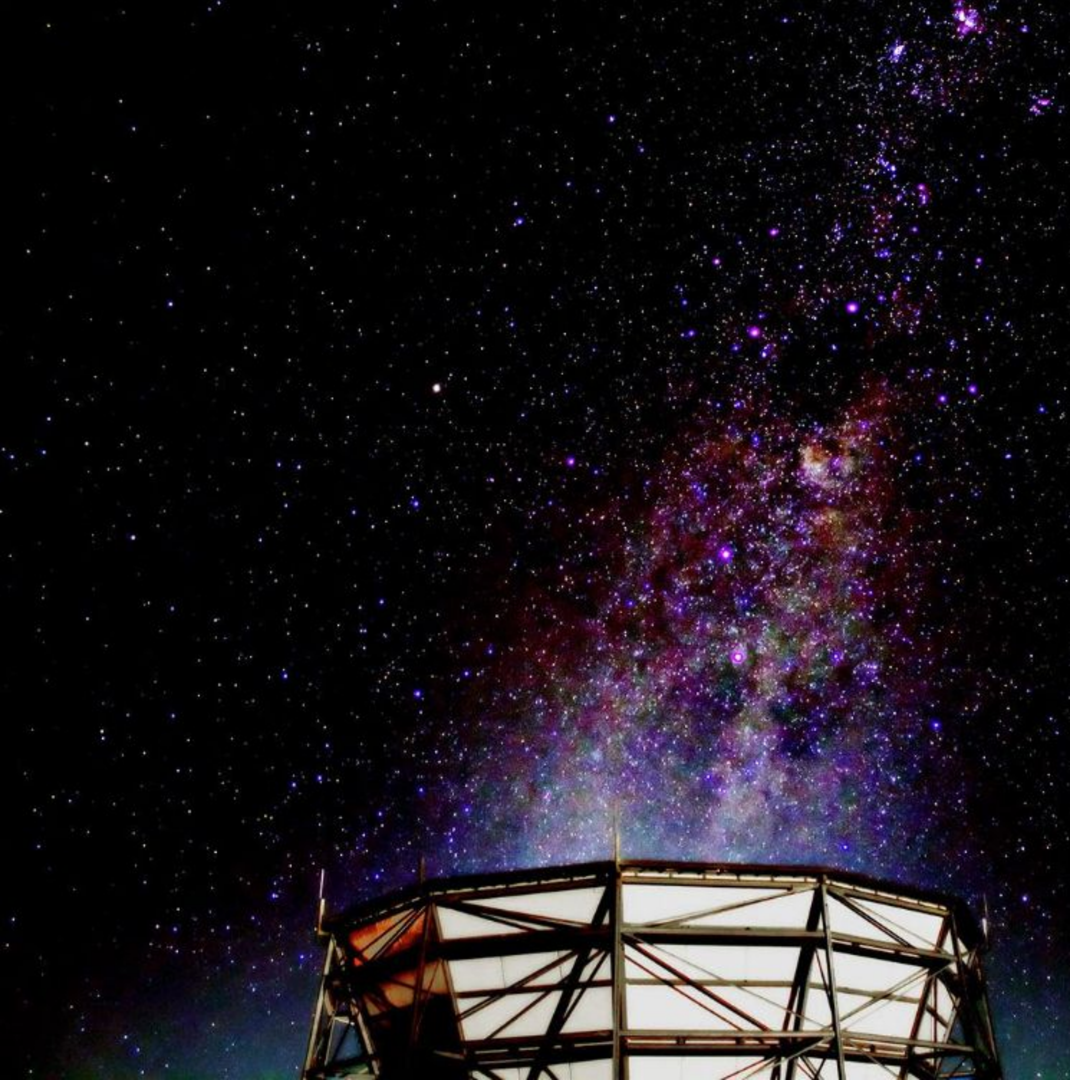


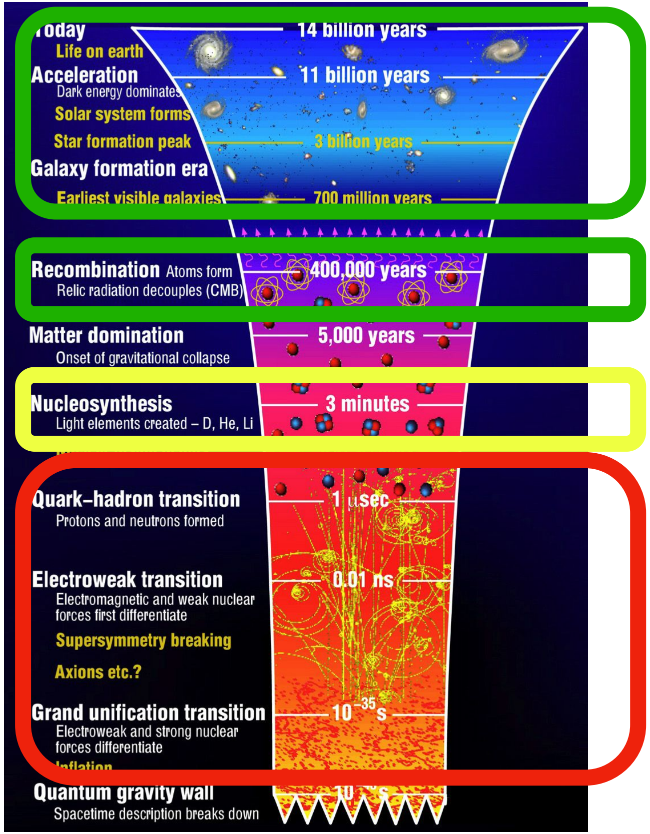
Inflationary Insights with the CMB

Joel Meyers

Michigan Cosmology Summer School 2023
6-6-2023

Image Credit: ACT / Princeton

History of the Universe



Direct Observations

Indirect Sensitivity

Discovery Space

Cosmology describes 13.8 billion years of cosmic history

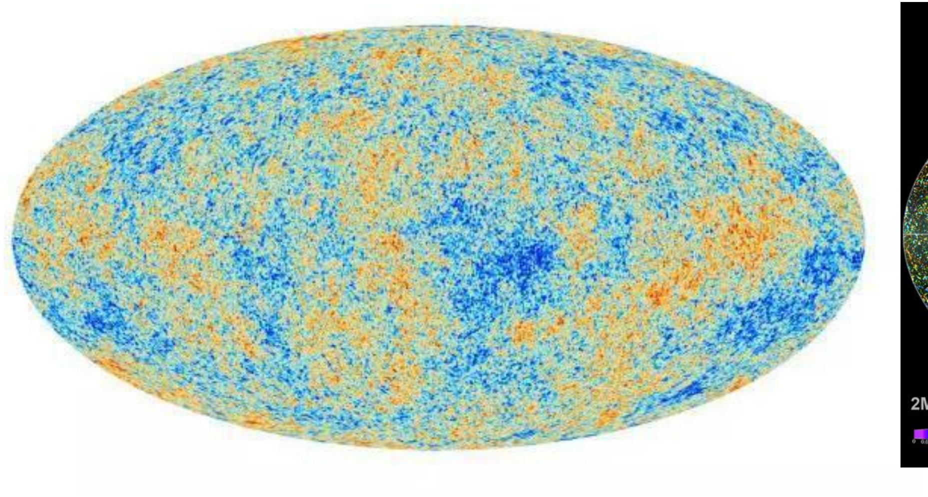
Direct observations are limited to a relatively small range of redshifts and energy scales

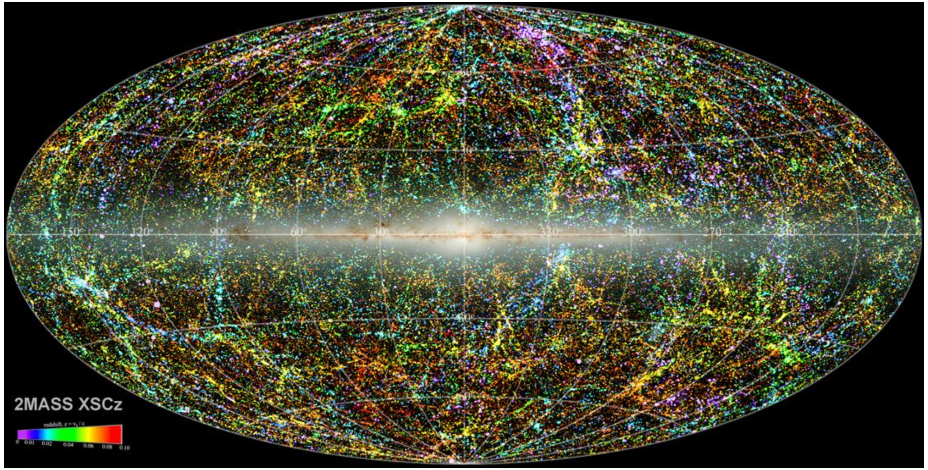
The CMB carries imprints from much earlier times and much higher energies:

- Lecture 1: Light Relics
- **Lecture 2: Inflation**

Image Credit: Center for Theoretical Cosmology, Cambridge

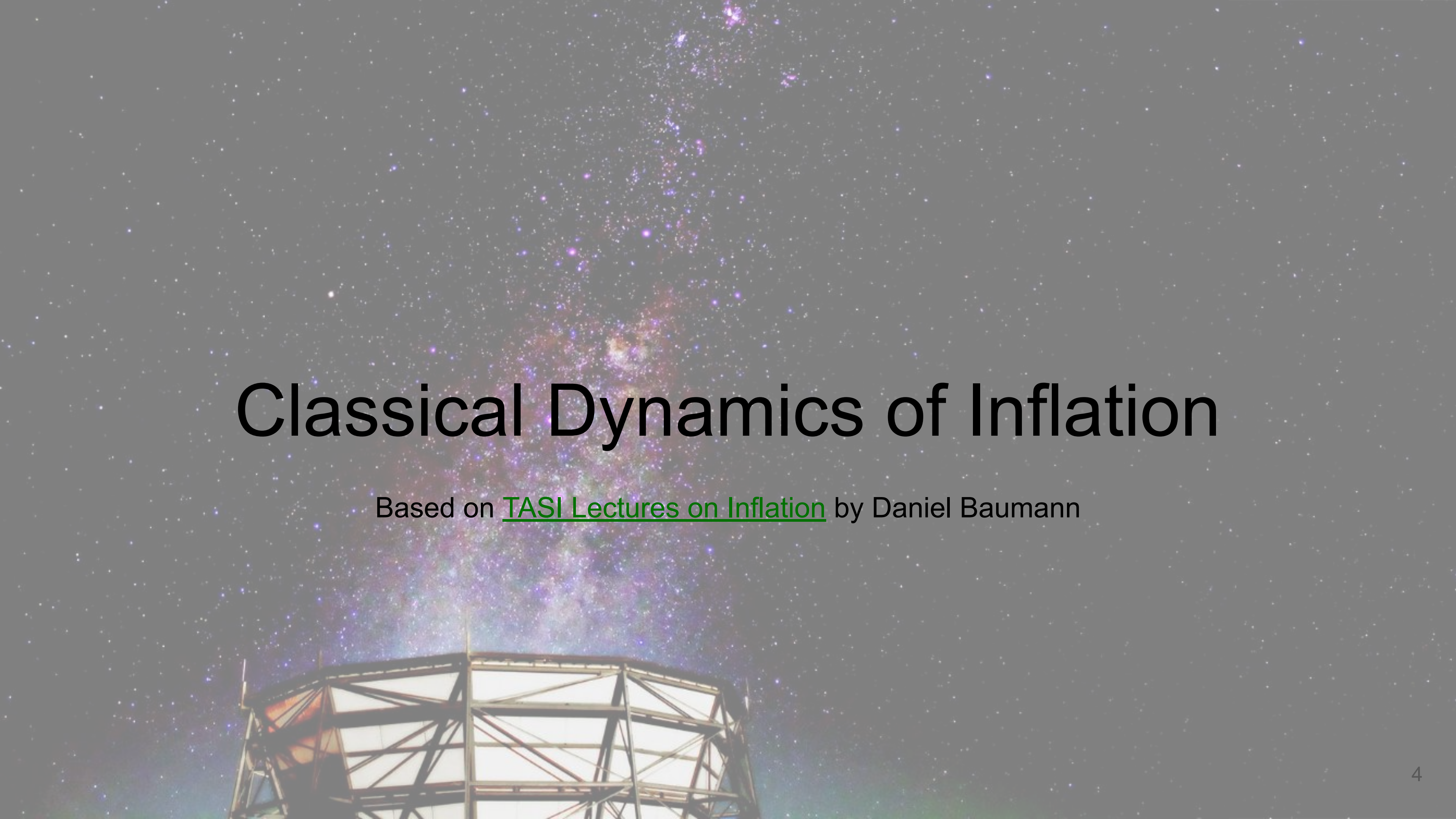
The Question of Initial Conditions





- Why is the universe so homogenous on large scales?
- What sourced the fluctuations in density that seeded structure growth?

Image Credits: Planck, 2MASS



Classical Dynamics of Inflation

Based on TASI Lectures on Inflation by Daniel Baumann

Horizon Problem

- In the classical Hot Big Bang model, where the universe always contained only matter and radiation, the comoving Hubble sphere always grows:

$$\tau \equiv \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da}{Ha^2} = \int_0^a d\ln a \left(\frac{1}{aH}\right)$$

$$(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3w)} \qquad \tau \propto a^{\frac{1}{2}(1+3w)}$$

- In such a scenario, one would expect ~10^5 causally disconnected regions at the surface of last scattering

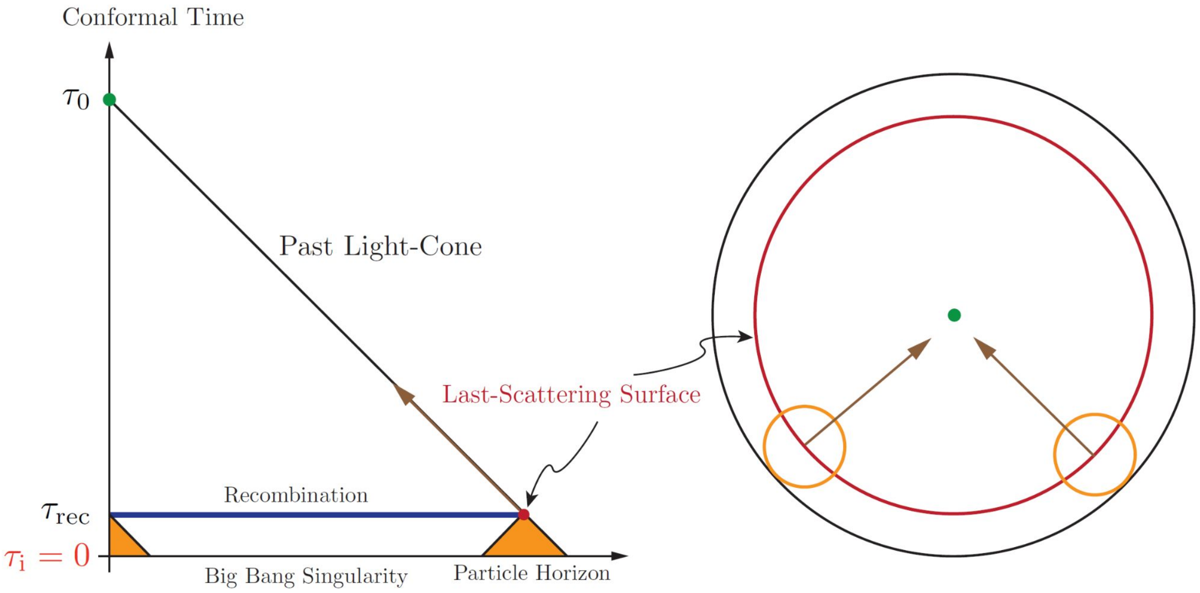
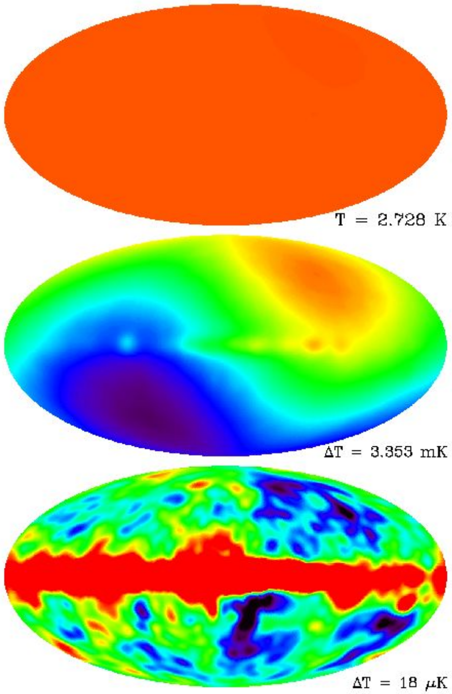


Image Credit: Baumann

The CMB is Observed to Be Very Uniform



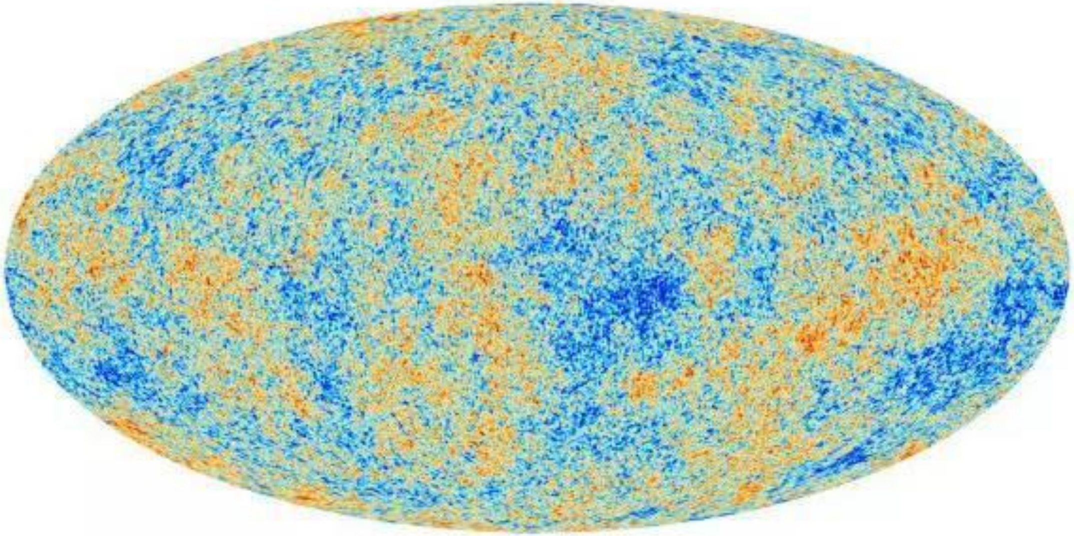


Image Credits: COBE DMR, Planck

Flatness Problem

- In the classical Hot Big Bang model, the deviation from spatial flatness grows with time

$$1 - \Omega(a) = \frac{-k}{(aH)^2}$$

$$\frac{d|\Omega - 1|}{d \ln a} > 0 \quad \Leftrightarrow \quad 1 + 3w > 0$$

- The observed flatness of the current universe would require extreme flatness in the distant past

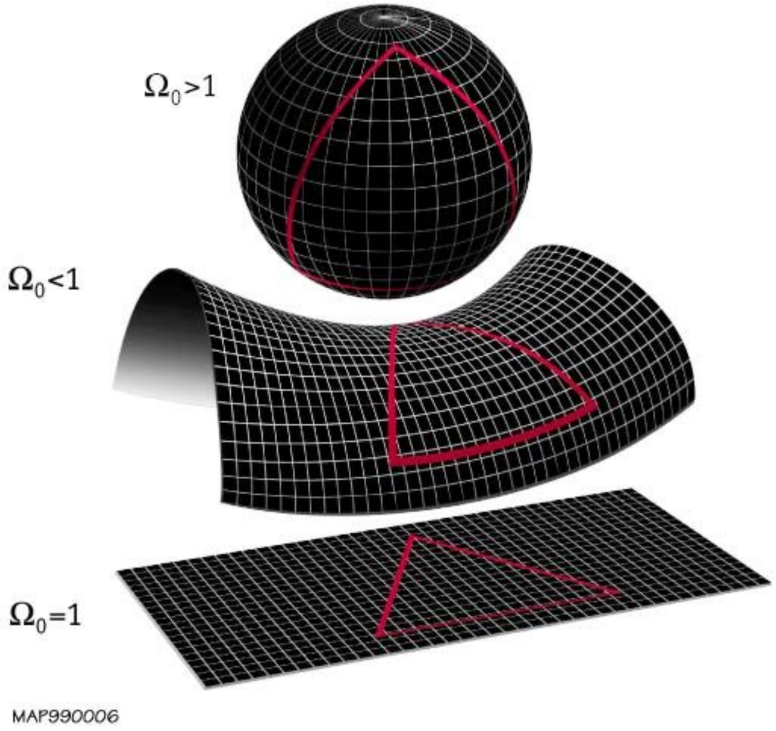


Image Credit: NASA / WMAP

Inflationary Solution - Shrinking Hubble Sphere

- Inflation solves the Horizon and Flatness problems by providing a period with a shrinking Hubble sphere preceding the Hot Big Bang evolution

$$\frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0 \quad \Rightarrow \quad \frac{d^2 a}{dt^2} > 0 \quad \Rightarrow \quad \rho + 3p < 0$$

- The Hubble sphere shrinks during accelerated expansion, which can be achieved with negative pressure

