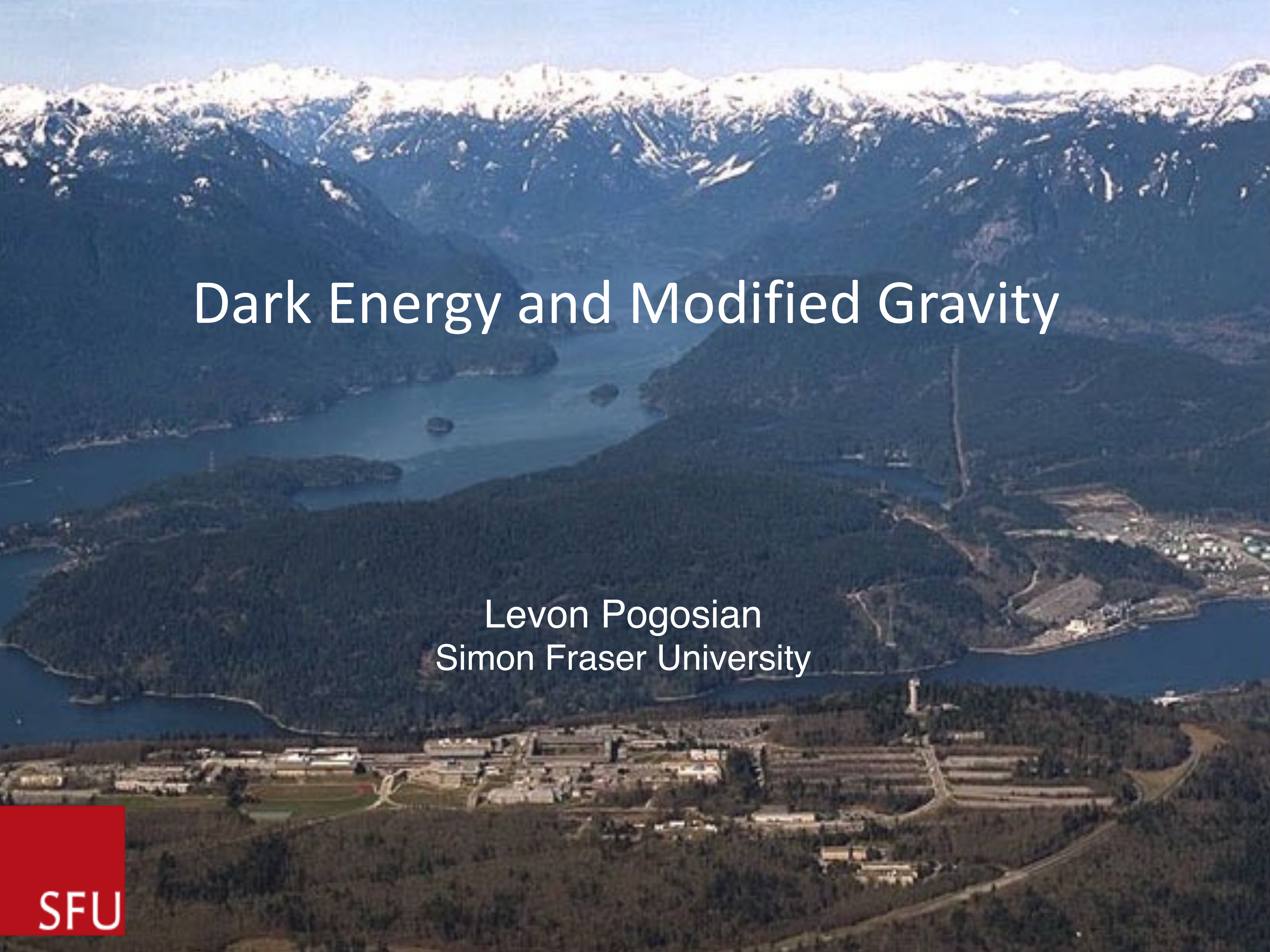

Dark Energy and Modified Gravity

Levon Pogosian
Simon Fraser University

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Part I: What is wrong with Lambda?

Part II: Dark Energy and its equation of state

Part III: Modified gravity and its phenomenology

Part IV: What can Cosmology tell us about gravity?

Part III

Modified Gravity

How special is General Relativity?

Lovelock (1971): *The only possible second-order, Euler-Lagrange equations obtainable in a 4D spacetime from an action containing solely the 4D metric and its derivatives are the Einstein field equations*

Weinberg (1965), Deser (1970): *A Lorentz invariant theory of a massless spin-2 particle must be General Relativity at low energies*

- To modify GR, we should either give graviton a mass, have extra dimensions, introduce new degrees of freedom or break Lorentz invariance

So far, such modifications tend to create more problems than they solve

Still, by exploring possible alternatives and their phenomenology we are gaining insights that may lead us to new discoveries

Astrophysics

[Submitted on 22 Jun 2003 (v1), last revised 10 Jul 2003 (this version, v2)]

Is Cosmic Speed-Up Due to New Gravitational Physics?

Sean M. Carroll, Vikram Duvvuri, Mark Trodden, Michael S. Turner

We show that cosmic acceleration can arise due to very tiny corrections to the usual gravitational action of General Relativity of the form R^n, with $n < 0$. This eliminates the need for dark energy, though it does not address the cosmological constant problem. Since a modification to the Einstein-Hilbert action of the form R^n, with $n > 0$, can lead to early-time inflation, our proposal provides a unified and purely gravitational origin for the early and late time accelerating phases of the Universe.

