The Galaxy-Halo Connection

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Michigan Cosmology Summer School 2023

OUTLINE

-ECTURE 1

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LECTURE

- A primer on Structure Formation
- The Halo Model
- Halo Occupation Modeling
- Halo Occupation Distribution (HOD)
- Conditional Luminosity Function (CLF)
- Subhalo Abundance Matching (SHAM)

- Empirical Constraints
 - L'IIpilical Constraints
- Galaxy-Halo Connection
- Cosmological Constraints
- Issues & Concerns

- Stellar Mass-Halo Mass Relation (SHMR)
- Scatter in SHMR

Galaxy clustering

Galaxy-Galaxy lensing

Satellite kinematics

- Satellite Galaxies
- The S₈ tension
- Artificial Subhalo Disruption & Orphans
- Baryonic Effects
- Assembly Bias

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Constraints from clustering

From Clustering to Galaxy-Halo Connection

Analytical Halo Model

$$\begin{split} \xi(r) &= \xi^{1\mathrm{h}}(r) + \xi^{2\mathrm{h}}(r) \\ \xi^{1\mathrm{h}}(r) &= \frac{1}{\overline{\rho}^2} \int \mathrm{d}M \, M^2 \, n(M) \int \mathrm{d}^3 \vec{y} \, u(\vec{x} - \vec{y} | M) u(\vec{x} + \vec{r} - \vec{y} | M) \\ \xi^{2\mathrm{h}}(r) &= \frac{1}{\overline{\rho}^2} \int \mathrm{d}M_1 \, M_1 \, b(M_1) \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, b(M_2) \, n(M_2) \times \\ &\int \mathrm{d}^3 \vec{y}_1 \int \mathrm{d}^3 \vec{y}_2 u(\vec{x} - \vec{y}_1 | M_1) \, u(\vec{x} + \vec{r} - \vec{y}_2 | M_2) \, \xi^{\mathrm{lin}}_{\mathrm{mm}}(\vec{y}_1 - \vec{y}_2) \end{split}$$

For given $\Phi(L \mid M)$ and $n_{sat}(r \mid M)$, compute the two-point correlation function and compare to observations





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From Clustering to Galaxy-Halo Connection

Analytical Halo Model

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For given $\Phi(L \mid M)$ and $n_{sat}(r \mid M)$, compute the two-point correlation function and compare to observations





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Testing with Mock Data



Populate dark matter halos in N-body simulation with mock galaxies using CLF

Construct mock SDSS

Measure clustering in mock SDSS data

Predict clustering using analytical model with same CLF and compare

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Testing with Mock Data



Populate dark matter halos in N-body simulation with mock galaxies using CLF

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Constraints on Halo Occupation Statistics



vdBosch, Yang & Mo 2004

2004

One of the first, clustering-based constraints, on galaxy-halo connection

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Constraints on Halo Occupation Statistics



Source: Zehavi et al. 2011

Simple HOD models can accurately fit the (projected) correlation functions for 9 different SDSS luminosity threshold samples...

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Key Premises

- Galaxies form and reside in halos (including subhalos)
- There exist some halo property(ies) that are tightly correlated with the properties of the galaxies they host i.e., *L* is strongly correlated with *M*



Ansatz

For satellite galaxies, their presentday luminosity/stellar mass is strongly correlated with their halo mass at accretion, M_{acc}

Motivation

Satellite galaxies quench after infall and tides mainly strip dark matter

 From N-body simulation, obtain cumulative mass function of halos + subhalos (for latter, use mass at infall)

• From a galaxy redshift survey, obtain cumulative mass or luminosity function of galaxies

Match rank orders.....done

Kravtsov et al. 2004; Vale & Ostriker 2006, 2007; Conroy et al. 2006



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Source: Conroy, Wechsler & Kravtsov 2006

...suggests that satellites indeed stop forming stars at infall (quenching)...