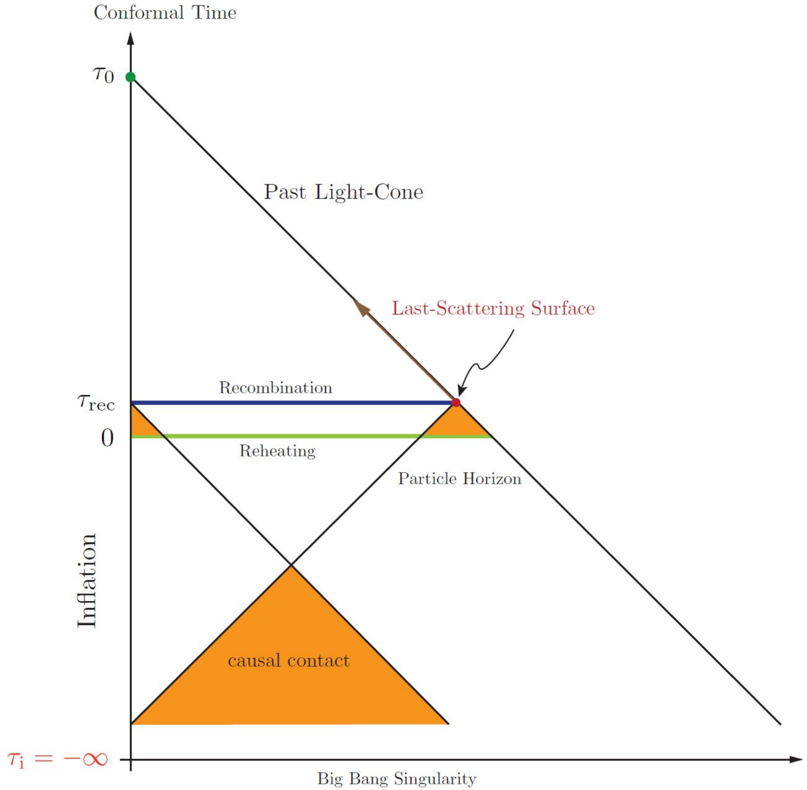


Image Credit: Baumann

Simple Model of Inflation - Single Scalar Field

- Negative pressure can be achieved with a scalar field whose energy density is dominated by its potential

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- A scalar field in an expanding universe acts like a ball rolling on a hill subject to friction

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

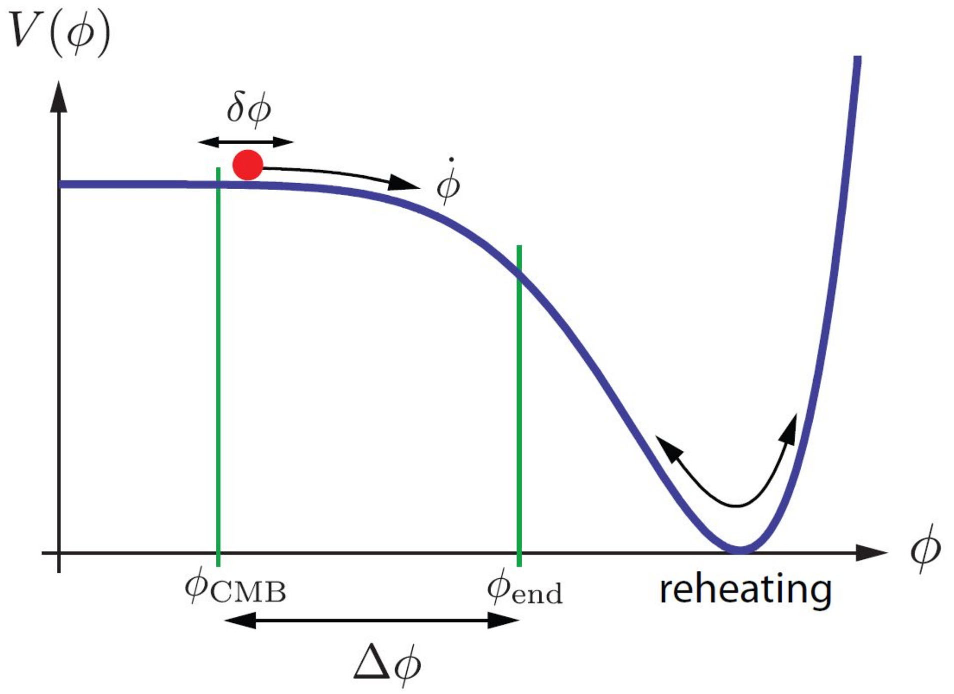


Image Credit: Baumann

Slow-Roll Inflation

- A sufficiently long period of accelerated expansion can be achieved if the scalar field obeys the slow-roll conditions

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{1}{2}\frac{\dot{\phi}^2}{H^2}$$

$$\varepsilon, |\eta| < 1$$

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} = \varepsilon - \frac{1}{2\varepsilon}\frac{d\varepsilon}{dN}$$

- The horizon and flatness problems are solved with enough inflationary expansion

$$N(\phi) \equiv \ln\frac{a_{\rm end}}{a} = \int_t^{t_{\rm end}} H\,\mathrm{d}t = \int_\phi^{\phi_{\rm end}} \frac{H}{\dot{\phi}}\mathrm{d}\phi$$

$$N_{\rm tot} \equiv \ln\frac{a_{\rm end}}{a_{\rm start}} \gtrsim 60$$

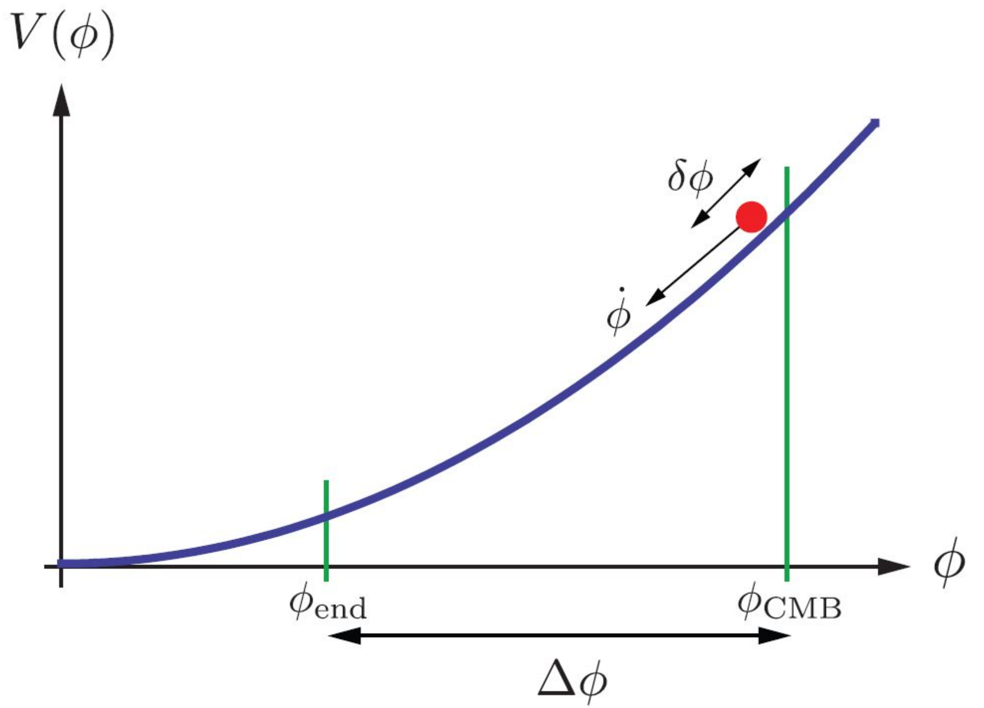


Image Credit: Baumann

Slow-Roll Approximation

- The conditions on the expansion can be reframed as conditions on the scalar field potential

$$\epsilon_V(\phi) \equiv \frac{M_{pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta_V(\phi) \equiv M_{pl}^2 \frac{V_{,\phi\phi}}{V}$$

$$\varepsilon \approx \epsilon_V \qquad \eta \approx \eta_V - \epsilon_V$$

- Successful slow-roll inflation requires a sufficiently flat potential
- Inflationary equations of motion simplify in slow-roll regime

$$\dot{\phi} \approx -\frac{V_{,\phi}}{3H}$$

$$H^2 \approx \frac{1}{3}V(\phi) \approx \text{const.}$$

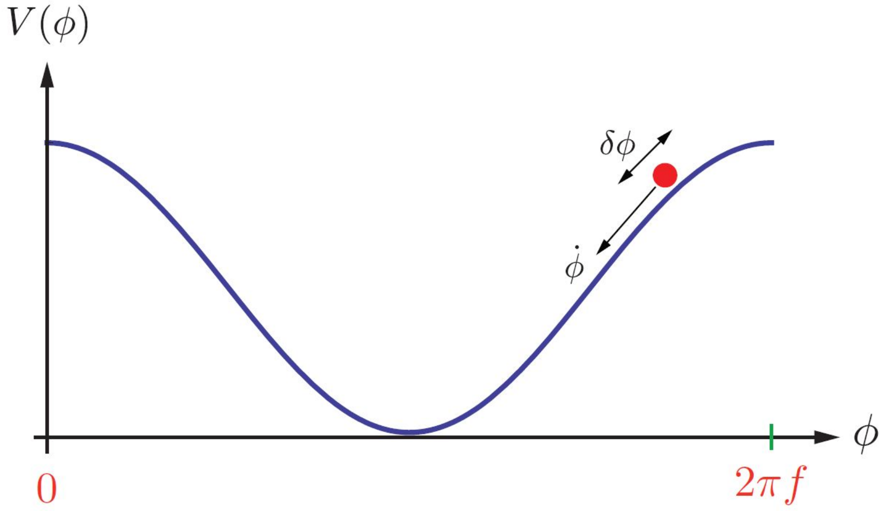


Image Credit: Baumann

Quantum Fluctuations During Inflation

Based on TASI Lectures on Inflation by Daniel Baumann

Fluctuations and Horizon Exit

- It is convenient to Fourier transform perturbations to the scalar field and the spacetime metric, since each mode evolves independently at linear order

$$\delta X(t, \mathbf{x}) \equiv X(t, \mathbf{x}) - \bar{X}(t)$$

$$X_{\mathbf{k}}(t) = \int \mathrm{d}^3\mathbf{x}\; X(t, \mathbf{x})\, e^{i\mathbf{k}\cdot\mathbf{x}}$$