Comments: 4 pages, 1 figure, RevTeX. Typos corrected, references updated

Subjects: **Astrophysics (astro-ph)**; General Relativity and Quantum Cosmology (gr-qc); High Energy Physics - Phenomenology (hep-ph); High Energy Physics - Theory (hep-th)

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Journal reference: Phys.Rev.D70:043528,2004

f(R) gravity

GR with Lambda

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} \{R - 2\Lambda\} + \mathcal{L}_M(g_{\mu\nu}, \psi) \right]$$

f(R) gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} f(R) + \mathcal{L}_M(g_{\mu\nu}, \psi) \right]$$

Modified Einstein Equation is 4th order in metric derivatives:

$$F(R)R_{\mu\nu}(g) - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \Box F(R) = \kappa^2 T^{(M)}_{\mu\nu}$$

$$F \equiv \frac{df}{dR} \qquad \kappa^2 \equiv 8\pi G$$

f(R) gravity – acceleration without Dark Energy?

f(R) gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} f(R) + \mathcal{L}_M(g_{\mu\nu}, \psi) \right]$$

The original f(R) by Carroll et al (2003)

$$f(R) = R - \frac{\mu^4}{R}$$

f(R) gravity – acceleration without Dark Energy?

f(R) gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} f(R) + \mathcal{L}_M(g_{\mu\nu}, \psi) \right]$$

A type of f(R) that's allowed (Hu & Sawicki 2007, Starobinsky 2007)

$$f(R) = R - m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}$$

f(R) gravity – acceleration without Dark Energy?

f(R) gravity
$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} f(R) + \mathcal{L}_M(g_{\mu\nu}, \psi) \right]$$

Original f(R) by Carroll et al (2003)
$$f(R) = R - \frac{\mu^4}{R}$$
not viable

Hu and Sawicki (2007)
Starobinsky (2007)
$$f(R) = R - m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}$$

One cannot get cosmic acceleration in f(R) gravity without reintroducing Lambda

Still, we have learned a lot from studying the phenomenology of f(R)

f(R) as a scalar-tensor theory

f(R) Einstein equation is 4th order in metric derivatives:

$$F(R)R_{\mu\nu}(g) - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = \kappa^2 T^{(M)}_{\mu\nu}$$

Recast it as a 2nd order equation for the metric plus a new field $F \equiv \frac{df}{dR}$

that obeys a 2nd order equation $3\,\Box F(R) + F(R)R - 2f(R) = \kappa^2 T$

The f(R) action can be written as a scalar-tensor action:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\varphi R - U(\varphi)\right] + \int \mathrm{d}^4x\mathcal{L}_M(g_{\mu\nu}, \Psi_M)$$

$$\phi = F(R)\ , \quad R(\phi) = F^{(-1)}(\phi)$$

$$U(\phi) = \frac{R(\phi)\phi - f(R(\phi))}{2\kappa^2}$$

What is the most general scalar-tensor theory?

Gregory Horndeski, *Talking About Gravity*

Most studied modified gravity models can be recast as scalar-tensor theories

The Horndeski Lagrangian

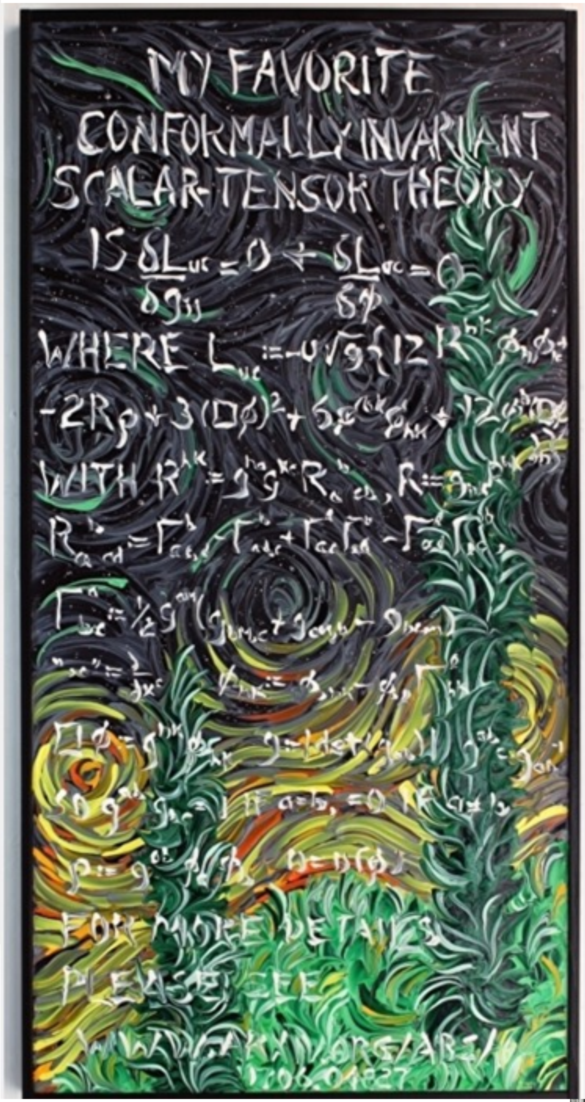
$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\sum_{i=2}^{5} \mathcal{L}_i + \mathcal{L}_\mathrm{m}[g_{\mu\nu}] \right]$$

