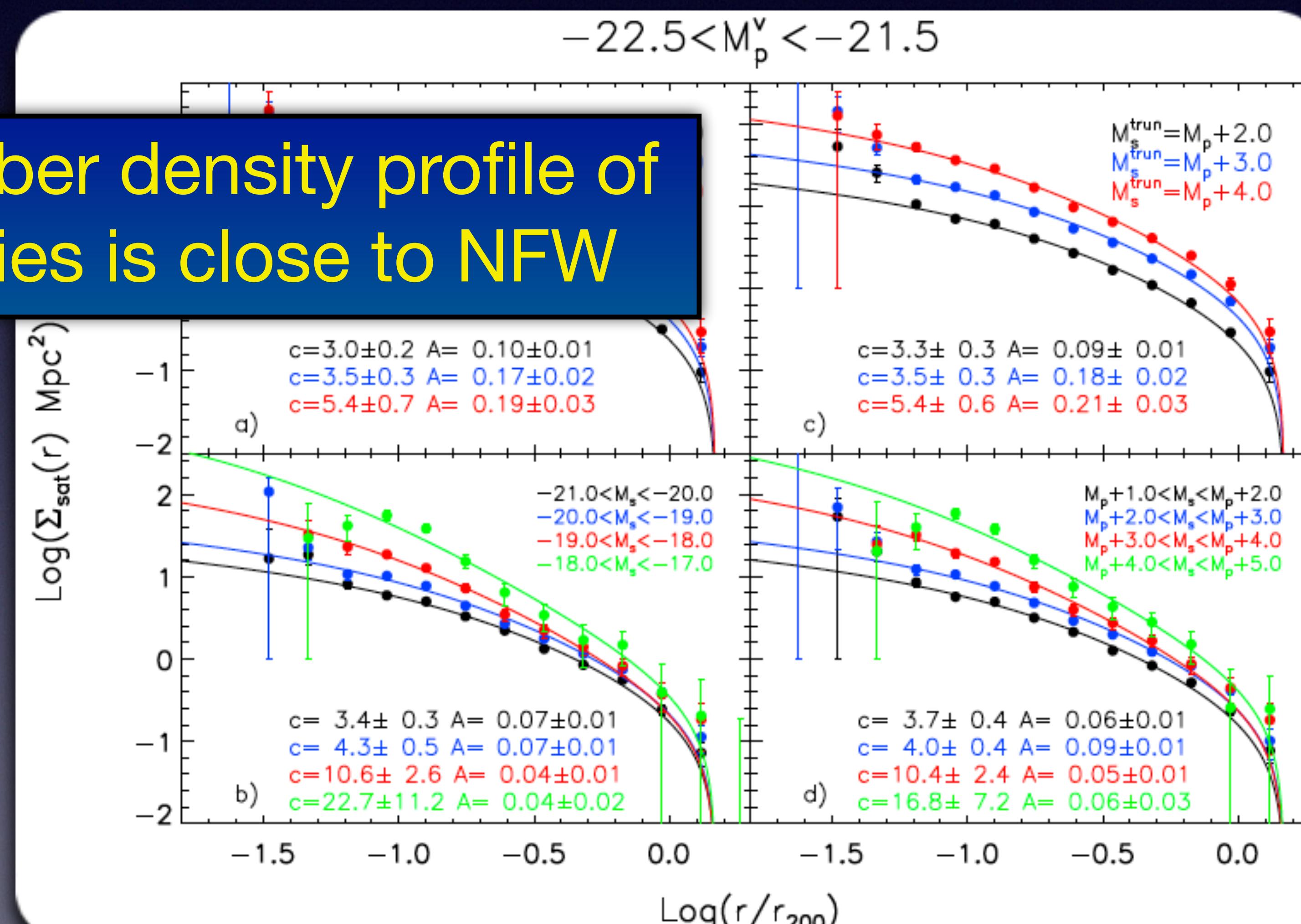
$$\mathcal{R} = c_{\rm sat}/c_{\rm dm}$$

For $\gamma = \mathcal{R} = 1$ satellites are unbiased tracer of mass distribution within individual halos