Source: Reddick et al. 2013

SHAM has only 2 degrees of freedom: halo property & scatter

Downsides of SHAM

- Only works for complete samples of galaxies
- Requires N-body simulations
- artificial disruption; SHAM to be complemented with orphan treatment

Campbell, vdB et al. 2018

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The mass associated with galaxies lenses background galaxies



Lensing causes correlated ellipticities, the tangential shear, γ_t , which is related to the excess surface density, $\Delta \Sigma$, according to

$$\gamma_{\rm t}(R)\Sigma_{\rm crit} = \Delta\Sigma(R) = \bar{\Sigma}(\langle R) - \Sigma(R)$$

 $\Delta\Sigma$ is line-of-sight projection of galaxy-matter cross correlation

$$\Sigma(R) = \bar{\rho} \int_0^{D_{\rm s}} [1 + \xi_{\rm g,dm}(r)] \,\mathrm{d}\chi$$

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Number of background sources per lens is limited

• Measuring shear with sufficient S/N requires stacking of many lenses



Mandelbaum et al. 2006

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Number of background sources per lens is limited

Measuring shear with sufficient S/N requires stacking of many lenses



Brighter galaxies reside in more massive haloes



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Because of stacking, it is not straightforward to interpret lensing signal

In order to model the data, what is required is:

$$P_{\rm cen}(M|L)$$
 $P_{\rm sat}(M|L)$ $f_{\rm sat}(L)$

These can all be computed from the CLF...

For a given $\Phi(L|M)$ we can predict the lensing signal $\Delta\Sigma(R|L_1, L_2)$

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Galaxy-Galaxy Lensing can constrain halo occupation statistics

Satellite Kinematics

Probing Halo Mass with Satellite Kinematics



Die Rotver

Die Rotverschiebung von extragalaktischen Nebeln von F. Zwicky. (16. II. 33.)

Um, wie beobachtet, einen mittleren Dopplereffekt von 1000 km/sek oder mehr zu erhalten, müsste also die mittlere Dichte im Comasystem mindestens 400 mal grösser sein als die auf Grund von Beobachtungen an leuchtender Materie abgeleitete¹). Falls sich dies bewahrheiten sollte, würde sich also das überraschende Resultat ergeben, dass dunkle Materie in sehr viel grösserer Dichte vorhanden ist als leuchtende Materie. Zwicky, 1933, Helv. Phys. Acta, 6, 110

Probing Halo Mass with Satellite Kinematics







Spherical
Jeans
Equations
$$M(r) = -\frac{r\sigma_r^2}{G} \left[\frac{d \ln n_{\text{sat}}}{d \ln r} + \frac{d \ln \sigma_r^2}{d \ln r} + 2\beta \right]$$
anisotropy parameter $\beta = 1 - \frac{\sigma_t^2}{2r^2}$

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Selecting Centrals & Satellites

Centrals (`primaries') are the brightest galaxies in their cylinder. Others are satellites (`secondaries')



Cylinder sizes are optimized for purity and completeness (using mocks)



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Satellite Kinematics



Select centrals and their satellites from a redshift survey Using redshifts, determine $\Delta V = V_{sat} - V_{cen}$ as function of L_c



Compute satellite velocity dispersion, σ_{sat} , as function of L_c Use $\sigma_{sat}(L_c)$ to constrain $\Phi(L_c | M)$

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Constraints on the Galaxy-Halo Connection

The Stellar Mass - Halo Mass Relation



Source: Behroozi, Wechsler & Conroy, 2013

Satellite Fractions



Source: van den Bosch et al. 2007

Source: Cacciato, vdB et al. 2013

Majority of galaxies of a given luminosity (or stellar mass) are centrals.

Scatter in the SHMR



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Scatter in SHMR



Source: Wechsler & Tinker 2018

Scatter in halo mass at fixed luminosity increases strongly with stellar mass

Beware when stacking galaxies by stellar mass

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Constraining Cosmology

Cosmology Dependence

14







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Cosmology Dependence



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2012

<u>a</u>l.