- The shrinking Hubble sphere during inflation causes fluctuation modes to exit the horizon during inflation and to re-enter the horizon after inflation

$$\text{subhorizon}: \quad k \gg aH$$

$$\text{superhorizon}: \quad k < aH$$

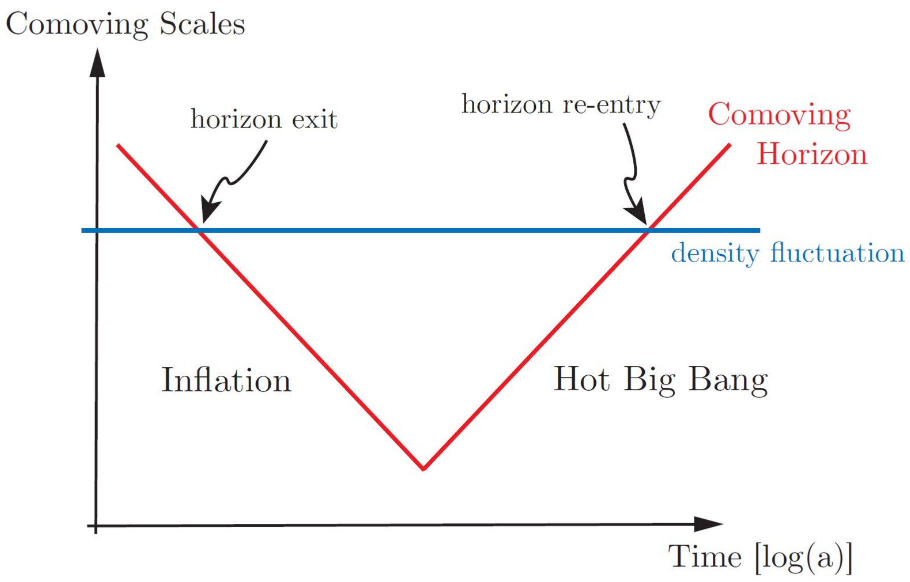


Image Credit: Baumann

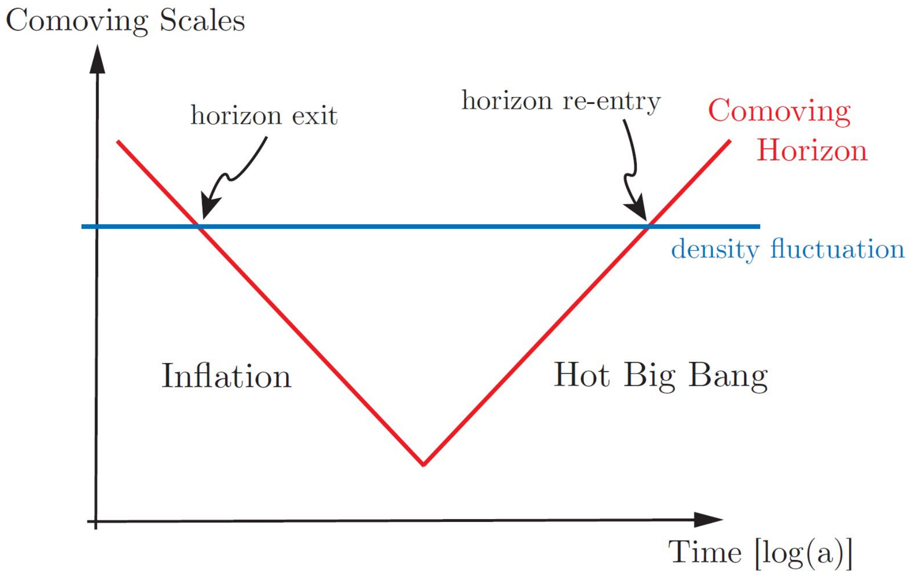
Superhorizon Conservation

- We are primarily interested in scalar and tensor perturbations, described by the gauge-invariant quantities

$$\mathcal{R} = \Psi + \frac{H}{\dot{\bar{\phi}}}\delta\phi \qquad\qquad h^i_i = \partial^i h_{ij} = 0$$

- These quantities are conserved on superhorizon scales during and after single-field inflation

$$\dot{\mathcal{R}} = -\frac{H}{\bar{\rho}+\bar{p}}\delta p_{en} + \frac{k^2}{(aH)^2}\left(\dots\right)$$



Weinberg (2004); Image Credit: Baumann

From Quantum Fluctuations to Cosmological Perturbations

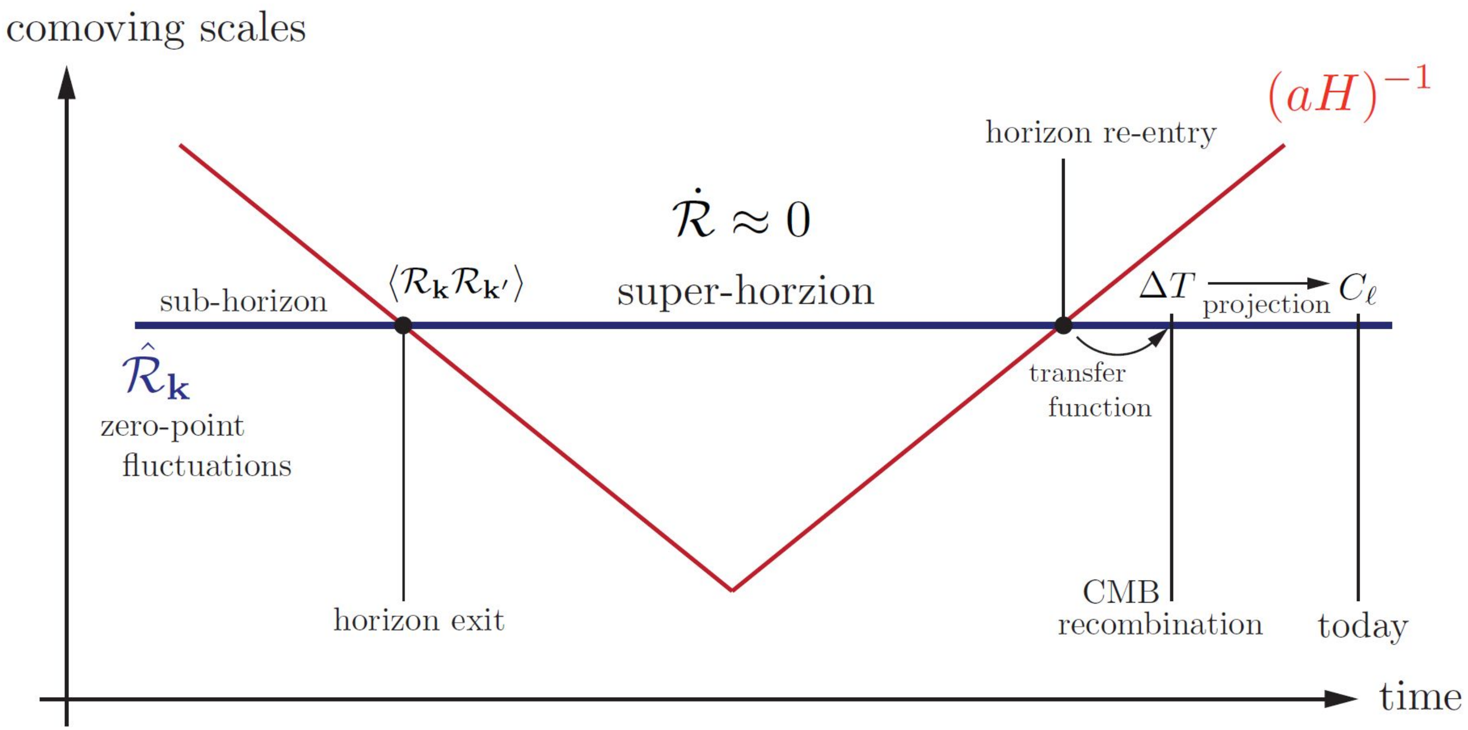


Image Credit: Baumann

Quantum Vacuum Fluctuations

- We are interested in the statistics of the curvature and metric perturbations at horizon exit

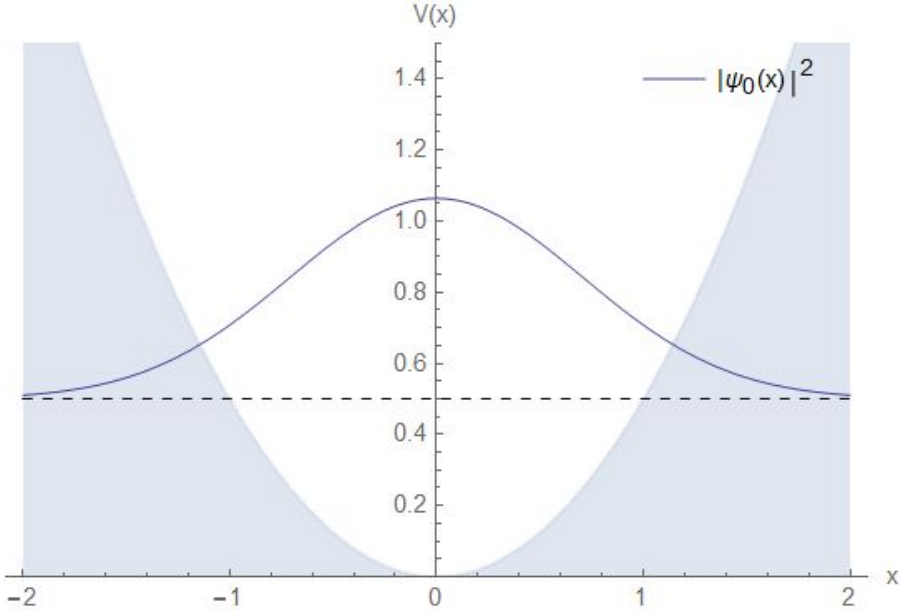
$$\langle \mathcal{R}\mathcal{R} \rangle = \int_0^\infty \Delta^2_{\mathcal{R}}(k)\, \mathrm{d}\ln k$$

- Scalar fields which are light during inflation (m<<H) experience quantum fluctuations with variance fixed by the Hubble rate

$$\Delta^2_{\delta\phi} = \left(\frac{H}{2\pi}\right)^2$$

- Perturbations to the inflaton change the duration of inflation

$$\mathcal{R} = H\frac{\delta\phi}{\dot{\phi}} \equiv -H\delta t$$



Quantum Harmonic Oscillator Ground State

Primordial Power Spectra

- The scalar spectrum is determined by the expansion rate and slow-roll parameter at horizon exit

$$\Delta^2_{\mathcal{R}}(k) = \frac{H_\star^2}{(2\pi)^2}\frac{H_\star^2}{\dot\phi_\star^2} = \frac{1}{8\pi^2}\frac{H^2}{M_{\rm pl}^2}\frac{1}{\varepsilon}\bigg|_{k=aH}$$

- Each polarization of tensor fluctuation acts just like a light scalar field, giving a fluctuation amplitude directly determined by the expansion rate

$$\Delta^2_{\rm t}(k) \equiv 2\Delta^2_h(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\rm pl}^2}\bigg|_{k=aH}$$