Source: Lin, Mohr & Stanford 2004

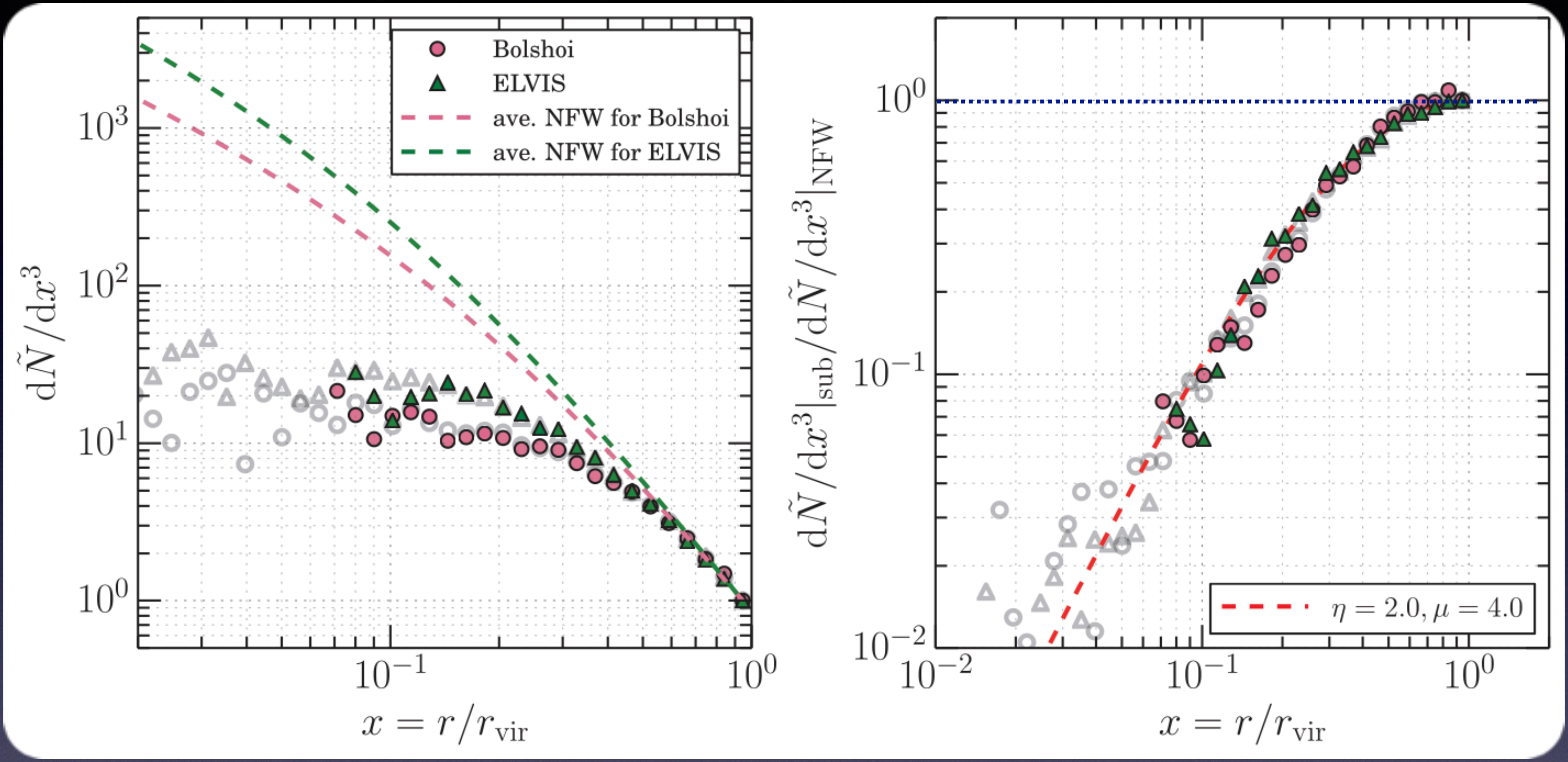
The radial number density profile of satellite galaxies is close to NFW



Source: Guo et al. 2012

Radial Number Density Profile of Subhalos

Subhalos do NOT follow NFW profile; their profile is inconsistent with that of satellites!