Finker et

Source:

15

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Cosmology Dependence



Observational constraints $\langle M/L \rangle_{cl} = 350 \pm 70 \ (M/L)_{\odot}$ Carlberg+96; Bahcall+00; suggest low values for Ω_m and σ_8

These were early manifestations of the S₈ tension

see also Yang et al. 2004; Tinker et al. 2005
Cosmology Dependence

Breaking degeneracies between cosmology and galaxy-halo connection requires methods that directly constrain halo mass

Galaxy-Galaxy lensing

Redshift-space distortions (RSD)

Satellite kinematics

Galaxy-Galaxy Lensing



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Combination of clustering & lensing can constrain cosmology!!!

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Leauthaud et al. (2017) BOSS-CMASS

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There is a persistent tension between observations of the low-z Universe and Planck CMB results.

- Systematic error in Planck analysis? there is also tension with respect to Hubble parameter...
- New physics beyond the 6-parameter "vanilla" LCDM model? neutrino mass, dark matter is warm/fuzzy/self-interacting,...
- Systematic errors in lensing shear measurements?
- Systematic errors in modeling of clustering and/or lensing? assembly bias, impact of baryons,...

The Dark Side of Halo Occupation Modelling

WARNING: things are about to get ugly and depressing

stop listening if you suffer from anxiety or heart conditions

Baryonic Effects

Throughout, it is assumed that dark matter halos follow a NFW profile.

This is motivated by results from N-body, dark-matter-only (DMO) simulations.

However, hydro-dynamical simulations show that galaxy formation physics impacts both the masses and density profiles of dark matter halos...

Zentner et al. 2008; van Daalen et al. 2011, Sawala et al. 2013, Martizzi et al. 2016 Springel et al. 2017, Schneider et al. 2019, Chisari et al. 2019, Beltz-Mohrmann & Berlind 2021



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Baryonic Effects vs S₈ Tension

'Feedback' reduces ESD on small scales (≤ 1 Mpc).

Not accounted for in most clustering/lensing analyses

Effect diminishes, but seems unable to fully solve "lensing is low"



source: Lange et al. (2019)

Main Challenge: • There is (and likely never will be) concensus among simulations as to the magnitude of the effect.

- Assume simulations bracket truth and marginalize over uncertainties
- Simultaneously model & observe stars (galaxies) and gas (CGM).



Halo Assembly Bias





source: Gao et al. 2005



source: van den Bosch (2002)

Halo bias, at fixed mass, depends on halo age

At low masses: halos that assemble earlier are more strongly clustered

Gao et al. 2005

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Halo Assembly Bias



Source: Wechsler et al. 2006

Halo bias also depends on halo concentration... Wechsler et al. 2006

In fact, N-body simulations have shown that halo bias depends on virtually <u>every</u> halo property...

- spin parameter (angular momentum)
- environment
- halo shape

Gao & White 2007, Lacerna & Padilla 2012 Villareal et al. 2017, Mao et al. 2018, Salcedo et al 2018

Clustering strength is not a clean indicator of halo mass

The Origin of Halo Assembly Bias

What is the origin of Halo Assembly Bias?

Correlations between halo properties.

Halos of a given mass that assemble earlier

- are more centrally concentrated
- have smaller spin parameter
- have less (surviving) substructure
- are more spherical

Wechsler et al. 2002; Vitvitska et al. 2002 Gao et al. 2004, vdBosch 2005 Allgood et al. 2006; Jiang & vdBosch 2017

Tides from nearby halos or large-scale structure [arrested development]

Desjacques 2008; Dalal et al. 2008; Hahn et al. 2009; Wang et al. 2011 Hearin et al. 2016; Paranjape et al. 2018; Mansfield & Kravtsov 2019

Contamination with Splashback halos

[environmental stripping]

Wang et al. 2009; Li et al. 2013; Wetzel et al. 2015; Hearin et al. 2015 Sunayama et al. 2016; Tucci et al 2020; Mansfield & Kravtsov 2020

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Halo Assembly Bias: the effect that the clustering of halos at fixed mass depends on secondary halo properties

aka secondary halo bias

Mass-Only Ansatz: the assumption, made up to now, that halo occupation statistics depend only on halo mass