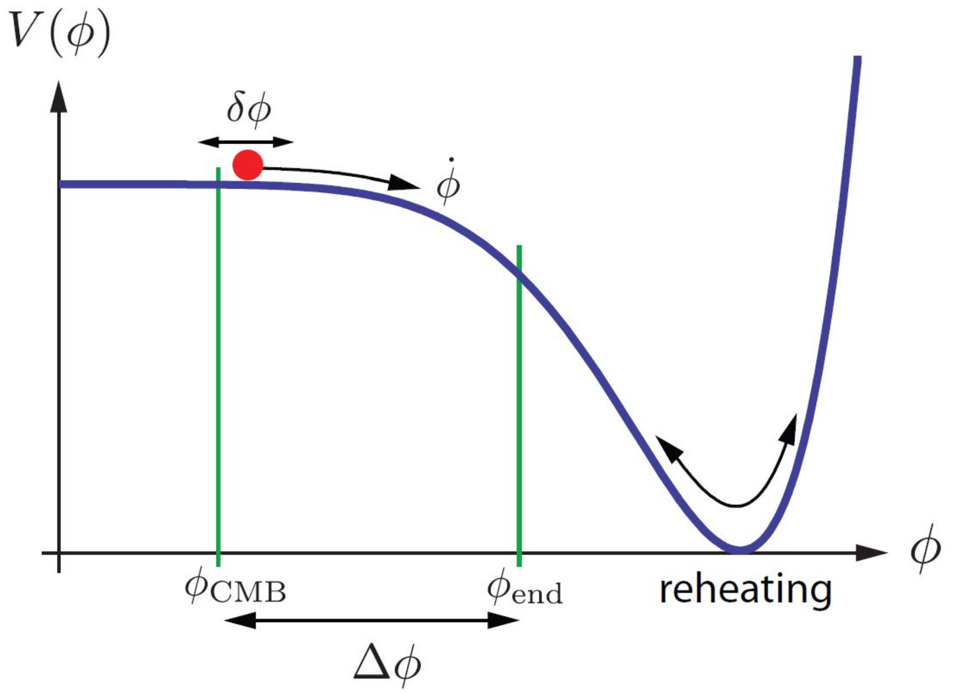


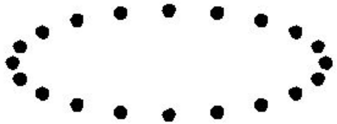
Image Credit: Baumann

Primordial Tensor Spectrum

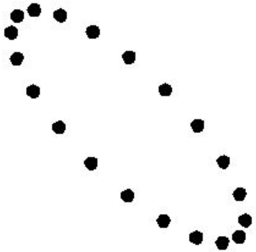
- A spectrum of primordial gravitational waves is a generic prediction of inflation
- Measuring the amplitude of primordial gravitational waves would indicate the energy scale of inflation
- The tensor-amplitude is often reported in terms of the tensor-to-scalar ratio

$$r \equiv \frac{\Delta_{\rm t}^2(k)}{\Delta_{\rm s}^2(k)} \qquad \Delta_{\mathcal{R}}^2 \sim 10^{-9}$$

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\,\mathrm{GeV}$$



Plus polarization



Cross polarization

Spectral Tilt

- The expansion rate and field velocity change during inflation, giving a scale-dependence to the fluctuation amplitude

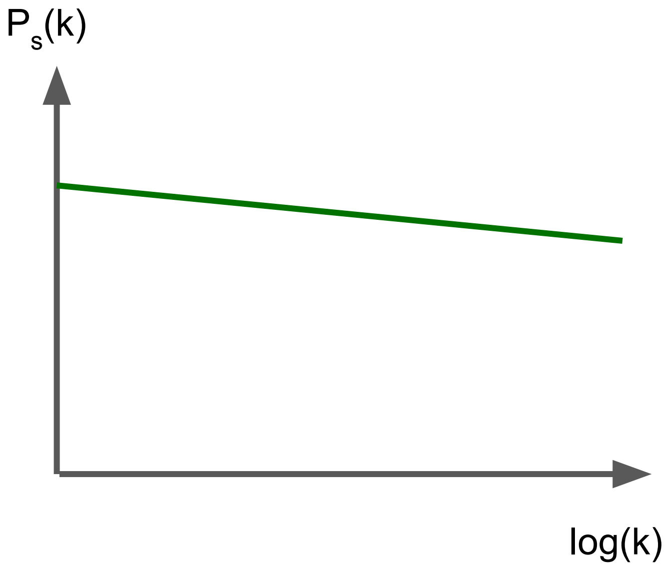
$$n_{\rm s} - 1 \equiv \frac{d \ln \Delta_{\rm s}^2}{d \ln k} \qquad n_{\rm t} \equiv \frac{d \ln \Delta_{\rm t}^2}{d \ln k}$$

- The spectral tilt can be straightforwardly calculated in the slow-roll approximation

$$n_{\rm s} - 1 = 2\eta_{\rm v}^\star - 6\epsilon_{\rm v}^\star \qquad n_{\rm t} = -2\epsilon_{\rm v}^\star$$

- All single-field slow-roll models of inflation obey a consistency relation

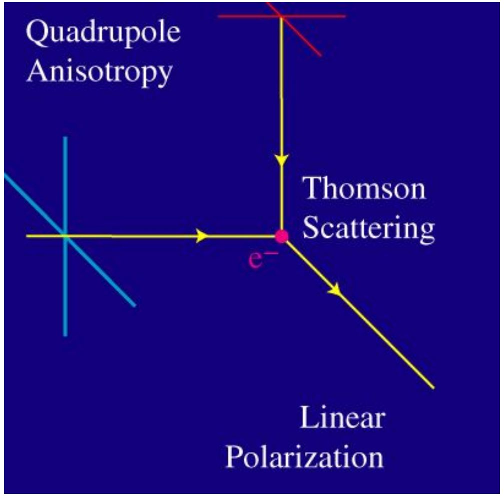
$$r = 16\epsilon_{\rm v}^\star \qquad r = -8n_{\rm t}$$



Signals of Inflation in the CMB

Linear Polarization of the CMB

- Thomson scattering of CMB photons induces a linear polarization when the incident radiation exhibits a quadrupole anisotropy
- The polarization of the CMB can be characterized in terms of the Stokes parameters Q and U



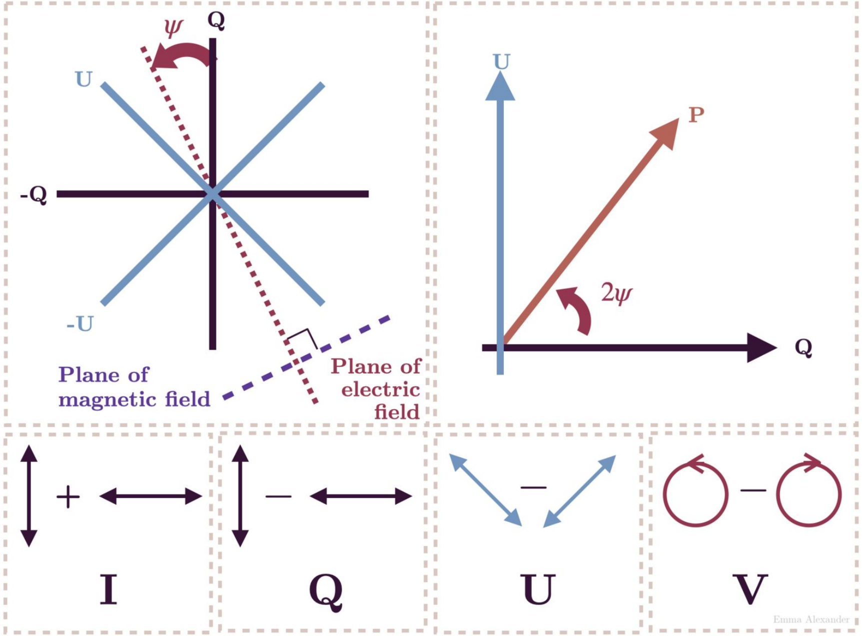
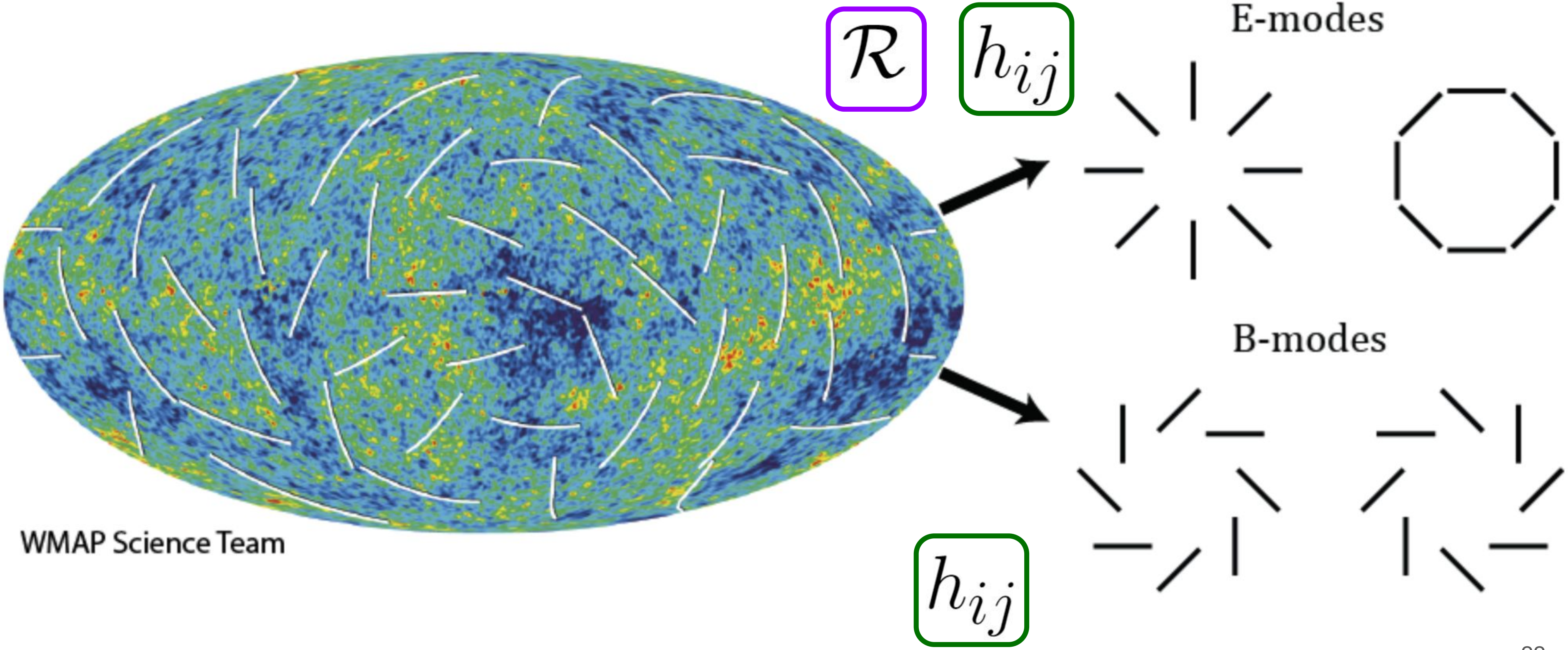


Image Credits: Alexander, Hu

E and B Modes



Kamionkowski, Kosowsky, Stebbins (1997); Zaldarriaga, Seljak (1997); Image Credit: PIPER (2014)

T, E, and B Spectra

- We have precise measurements of temperature and E-mode polarization spectra that agree with the primordial scalar spectrum from inflation (nearly scale-invariant with small red tilt)
- We also measure the B-mode polarization spectrum, though unfortunately not yet from primordial gravitational waves