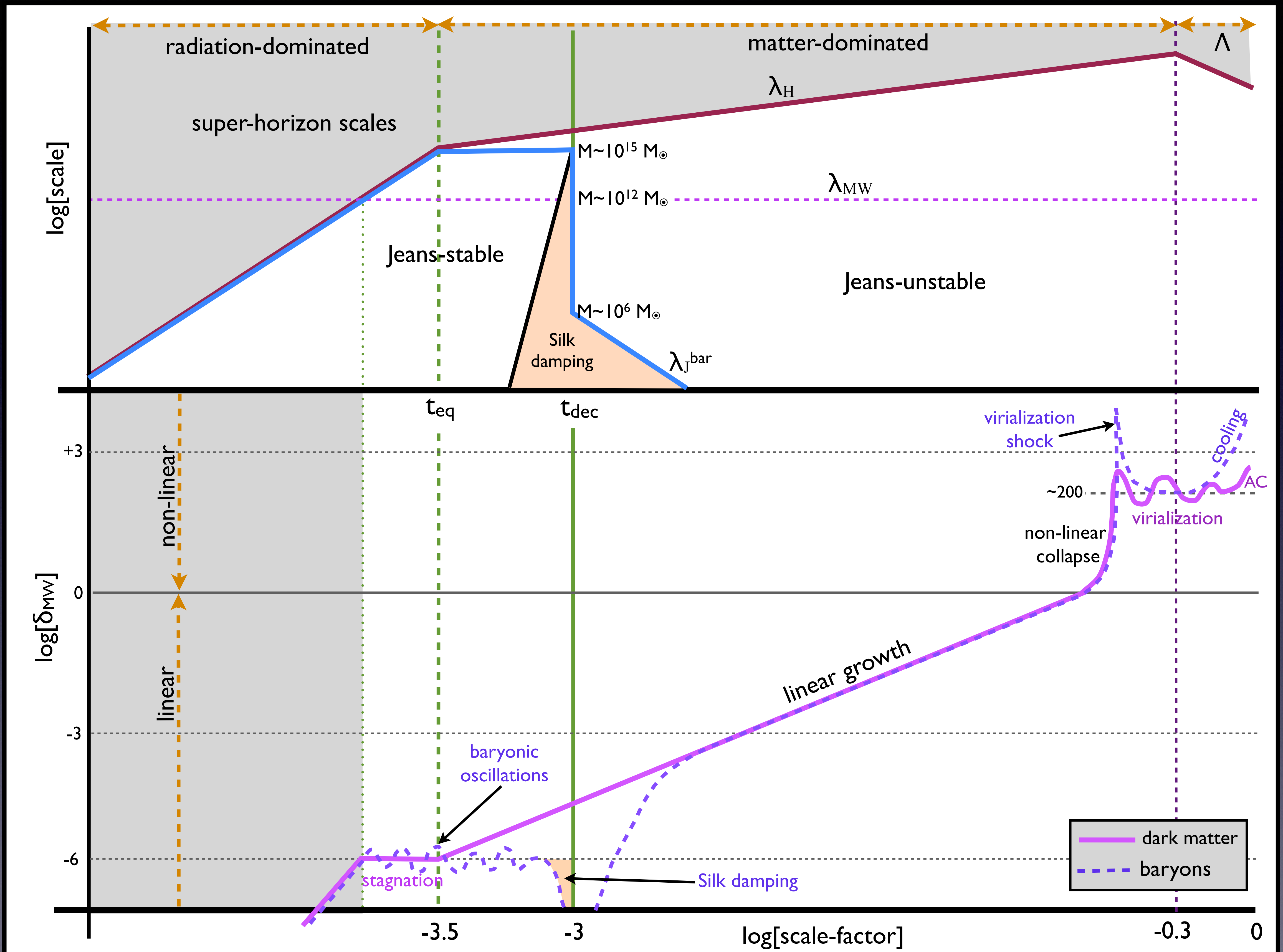
Source: Jiang & vdB 2017, MNRAS, 472, 657