Galaxy Assembly Bias: at fixed halo mass, the occupation statistics depend on secondary halo properties that show halo assembly bias



Galaxy Assembly Bias is a violation of the Mass-Only ansatz

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Why do we care about galaxy assembly bias?

provides important insight into galaxy formation

ignoring GAB can bias the inferred galaxy-halo connection

Empirical Fact: red galaxies are more strongly clustered than blue galaxies of same stellar mass. without GAB: red galaxies reside in more massive halos than their blue couterparts With GAB: red galaxies reside in <u>older halos</u> than their blue counterparts, but of <u>same mass</u>

See Zentner, Hearin & vdB (2014) for an explicit example

ignoring GAB can bias cosmological inference



might the S₈ tension be a manifestation of galaxy assembly bias

Modeling Galaxy Assembly Bias

How can one model galaxy assembly bias?

• generalize halo occupation models: $P(L \mid M) \rightarrow P(L \mid M, x)$ $< N \mid M > \rightarrow < N \mid M, x >$

quantify relevant halo bias

 $b(M) \rightarrow b(M,x)$

Bottleneck: we (currently) lack an analytical/theoretical model for *b(M,x)*



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Do we expect galaxy assembly bias?

- there is NO reason to assume that there is NO galaxy assembly bias
- it seems only logical that the properties of a galaxy depend on the assembly history, spin, and shape of its dark matter halo
- SAMs, SIMs and empirical models all `predict' different levels of GAB Artale et al. 2018; Zehavi et al. 2018; Bose et al. 2019; Contreras et al. 2019; Xu & Zheng 2020



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Have we observationally detected galaxy assembly bias?

 direct detection: requires demonstration that different subsets of the same halo mass are clustered differently



Have we observationally detected galaxy assembly bias?



source: Miyatake et al. 2016



Miyatake et al. 2016

two cluster samples with equal g-g lensing (=equal mass) display different clustering.

signal larger than expected

effect can be explained with projection effects Busch et al. 2017; Zu et al. 2017; Sunayama et al. 2020

Have we observationally detected galaxy assembly bias?

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Beyond halo mass: galactic conformity as a smoking gun of central galaxy assembly bias

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ABSTRACT

Quenched central galaxies tend to reside in a preferentially quenched large-scale environment, a phenomenon that has been dubbed *galactic conformity*. Remarkably, this tendency persists out to scales far larger than the virial radius of the halo hosting the central. Therefore, conformity manifestly violates the widely adopted assumption that the dark matter halo mass $M_{\rm vir}$ exclusively governs galaxy occupation statistics. This paper is the first in a series studying the implications of the observed conformity on scales $r \sim 1-5$ Mpc imply that central galaxy quenching statistics cannot be correctly predicted with the knowledge of $M_{\rm vir}$ alone. We also demonstrate that ejected (or 'backsplash') satellites cannot give rise to the signal. We then invoke the age matching model, which is predicated on the co-evolution of galaxies and haloes. We find that this model produces a strong signal, and that central galaxies are solely responsible. We conclude that large-scale 'two-halo' conformity represents a smoking gun of *central galaxy assembly bias*, and indicates that contemporary models of satellite quenching have systematically overestimated the influence of post-infall processes.

Key words: galaxies: evolution – galaxies: haloes – cosmology: theory – dark matter – large-scale structure of Universe.

Galactic Conformity:

red/quenched galaxies are typically surrounded by red/quenched galaxies

Weinmann, vdB et al. 2006; Kauffmann et al 2013 Kawinwanichakij et al. 2016; Berti et al. 2017

on small scales: 1-halo conformity on large scales: 2-halo conformity

Hearin et al. 2015

Group finder errors and projection effects can masquerade as conformity

Campbell, vdB et al. 2015; Paranjape et al. 2015 Calderon et al. 2018; Zu & Mandelbaum 2017

Ongoing debate..... Challenge is to control for halo mass.

Have we observationally detected galaxy assembly bias?