$$\mathcal{L}_2 = K(\phi,\, X)\,,$$
$$\mathcal{L}_3 = -G_3(\phi,\, X)\Box\phi\,,$$
$$\mathcal{L}_4 = G_4(\phi,\, X)R + G_{4X}(\phi,\, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]\,,$$
$$\mathcal{L}_5 = G_5(\phi,\, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi,\, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]$$

$$X = -\phi^{;\mu}\phi_{;\mu}/2$$

G. W. Horndeski, Int. J. Theor. Phys (1974)
C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, PRD (2011)

GREGORY W. HORNDESKI



https://www.horndeskicontemporary.com

The generalized Brans-Dicke (GBD) subset of Horndeski

GBD are the scalar-tensor theories with the usual kinetic energy term

$$K(\phi, X) = X - V(\phi)$$
$$G_4(\phi, X) = \frac{A^{-2}(\phi)}{16\pi G}$$
$$G_3 = G_5 = 0$$

$$S = \int d^4x\sqrt{-g}\left[\frac{A^{-2}(\phi)}{16\pi G}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_M(g_{\mu\nu}, \psi)\right]$$

Note that we get quintessence Dark Energy with *A=1*

The fifth force

$$S = \int d^4x\sqrt{-g}\left[\frac{A^{-2}(\phi)}{16\pi G}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_M(g_{\mu\nu},\psi)\right]$$

Additional force mediated by the scalar: $$\vec{f} = -\vec{\nabla}\Psi - \frac{d\ln A(\phi)}{d\phi}\vec{\nabla}\phi$$

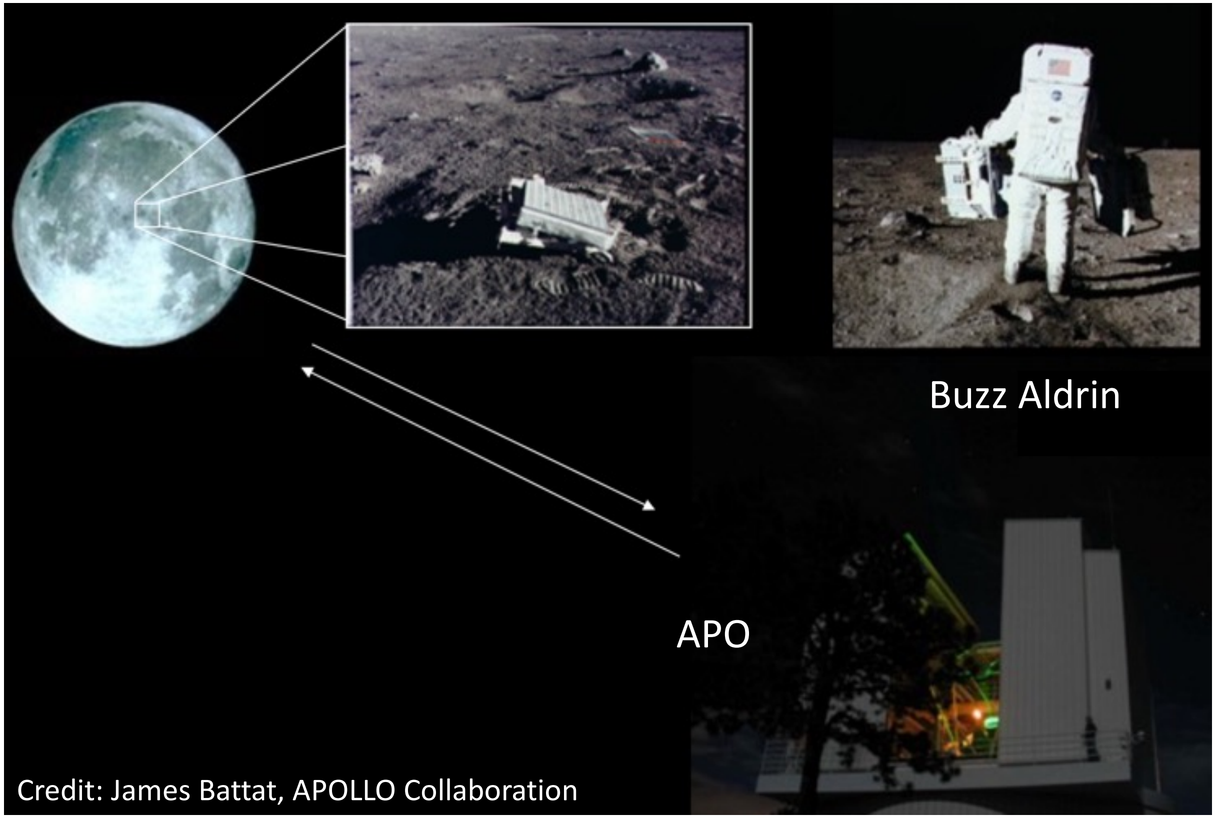
The range set by the mass: $$m^2 = \frac{d^2V_{\rm eff}}{d\phi^2}$$

The coupling strength: $$\beta = m_{\rm Pl}\frac{d\ln A}{d\phi}$$

This "fifth force" only affects non-relativistic matter (CDM and baryons) and has no effect on relativistic species (photons, neutrinos)

Matter and light follow different geodesics!

Lunar ranging tests of GR



Tests of the equivalence principle, variation of G, inverse square law ..

Not much room for alternative theories in our Solar System
(we want our GPS to work!!)