- This is an outcome of artificial subhalo disruption — van den Bosch & Ogiya, 2018
- This directly affects any method that uses subhalos in simulations to model satellite galaxies (SHAM and sim-based HOD/CLF modeling)
- Requires treatment of orphan galaxies

Guo et al. 2010, Pujol et al. 2017; Diemer et al. 2023

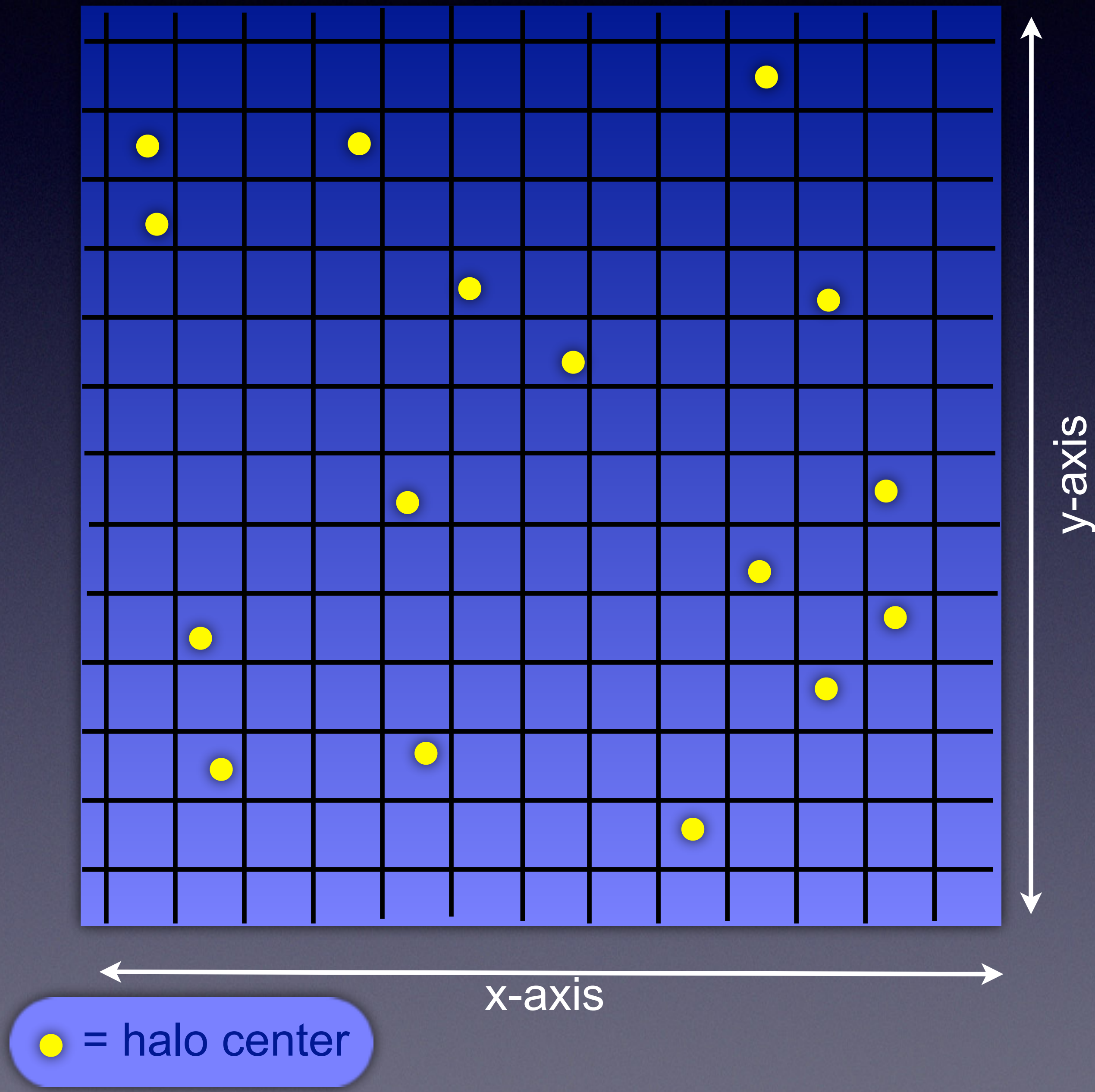
Extra Slides

Galaxy Formation in a Nutshell



The Halo Model

Imagine space divided into many small volumes, ΔV_i ,which are so small that none of them contain more than one halo center.