- direct detection: requires demonstration that different subsets of the same halo mass are clustered differently
- indirect detection: include GAB in occupation models and see if it improves fits to data. This typically requires multiple types of data (i.e., clustering + lensing) to break degeneracies.

Hearin & Watson 2013; Hearin+14; Lehmann+17; Zentner+19; Lange +19 Vakili & Hahn 2019; Salcedo+19; Yuan+21; Wang+22; Beltz-Mohrmann+23

Age Matching[‡]

Step 1: using an N-body simulations, abundance match L to V_{max} . Step 2: in bins of L, abundance match color to halo assembly time; reddest to oldest Step 3: compute clustering as function of L and color and compare to data



Hearin & Watson 2013; Hearin et al. 2014

Age Matching accurately fits clustering of red and blue galaxies.

Proof of galaxy assembly bias ??

‡an example of conditional abundance matching (CAM)]

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Satellite Kinematics

source: More, vdB et al. 2011

Satellite kinematics and galaxy-galaxy lensing shows that red centrals live in more mass halos, which can also explain their different clustering...

see also Rodríguez-Puebla et al. 2015; Zu & Mandelbaum 2015; Lange, vdB et al. 2019

Degeneracies are common and make it difficult to find unambiguous evidence for galaxy assembly bias

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Hearin & Watson 2013; Hearin+14; Lehmann+17; Zentner+19; Lange +19 Vakili & Hahn 2019; Salcedo+19; Yuan+21; Wang+22; Beltz-Mohrmann+23

1.0 data that is particularly good • $w_{\rm p}(r_{\rm p})$ $w_{\rm p}(r_{\rm p}) + \Delta \Sigma(r_{\rm p}) + VPF(r) + P(N_{
m CIC}) + P(N_{
m CIA}) + P(N_2/N_5)$ $w_{\rm p}(r_{\rm p}) + {\rm VPF}(r)$ $w_{\rm p}(r_{\rm p}) + P(N_{\rm CIC})$ $w_{\rm p}(r_{\rm p}) + P(N_{\rm CIA})$ $w_{\rm p}(r_{\rm p}) + \Delta \Sigma(r_{\rm p})$ $v_{\rm p}(r_{\rm p}) + P(N_2/N_5)$ at breaking degeneracies 0.8 void-probability function $\sigma(A_{\rm sat})$ Walsh & Tinker 2019 counts-in-cells Wang et al. 2019 0.2 0.0

source: Wang et al. 2019
Assembly Bias vs. S₈ Tension



Assembly bias unable to explain the lensing discrepancy?

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Assembly bias is degenerate with cosmological parameters



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Assembly Bias vs. S₈ Tension



Source: Contreras et al. 2023

SHAMe: includes treatment of orphan galaxies secondary halo property used is large-scale environmental density

Assembly bias is degenerate with cosmological parameters and is flexible enough to solve the S8 tension?

see also Chaves-Montero et al. 2023

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Assembly Bias vs. S₈ Tension



Source: Yuan et al. 2022

YOYO

flexible HOD + emulator: flexible radial profile of satellites HOD has environment dependence to model galaxy assembly bias

> Inconsistent with results of Contreras et al. 2023 The tension continues....

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lesson learned (again)

"Inconsistency is the only thing in which men are consistent"

[Horace Smith]

The Galaxy-Halo Connection

Constrain Cosmology	Fix Cosmology
marginalize over galaxy formation	learn about galaxy formation
Assembly Bias is a condemnation	Assembly Bias is a blessing
N-body simulations are a must	N-body simulations are a must
but what about orphans?	but what about orphans?
Emulators & Cosmological Evidence Modeling	Only a few simulations needed
lots of progress in recent years	to beat cosmic variance
Will we be able to break degeneracies?	Will we be able to break degeneracies?
uncertain	probably
How to deal with baryonic effects?	How to deal with baryonic effects?
marginalization or include gas?	marginalization or include gas?
Can we (ever) reach 1% accuracy? maybe; beat cosmic variance [see Angular & Pontzen 2016]	No need for 1% accuracy
How to convince critics that marginalization is sufficiently general?	No marginalization

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