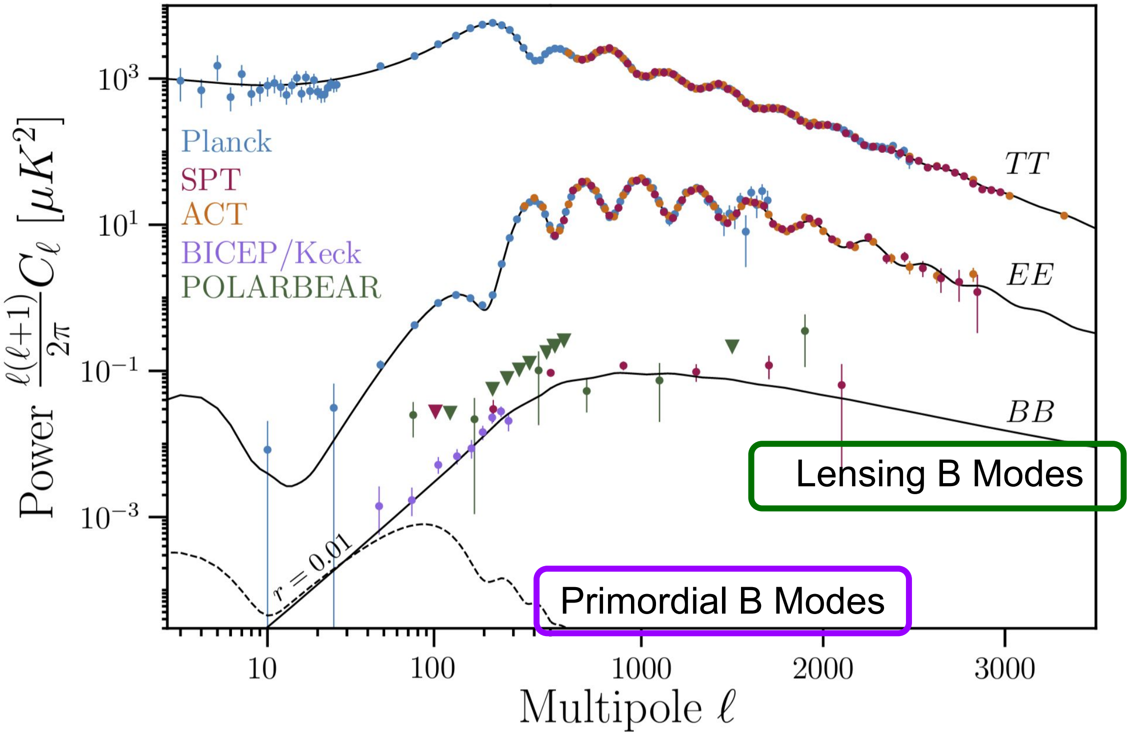
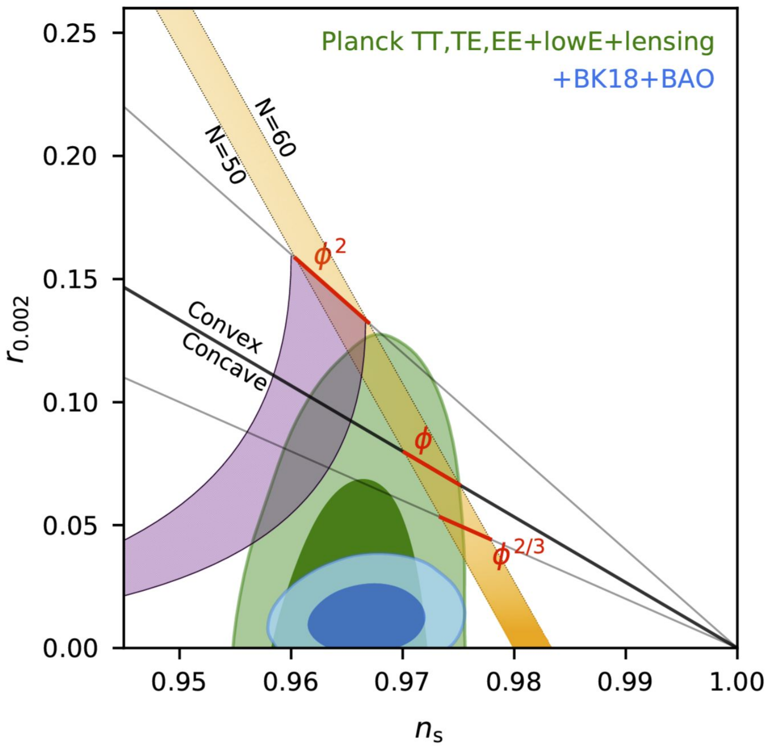


Image Credit: Chang, Huffenberger, et al (2022)

Current Constraints

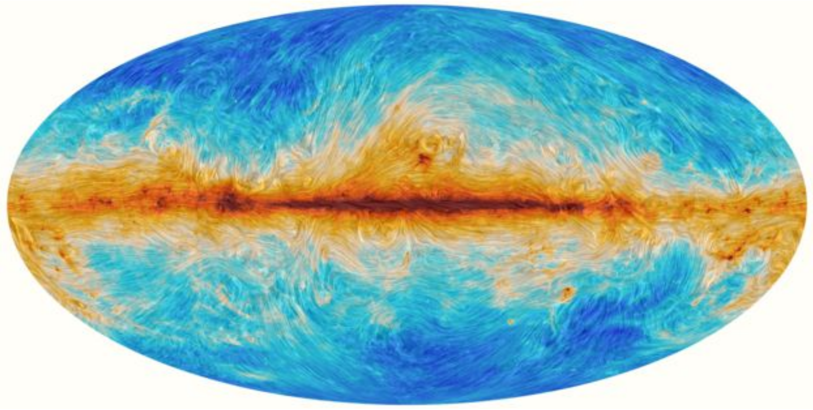
- The scalar spectral tilt n_s and tensor-to-scalar ratio r are predicted by each inflation model, specifying a point in the n_s - r plane
- Observational constraints therefore test specific models of inflation
- The simplest ϕ^2 model of inflation is already strongly disfavored by current observations



BICEP / Keck / Planck (2021)

Challenges for Detecting Primordial B Modes

- **Small Signal** - The primordial B-mode power is inherently small, and we do not know the amplitude
  - Requires very deep observations
- **Polarized Foregrounds** - Astrophysical foregrounds such as Galactic dust produce polarized emission
  - Requires multi-frequency observations
- **Gravitational Lensing** - Deflection of CMB photons converts E modes to B modes (See Gil's Lecture after lunch)
  - Requires delensing



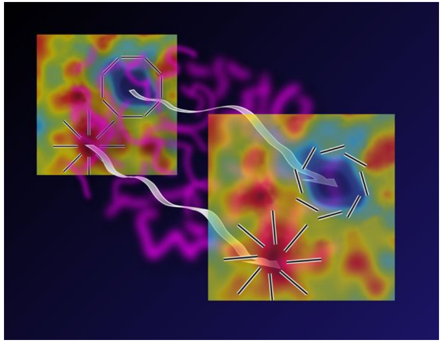
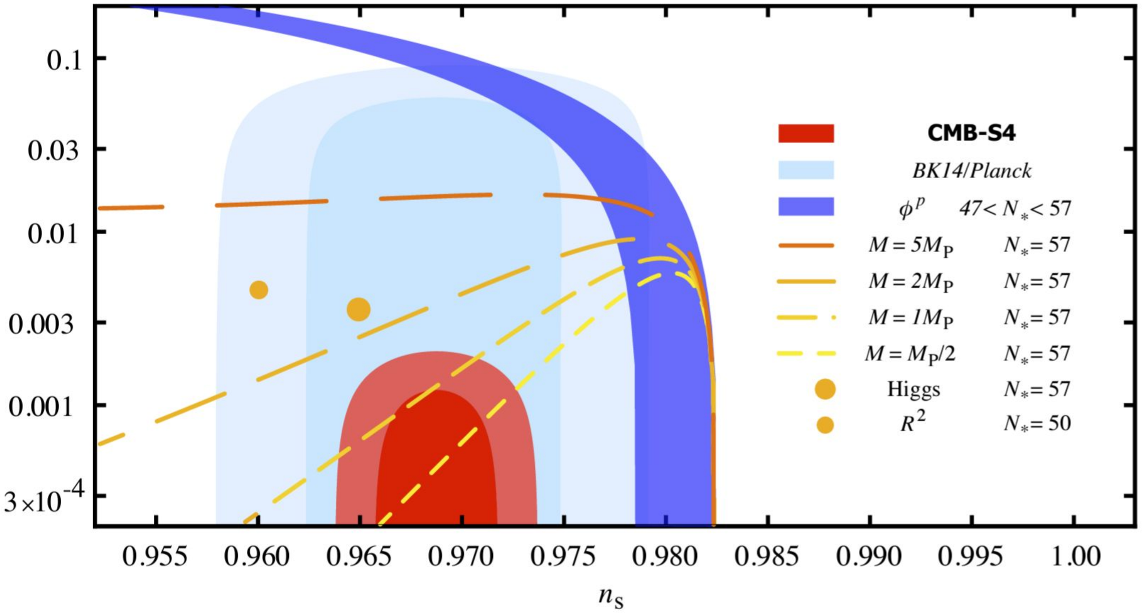


Image Credits: Planck, APS

Future Prospects for Detecting Primordial B Modes

- Future surveys are designed to detect primordial gravitational waves or rule out a broad class of models that naturally explains the deviation from scale invariance
- CMB-S4 is designed to either achieve a 5σ measurement of r>0.003 or set an upper limit of r<0.001 at 95% confidence



CMB-S4 (2016)

Impacts of Detecting Primordial Gravitational Waves

- Direct probe of earliest times
- Evidence of quantization of gravity
- Strong evidence for inflation
- Determines energy scale of inflation
- New characteristic energy scale near GUT scale

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\,\mathrm{GeV}$$

- Rules out small-field models of inflation

$$r = \frac{8}{M_{\rm pl}^2}\left(\frac{d\phi}{dN}\right)^2 \qquad \frac{\Delta\phi}{M_{\rm pl}} = \int_{N_{\rm end}}^{N_{\rm cmb}} \mathrm{d}N \sqrt{\frac{r}{8}} = \mathcal{O}(1) \times \left(\frac{r}{0.01}\right)^{1/2}$$