Screening mechanisms

Modified Gravity implies additional force(s)

Must screen the 5th force to restore GR inside the Solar System

- make the mass of the scalar field large, or make the coupling small, at high matter densities (e.g. chameleon, symmetron)

- the force law modified close to a matter source (Vainshtein mechanism)

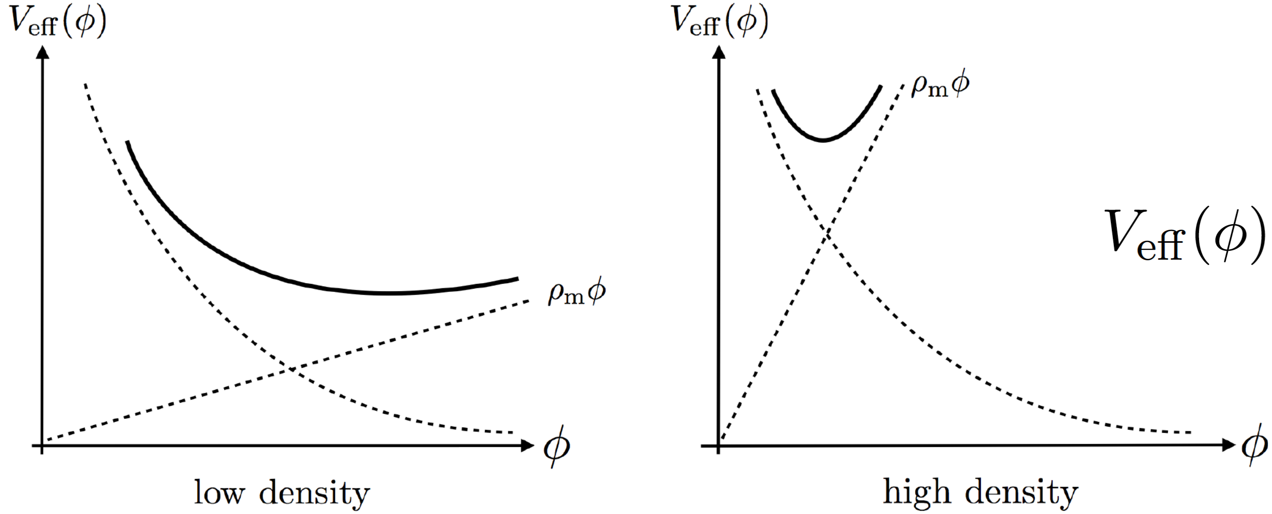
Rich astrophysical phenomenology!

Chameleon Screening

$$\Box\phi = V_{,\phi} - A^3(\phi)A_{,\phi}\tilde{T}$$

matter density

$$\Box\phi = V_{\mathrm{eff}\,,\phi}(\phi)$$



$$V_{\mathrm{eff}}(\phi) = V(\phi) + A(\phi)\rho$$

The scalar field mass is large (the range of the force is small) in high density regions

J. Khoury and A. Weltman, Phys.Rev.Lett. 93 (2004)

Vainshtein Screening

E.g., consider the Cubic Galileon theory:

$$\mathcal{L} = -3(\partial\phi)^2 - \frac{1}{\Lambda^3}\Box\phi(\partial\phi)^2 + \frac{g}{M_{\rm Pl}}\phi T^\mu_\mu$$

matter density

Static solution for a point mass M $\quad \vec{\nabla}\cdot\left(6\vec{\nabla}\phi + \hat{r}\frac{4}{\Lambda^3}\frac{(\vec{\nabla}\phi)^2}{r}\right) = \frac{gM}{M_{\rm Pl}}\delta^{(3)}(\vec{x})$

$$\phi'(r) = \frac{3\Lambda^3 r}{4}\left(-1 + \sqrt{1 + \frac{1}{9\pi}\left(\frac{r_{\rm V}}{r}\right)^3}\right)$$

$$\phi'(r \gg r_{\rm V}) \simeq \frac{g}{3}\cdot\frac{M}{8\pi M_{\rm Pl} r^2}$$

$$\left.\frac{F_\phi}{F_{\rm gravity}}\right|_{r\gg r_{\rm V}} \simeq \frac{g^2}{3}$$

$$r_{\rm V} \equiv \frac{1}{\Lambda}\left(\frac{gM}{M_{\rm Pl}}\right)^{1/3}$$

Vainshtein radius

$$\phi'(r \ll r_{\rm V}) \simeq \frac{\Lambda^3 r_{\rm V}}{2}\sqrt{\frac{r_{\rm V}}{r}} \sim \frac{1}{\sqrt{r}}$$

$$\left.\frac{F_\phi}{F_{\rm gravity}}\right|_{r\ll r_{\rm V}} \sim \left(\frac{r}{r_{\rm V}}\right)^{3/2} \ll 1$$

The scalar force contributes far away from the source, negligible close to it

A. I. Vainshtein, Phys. Lett. B (1972)

Astrophysical tests of screening

Modified properties of dark matter halo distribution

Falck, K. Koyama, and G.-B. Zhao, JCAP(2015), 1503.06673

- dependence on the morphology of the cosmic web
- difference between the halo lensing mass and the dynamical mass

Altered stellar evolution

P. Chang and L. Hui, Astrophys. J. (2011) arXiv:1011.4107

A-C. Davis, E. A. Lim, J. Sakstein and D. Shaw, Phys. Rev. D (2012) arXiv:1102.5278

- stars progress more rapidly through their evolutionary tracks
- stars of a given mass brighter and hotter than in GR, burn at a faster rate
- altered universal relations such as Period-Luminosity relation of cepheids