Let $\mathcal{N}_i$ be the occupation number of dark matter haloes in cell i

Then we have that $\mathcal{N}_i = 0, 1$
and therefore $\mathcal{N}_i = \mathcal{N}_i^2 = \mathcal{N}_i^3 =$

This allows us to write the matter density field as a summation:

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i \, M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i\, M_i\, u(\vec{x} - \vec{x}_i | M_i)$$

$$\overline{\rho} = \int \rho(\vec{x})\, \mathrm{d}^3\vec{x} = \langle \rho(\vec{x}) \rangle = \left\langle \sum_i \mathcal{N}_i\, M_i\, u(\vec{x} - \vec{x}_i | M_i) \right\rangle$$

ergodicity

$$= \sum_i \langle \mathcal{N}_i\, M_i\, u(\vec{x} - \vec{x}_i | M_i) \rangle$$

halo mass function

$$= \sum_i \int \mathrm{d}M\, M\, n(M)\, \Delta V_i\, u(\vec{x} - \vec{x}_i | M)$$

$$= \int \mathrm{d}M\, M\, n(M) \int \mathrm{d}^3\vec{y}\, u(\vec{x} - \vec{y} | M)$$

$$= \int \mathrm{d}M\, M\, n(M)$$

$$= \overline{\rho}$$

Q.E.D.

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i \, M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

Now that we can write the density field in terms of the halo building blocks, let's focus on two-point statistics: $\xi_{\rm mm}(r) \equiv \langle \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) \rangle = \frac{1}{\bar{\rho}^2} \langle \rho(\vec{x})\, \rho(\vec{x}+\vec{r}) \rangle - 1$

$$\langle \rho(\vec{x})\, \rho(\vec{x}+\vec{r}) \rangle = \left\langle \sum_i \mathcal{N}_i \, M_i \, u(\vec{x}_1 - \vec{x}_i | M_i) \cdot \sum_j \mathcal{N}_j \, M_j \, u(\vec{x}_2 - \vec{x}_j | M_j) \right\rangle$$

$\vec{x}_2 = \vec{x}_1 + \vec{r}$

$$= \sum_i \sum_j \langle \mathcal{N}_i \, \mathcal{N}_j \, M_i M_j \, u(\vec{x}_1 - \vec{x}_i | M_i) u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

We split this in two parts: the 1-halo term $(i = j)$, and the 2-halo term $(i \neq j)$

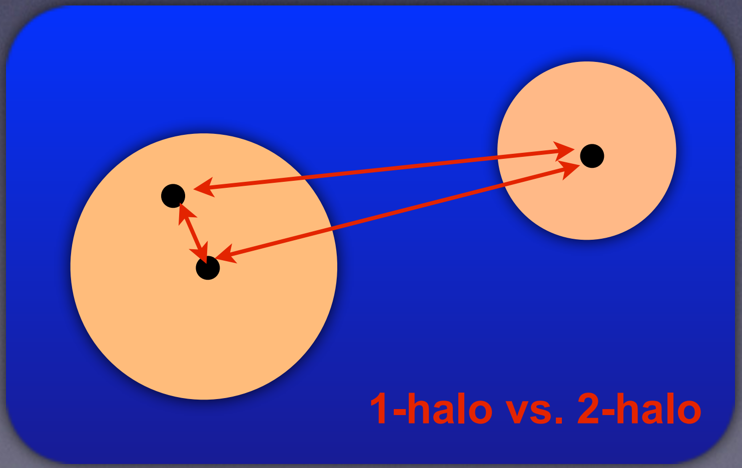
For the 1-halo term we obtain:

$\mathcal{N}_i^2 = \mathcal{N}_i$

$$\langle \rho(\vec{x})\, \rho(\vec{x}+\vec{r}) \rangle_{\rm 1h} = \sum_i \langle \mathcal{N}_i \, M_i^2 \, u(\vec{x}_1 - \vec{x}_i | M_i) u(\vec{x}_2 - \vec{x}_i | M_i) \rangle$$

$$= \sum_i \int {\rm d}M \, M^2 \, n(M) \, \Delta V_i \, u(\vec{x}_1 - \vec{x}_i | M) u(\vec{x}_2 - \vec{x}_i | M)$$

$$= \int {\rm d}M \, M^2 \, n(M) \int {\rm d}^3\vec{y} \, u(\vec{x}_1 - \vec{y} | M) u(\vec{x}_2 - \vec{y} | M)$$



convolution integral

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i \, M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\langle \rho(\vec{x})\, \rho(\vec{x} + \vec{r}) \rangle_{2\mathrm{h}} = \sum_i \sum_{j \neq i} \langle \mathcal{N}_i \mathcal{N}_j \, M_i \, M_j \, u(\vec{x}_1 - \vec{x}_i | M_i)\, u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

$$\overset{?}{\neq} \sum_i \sum_{j \neq i} \int \mathrm{d}M_1 \, M_1 \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \, \Delta V_i \, \Delta V_j \; \times$$

[redacted] $u(\vec{x}_1 - \vec{x}_i | M_1)\, u(\vec{x}_2 - \vec{x}_j | M_2) \; = \bar{\rho}^2$

NO: dark matter haloes themselves are clustered; needs to be taken into account.

Clustering of dark matter haloes is characterized by halo-halo correlation function:

$$\xi_{\mathrm{hh}}(r | M_1, M_2) = b(M_1)\, b(M_2)\, \xi^{\mathrm{lin}}_{\mathrm{mm}}(r)$$

Here $b(M)$ is the halo bias function.

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i \, M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\langle \rho(\vec{x}) \, \rho(\vec{x}+\vec{r}) \rangle_{2\mathrm{h}} = \sum_i \sum_{j \neq i} \langle \mathcal{N}_i \, \mathcal{N}_j \, M_i \, M_j \, u(\vec{x}_1 - \vec{x}_i | M_i) \, u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

$$= \sum_i \sum_{j \neq i} \int \mathrm{d}M_1 \, M_1 \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \, \Delta V_i \, \Delta V_j \; \times$$

$$[1 + \xi_{\mathrm{hh}}(\vec{x}_i - \vec{x}_j | M_1, M_2)] \, u(\vec{x}_1 - \vec{x}_i | M_1) \, u(\vec{x}_2 - \vec{x}_j | M_2)$$

$$= \overline{\rho}^2 + \int \mathrm{d}M_1 \, M_1 \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \; \times$$

$$\int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2 \, u(\vec{x}_1 - \vec{y}_1 | M_1) \, u(\vec{x}_2 - \vec{y}_2 | M_2) \, \xi_{\mathrm{hh}}(\vec{y}_1 - \vec{y}_2 | M_1, M_2)$$

$$= \overline{\rho}^2 + \int \mathrm{d}M_1 \, M_1 \, b(M_1) \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, b(M_2) \, n(M_2) \times$$

$$\int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2 u(\vec{x}_1 - \vec{y}_1 | M_1) \, u(\vec{x}_2 - \vec{y}_2 | M_2) \, \xi^{\mathrm{lin}}_{\mathrm{mm}}(\vec{y}_1 - \vec{y}_2)$$

convolution integral

The Halo Model: Summary

$$\xi(r) = \xi^{1\mathrm{h}}(r) + \xi^{2h}(r)$$

$$\xi^{1\mathrm{h}}(r) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M\, M^2\, n(M) \int \mathrm{d}^3\vec{y}\, u(\vec{x}-\vec{y}|M) u(\vec{x}+\vec{r}-\vec{y}|M)$$

$$\xi^{2\mathrm{h}}(r) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M_1\, M_1\, b(M_1)\, n(M_1) \int \mathrm{d}M_2\, M_2\, b(M_2)\, n(M_2) \times \int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2 u(\vec{x}-\vec{y}_1|M_1)\, u(\vec{x}+\vec{r}-\vec{y}_2|M_2)\, \xi^{\mathrm{lin}}_{\mathrm{mm}}(\vec{y}_1-\vec{y}_2)$$