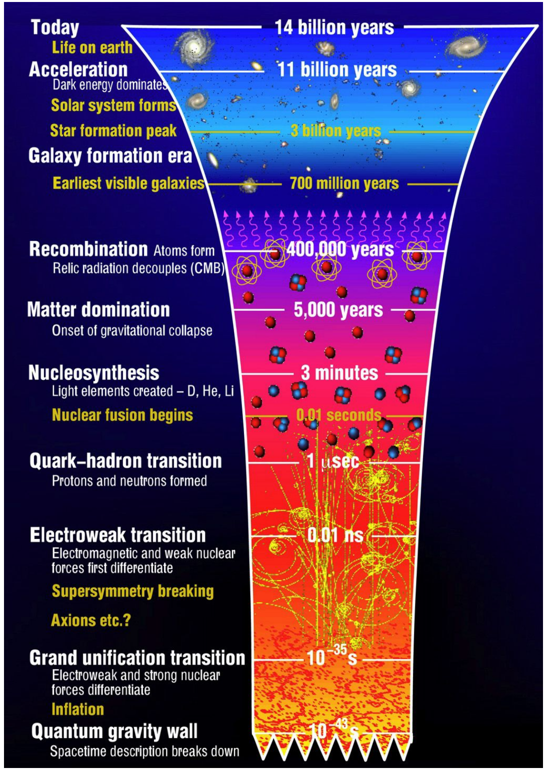


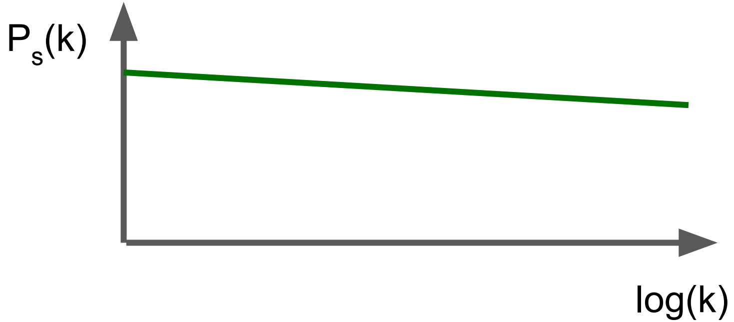
Image Credit: Center for Theoretical Cosmology, Cambridge

Primordial Non-Gaussianity

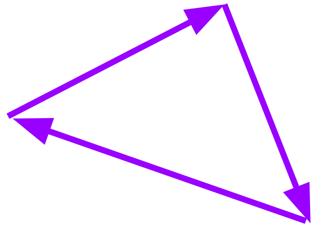
Beyond the Power Spectrum

- The simplest models of inflation predict nearly Gaussian scalar fluctuations
- Fluctuations can be non-Gaussian, predicting higher-order correlations of curvature perturbations
- In principle, much more information about the details of inflation is contained in the higher-order correlations

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) P_{\mathcal{R}}(k_1)$$



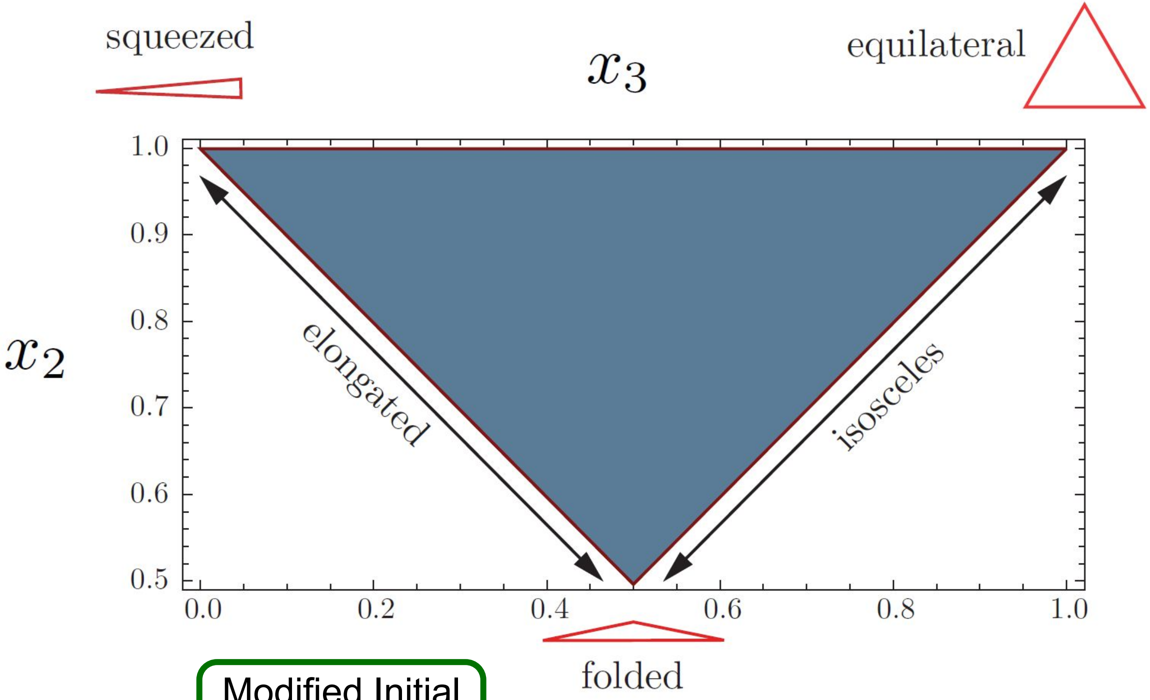
$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_{\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$



Bispectrum Shapes

Multiple Fields

Derivative Interactions

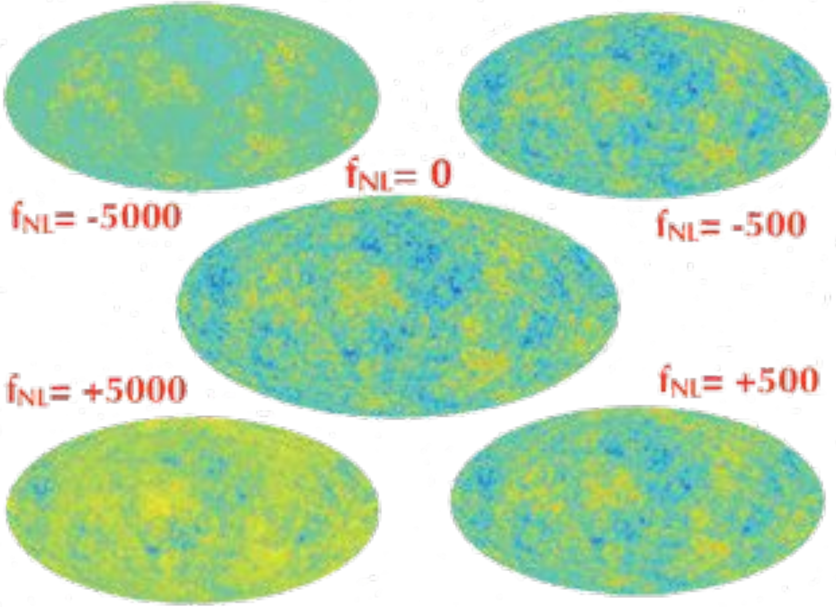


Modified Initial States

Image Credit: Baumann

Local (Squeezed) Non-Gaussianity

$$\mathcal{R}(\mathbf{x}) = \mathcal{R}_g(\mathbf{x}) + \frac{3}{5} f_{\rm NL}^{\rm local} \mathcal{R}_g(\mathbf{x})^2$$



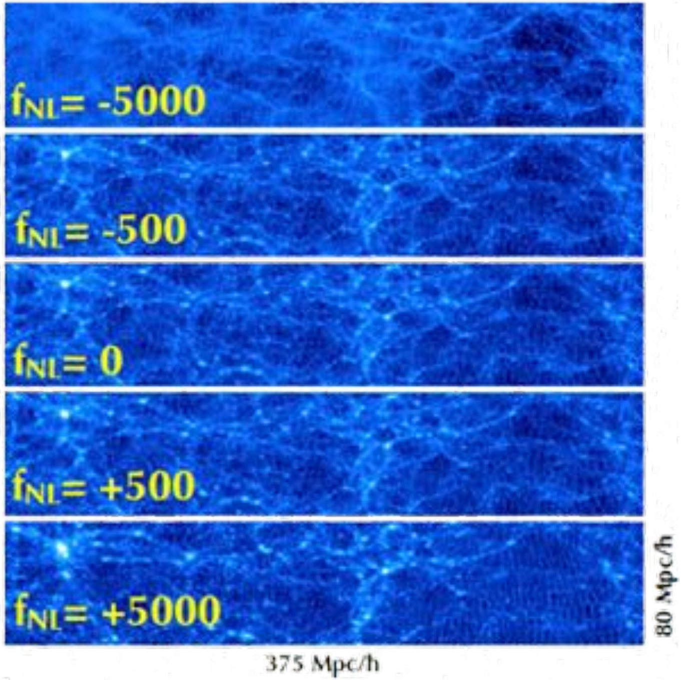
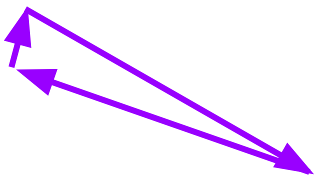
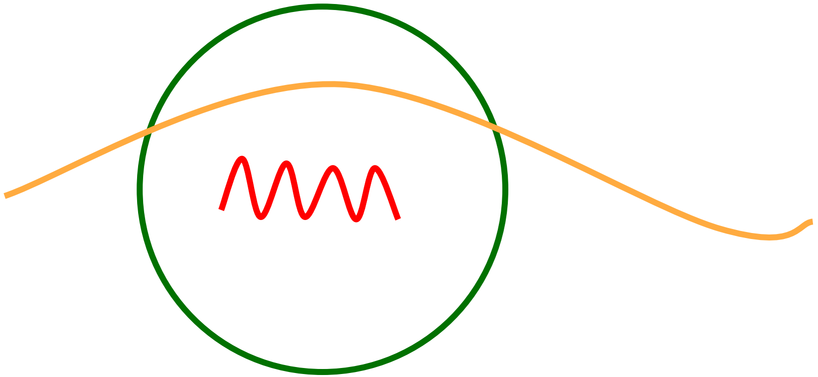


Image Credit: U Michigan Workshop 2011

Single-Field Consistency Condition

- *All* models of inflation with a single clock predict negligible squeezed bispectrum
- Long wavelength curvature fluctuations are locally indistinguishable from a rescaling of coordinates
- Small-scale power cannot correlate with long wavelength fluctuations apart from projection effects
- A detection of local (squeezed) non-Gaussianity would rule out all single field models of inflation



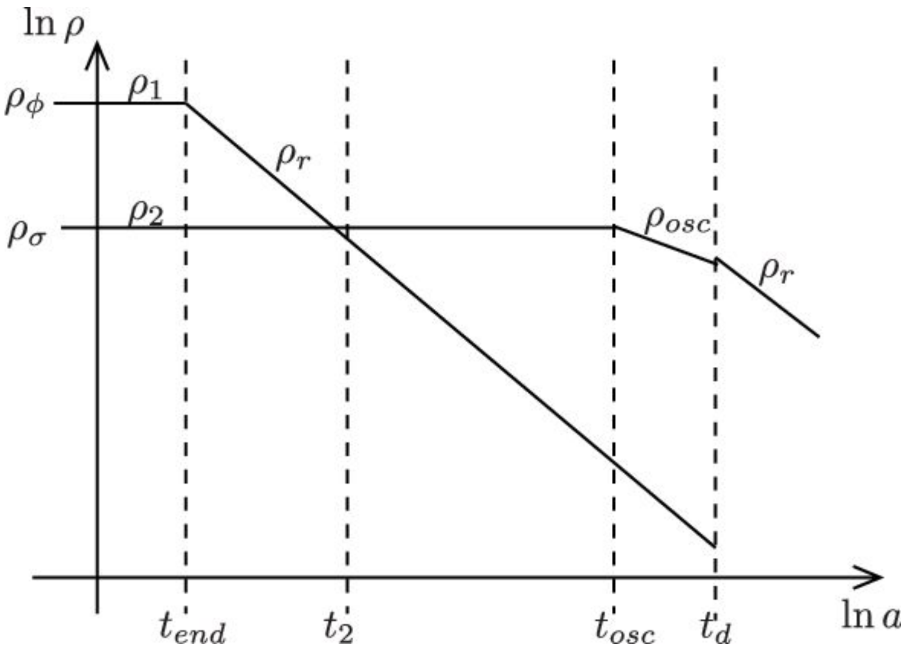


Maldacena (2002); Creminelli, Zaldarriaga (2004)