Broken equivalence principle for black holes, stars and gas in galaxies

L. Hui and A. Nicolis, Phys. Rev. Lett. (2012) arXiv:1201.1508

V. Vikram, A. Cabre, B. Jain and J. VanderPlas, arXiv:1303.0295

- they can rotate at different speeds.
- they can segregate and the disk can warp
- supermassive black holes shifted off-center of the galaxy

The rage of tests

- Earth-based and inside the Solar System

GR works very well, very accurate tests

- Astrophysics
  - Black holes
  - Stars
  - Galaxies

Room for testable modifications of GR

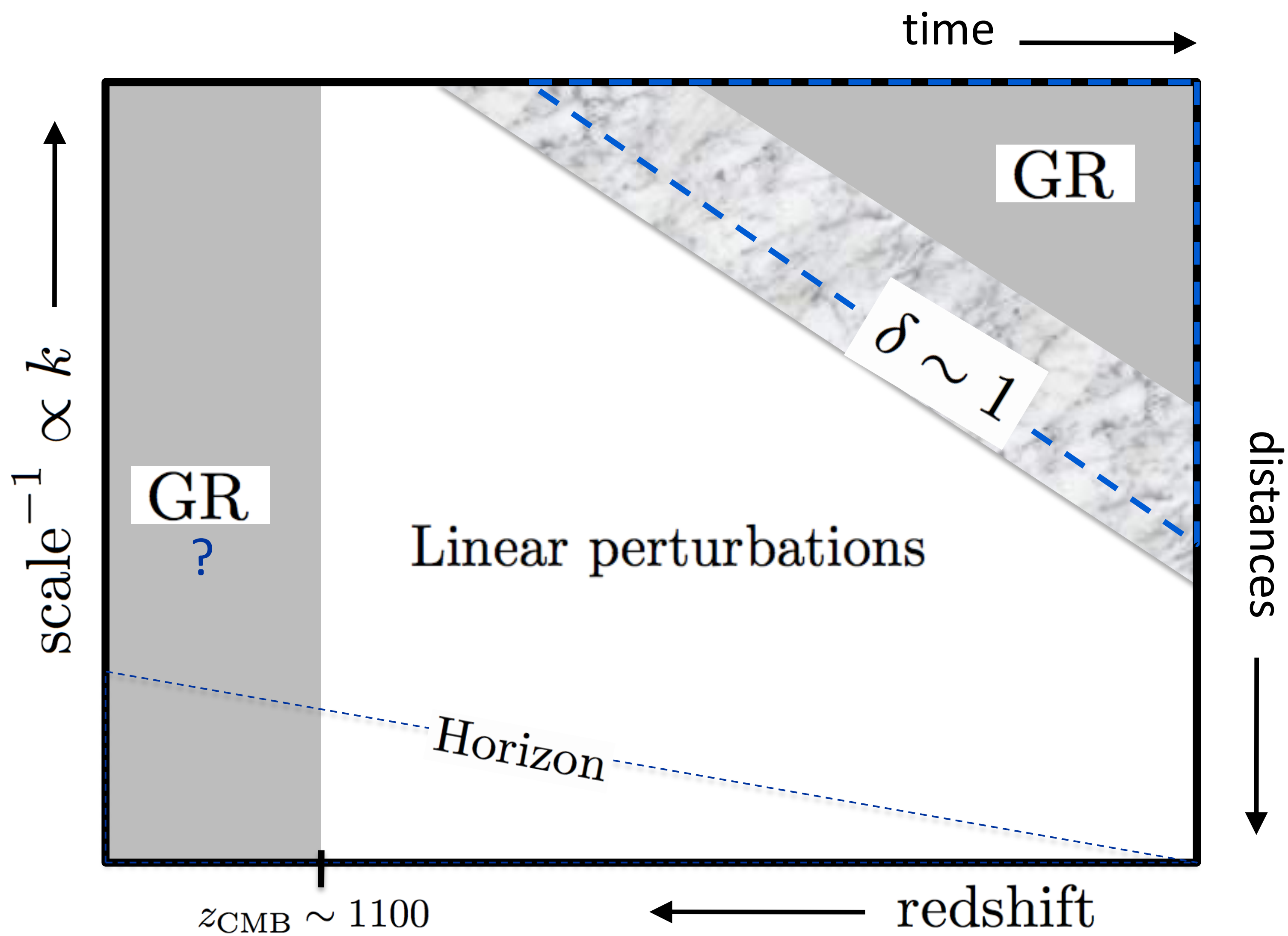
- Cosmology
  - Expansion history
  - Evolution of large-scale structure

The main motivation of modifying GR.
Room for testable modifications of GR

Part IV

What can cosmology tell us about gravity?

The testing ground



Cosmological Perturbation Theory in the late universe

$$ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)d\mathbf{x}^2]$$

density perturbations	$\delta(\vec{x},t)$
peculiar velocities	$V(\vec{x},t)$
gravitational potential	$\Psi(\vec{x},t)$
curvature perturbation	$\Phi(\vec{x},t)$

$$\text{Fourier Space} \;\rightarrow\; \delta(k,t), V(k,t), \Psi(k,t), \Phi(k,t)$$

Conservation of matter energy-momentum

$$\delta' + \frac{k}{aH}V - 3\Phi' = 0$$

$$V' + V - \frac{k}{aH}\Psi = 0$$

Einstein's General Relativity (ignoring radiation)

$$\begin{aligned} k^2\Phi &= -4\pi G a^2 \rho\delta \\ \Psi &= \Phi \end{aligned}$$

What can we expect in modified gravity?