Halo Model Ingredients:

- the halo density profiles $\rho(r|M) = M\,u(r|M)$
- the halo mass function $n(M)$
- the halo bias function $b(M)$
- the linear correlation function of matter $\xi^{\mathrm{lin}}_{\mathrm{mm}}(r)$

All of these are (reasonably) well calibrated against numerical simulations.

The Halo Model in Fourier Space

$$P(k) = P^{1h}(k) + P^{2h}(k)$$

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M \, M^2 \, n(M) \, |\tilde{u}(k|M)|^2$$

$$P^{2h}(k) = P^{lin}(k) \left[\frac{1}{\bar{\rho}} \int \mathrm{d}M \, M \, b(M) \, n(M) \, \tilde{u}(k|M) \right]^2$$

$$P^{lin}(k) = P_{\mathrm{i}}(k) \, T^2(k) = k^{n_s} \, T^2(k)$$

$$\tilde{u}(\vec{k}|M) = \int u(\vec{x}|M) e^{-i\vec{k}\cdot\vec{x}} \, \mathrm{d}^3\vec{x} = 4\pi \int_0^\infty u(r|M) \frac{\sin kr}{kr} \, r^2 \, \mathrm{d}r$$

Convolutions in real-space <—> Multiplications in Fourier space.

Computing power spectrum, $P(k)$, is much easier.

Two-point correlation function, $\xi(r)$, is obtained by Fourier transforming $P(k)$

For a detailed review article on the Halo Model: see Cooray & Sheth, 2002, Phys. Rep. 372, 1

The Halo Model: complications

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M \, M^2 \, n(M) \, |\tilde{u}(k|M)|^2$$

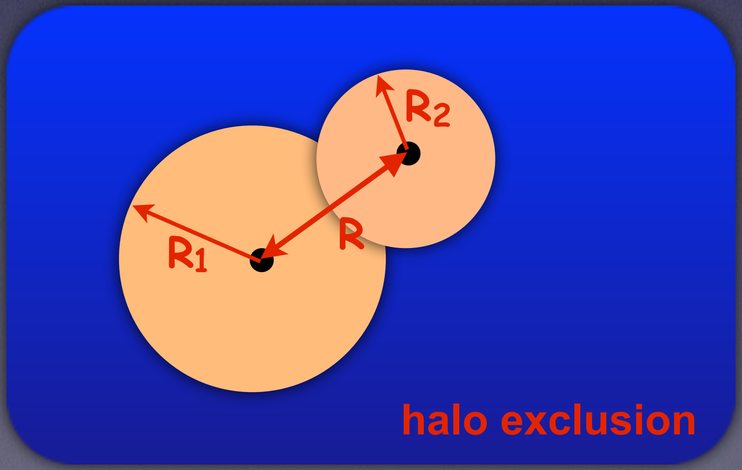
$$P^{2h}(k) = P^{\rm lin}(k) \left[\frac{1}{\bar{\rho}} \int \mathrm{d}M \, M \, b(M) \, n(M) \, \tilde{u}(k|M) \right]^2$$

However, this is ONLY true under the simplifying assumption that

$$\xi_{\rm hh}(r|M_1, M_2) = b(M_1) \, b(M_2) \, \xi^{\rm lin}_{\rm mm}(r)$$

In reality, on small scales, in the (quasi)-linear regime, this description of the halo-halo correlation function becomes inadequate for two reasons:

- $\xi^{\rm lin}_{\rm mm}(r)$ is no longer adequate (Tinker et al. 2005)
- halo exclusion (Smith et al. 2007, van den Bosch et al. 2013)
- halo triaxiality (van Daalen et al. 2012)



Properly accounting for these effects is complicated