Models Predicting Local Non-Gaussianity

- **Curvaton Scenario** - Spectator field is frozen due to Hubble friction during inflation, oscillates and redshifts like matter after inflation, decays to radiation

- **Modulated Reheating** - Value of spectator field determines rate of decay of inflation into radiation



Linde, Mukhanov (1997); Enqvist, Sloth (2002); Lyth, Wands (2002)
Kofman (2003); Dvali, Gruzinov, Zaldarriaga (2004)

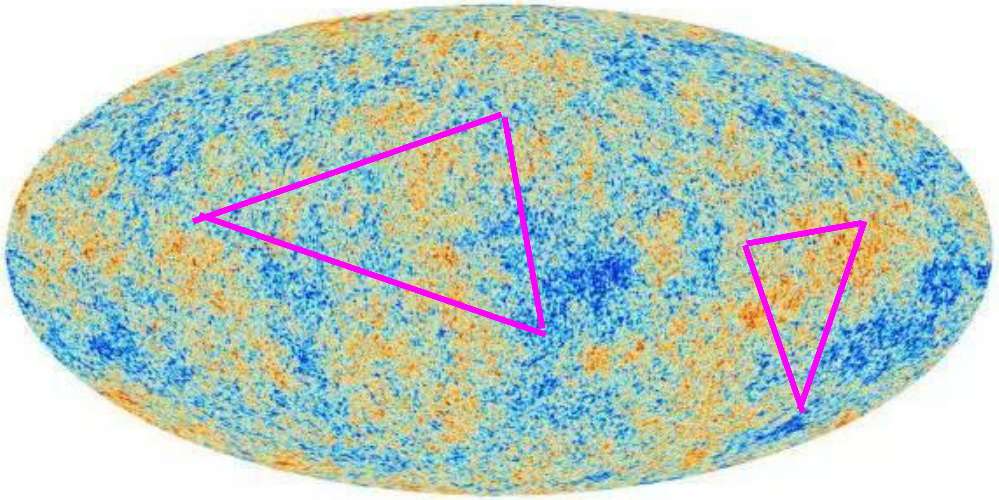
Measuring Non-Gaussianity in the CMB

Current Planck constraints:

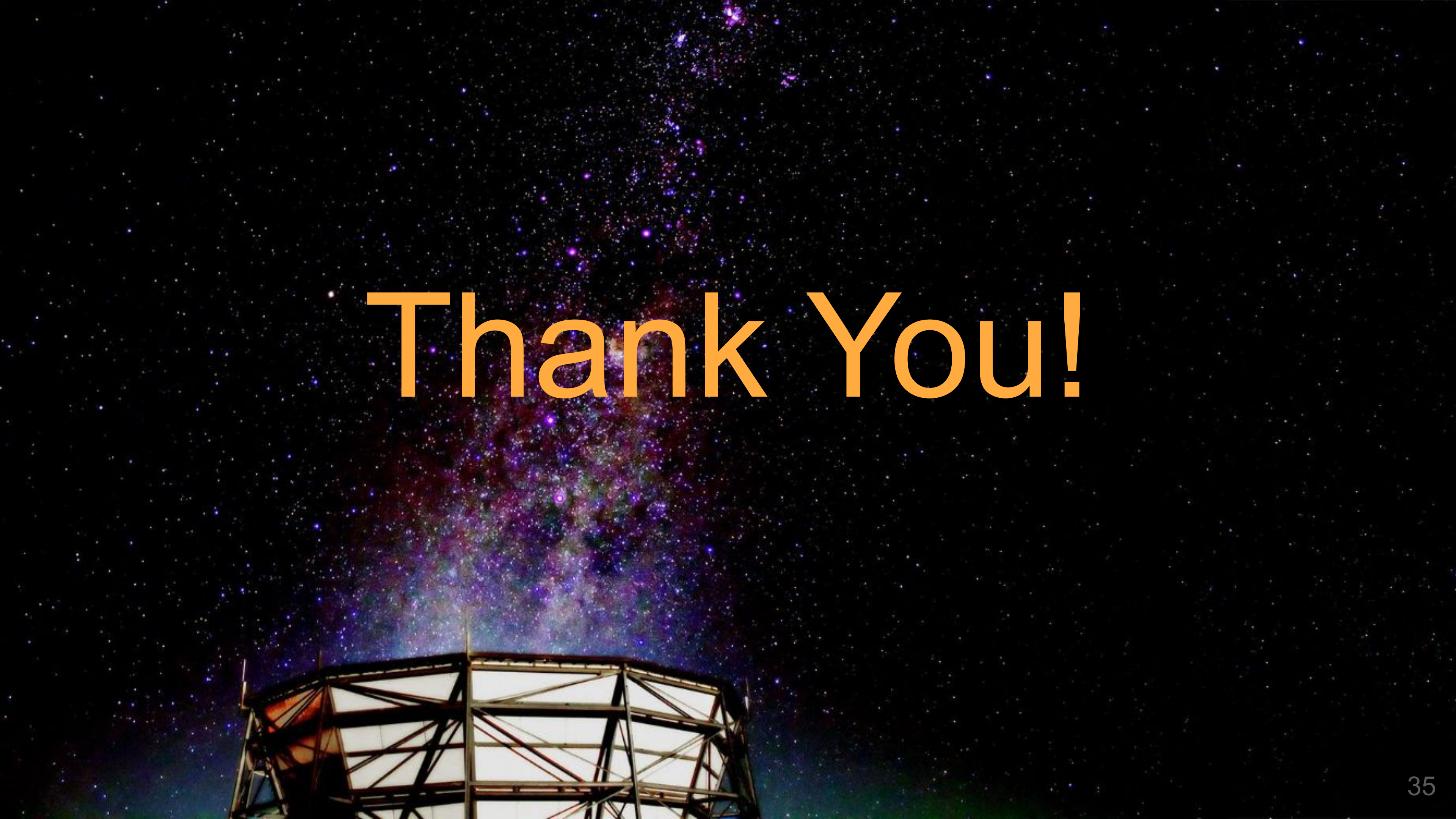
$$f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$$
$$f_{\rm NL}^{\rm equil} = -26 \pm 47$$
$$f_{\rm NL}^{\rm ortho} = -38 \pm 24$$

Future CMB-S4 constraints:

$$\sigma(f_{\rm NL}^{\rm local}) = 2.6$$
$$\sigma(f_{\rm NL}^{\rm equil}) = 21.2$$
$$\sigma(f_{\rm NL}^{\rm ortho}) = 9.1$$

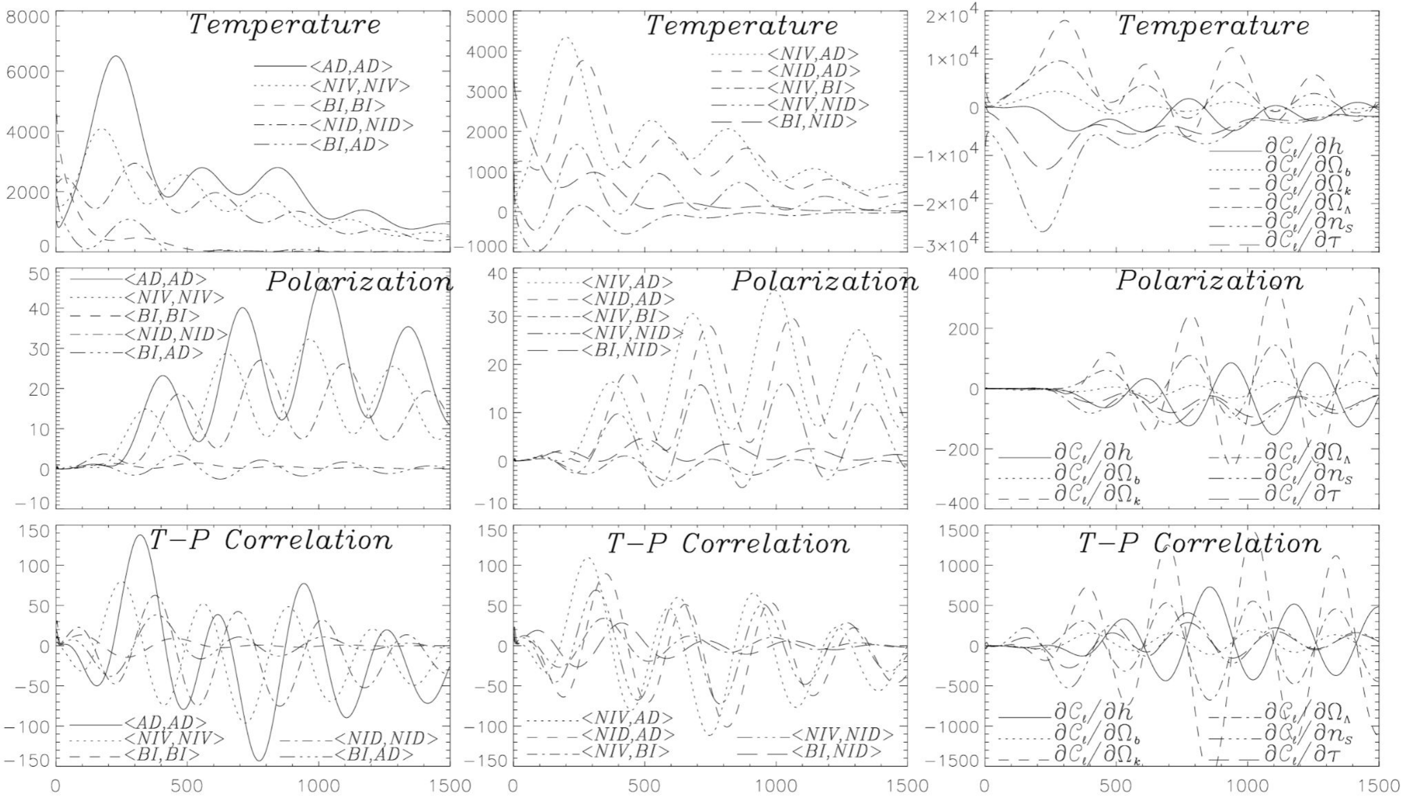


Planck (2018); CMB-S4 (2016)


Thank You!

Backup Slides

Single Field Inflation Predicts Adiabatic Fluctuations



Bucher, Moodley, Turok (2000)