- An attractive "fifth" force acting on matter

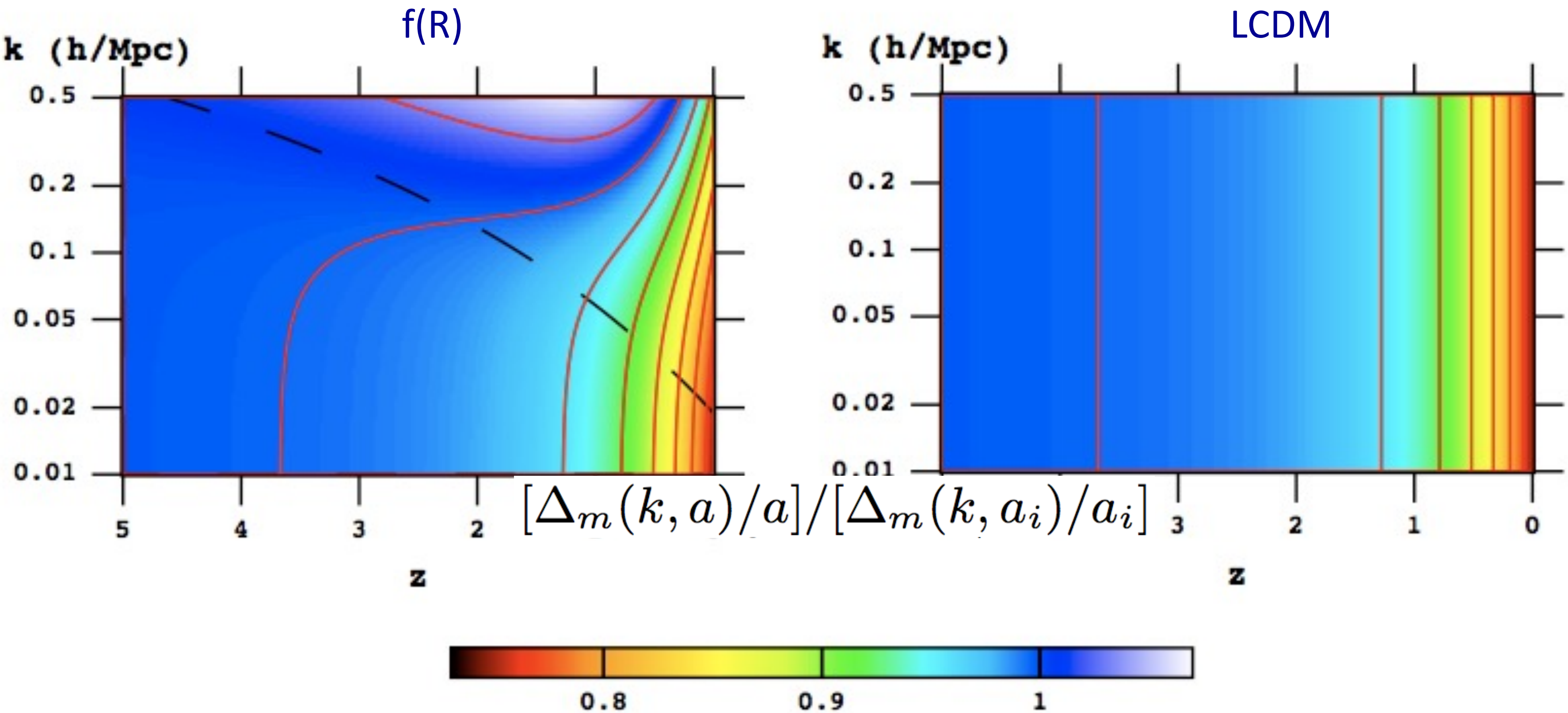
$$\vec{f} = -\vec{\nabla}\Psi - \frac{d\ln A(\phi)}{d\phi}\vec{\nabla}\phi$$

  - Enhanced growth rate of matter perturbations
  - Scale-dependence of the growth factor

- $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are not the same in scalar-tensor theories

  - Photons and matter respond to different spacetimes
    (matter responds to $\boldsymbol{\Psi}$, photons respond to $(\boldsymbol{\Phi}+\boldsymbol{\Psi})/2$)

The growth of cosmological perturbations in f(R)



In scalar-tensor theories, the growth is enhanced on scales below the Compton wavelength of the scalar field

L.P. and A. Silvestri, 0709.0296

Things we can agree to keep

FRW background with small perturbations:

$$ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)d\mathbf{x}^2]$$

Conservation of matter energy-momentum:

$$D_\mu T^{\mu\nu} = 0 \longrightarrow \begin{aligned} \delta' + \frac{k}{aH}V - 3\Phi' &= 0 \\ V' + V - \frac{k}{aH}\Psi &= 0 \end{aligned}$$

Need two additional equations to close the system of four variables

Einstein's General Relativity

$$\begin{aligned} k^2\Phi &= -4\pi G a^2 \rho\delta \\ \Psi &= \Phi \end{aligned}$$

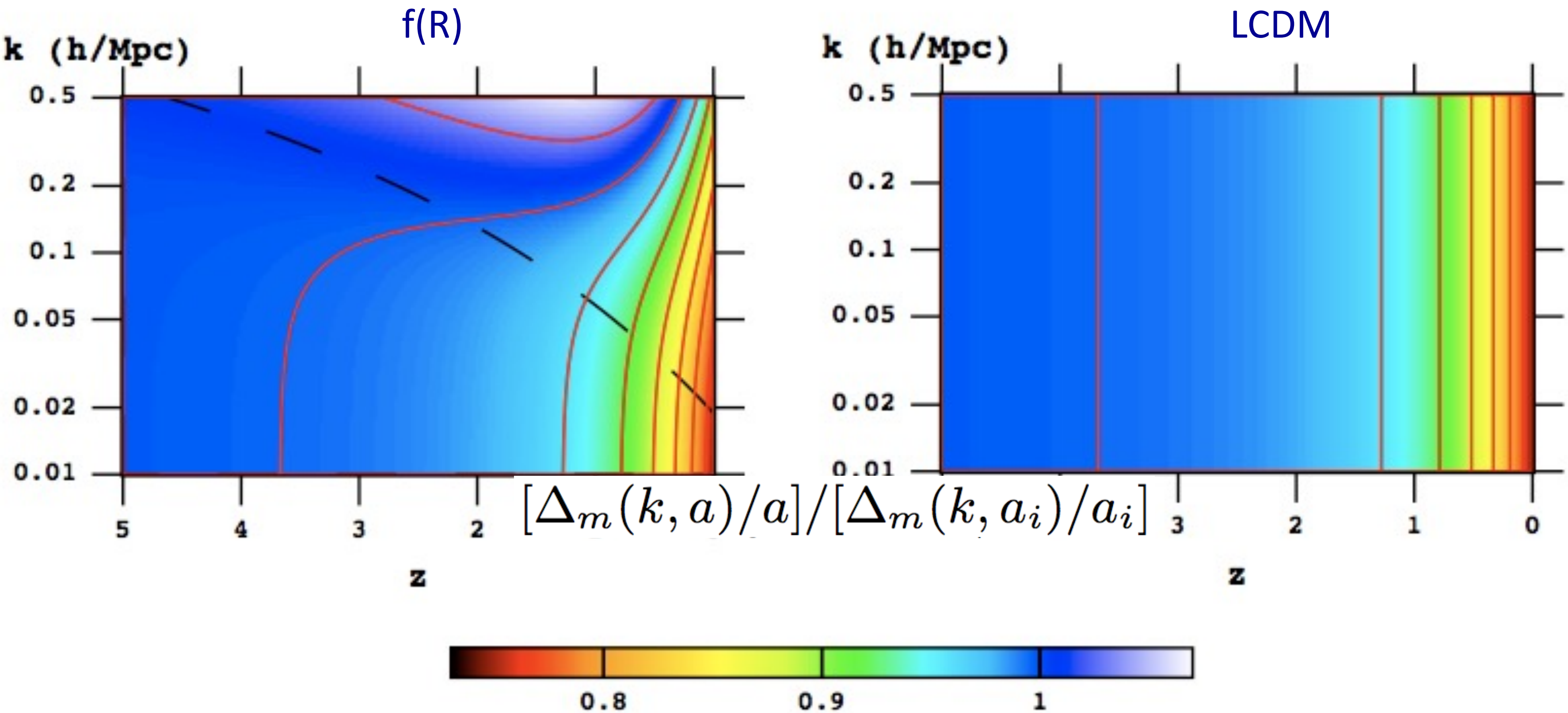
Modified equations

$$\begin{aligned} k^2\Psi &= -\mu(a,k)\; 4\pi G a^2 \rho\Delta \\ \Phi &= \gamma(a,k)\; \Psi \end{aligned}$$

GR+ΛCDM

$$\mu = \gamma = 1$$

The growth of cosmological perturbations in f(R)



$$\begin{aligned} k^2\Psi &= -\mu(a,k)\ 4\pi G a^2 \rho \Delta \\ \Phi &= \gamma(a,k)\ \Psi \end{aligned}$$

$$\mu(a,k) \approx \frac{1+4/3k^2\lambda_C^2}{1+k^2\lambda_C^2} \rightarrow_{k^{-1}\ll\lambda_C} \frac{4}{3}$$

$$\gamma(a,k) \approx \frac{1+2/3k^2\lambda_C^2}{1+4/3k^2\lambda_C^2} \rightarrow_{k^{-1}\ll\lambda_C} \frac{1}{2}$$

The phenomenology of modified gravity

Expansion history:
effective dark energy density X(a)
(X(a) = 1 in LCDM)

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_M}{a^3} + \Omega_{DE} X(a)$$

Linear perturbations:
Modified Einstein equations:
(μ = Σ = γ = 1 in LCDM)

$$\begin{aligned} -k^2\Psi &= 4\pi\, \underbrace{\mu(a,k)G}_{\text{“}G_{matter}\text{”}}\, a^2\delta\rho \\ \Phi &= \gamma(a,k)\, \Psi \\ -k^2\left(\frac{\Phi+\Psi}{2}\right) &= 4\pi\, \underbrace{\Sigma(a,k)G}_{\text{“}G_{light}\text{”}}\, a^2\delta\rho \end{aligned}$$

A smoking gun of new gravitational physics:

$$\boxed{G_{matter} \neq G_{light} \quad \text{or} \quad \Phi \neq \Psi}$$

(the “gravitational slip” γ is also known as η)

Procedure for testing GR on linear scales

- Choose a parametric form for $\rho_{eff}(a)$, $\mu(a,k)$ and $\gamma(a,k)$
- Use CosmoMC/Cobaya with MGCAMB to fit the parameters to data

  https://github.com/sfu-cosmo/MGCAMB

- Check for any evidence of $\rho_{eff}(a) \neq$ const, $\mu \neq 1$, $\gamma \neq 1$ or $\Sigma \neq 1$*
- Interpret

* Note that $\mu = \frac{2\Sigma}{\gamma + 1}$

Other methods and tools

Consistency tests:

The growth index γ_L

Wang and Steinhardt (1998)

Linder & Cahn (2007)

$$f \equiv \frac{d}{d\ln a}\left(\ln\frac{\Delta(k,a)}{\Delta(k,a_i)}\right) = \Omega_m(a)^\gamma$$

E_G= (Galaxy X Lensing) / (Galaxy X Velocity)

$$\langle \hat{E}_G \rangle = \left[\frac{\nabla^2(\Psi-\Phi)}{3H_0^2 a^{-1}\beta\delta}\right]_{k=\ell/\bar{\chi}}$$

Zhang, Liguori, Bean and Dodelson (2007)

MGCAMB, ISiTGR, MGCLASS constrain μ-Σ-γ parameterized in different ways

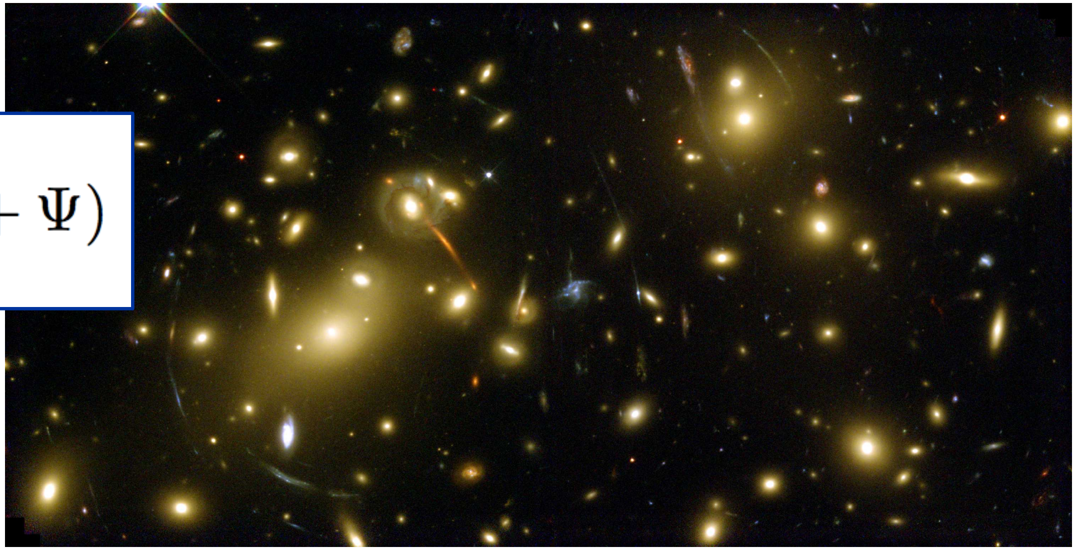
Links with theory in the quasi-static limit

Can also be purely phenomenological

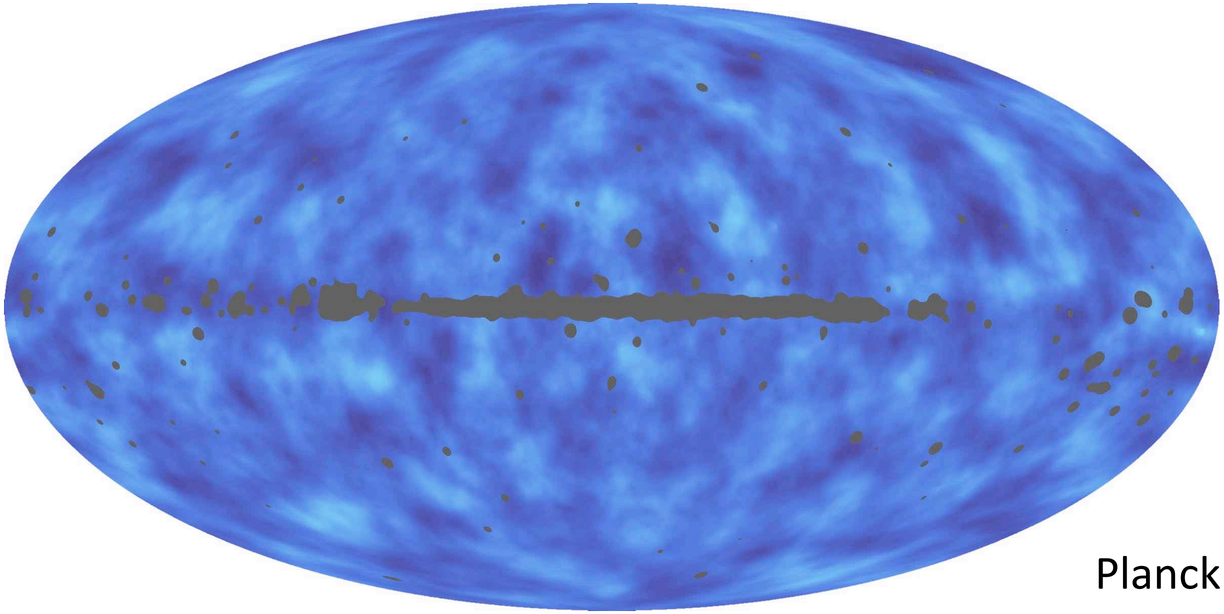
EFTCAMB, hi_class constrain (Horndeski) scalar-tensor theories

Gravitational Lensing

$$\text{Distortion} \propto \int dz\ \partial_\perp(\Phi + \Psi)$$



Hubble

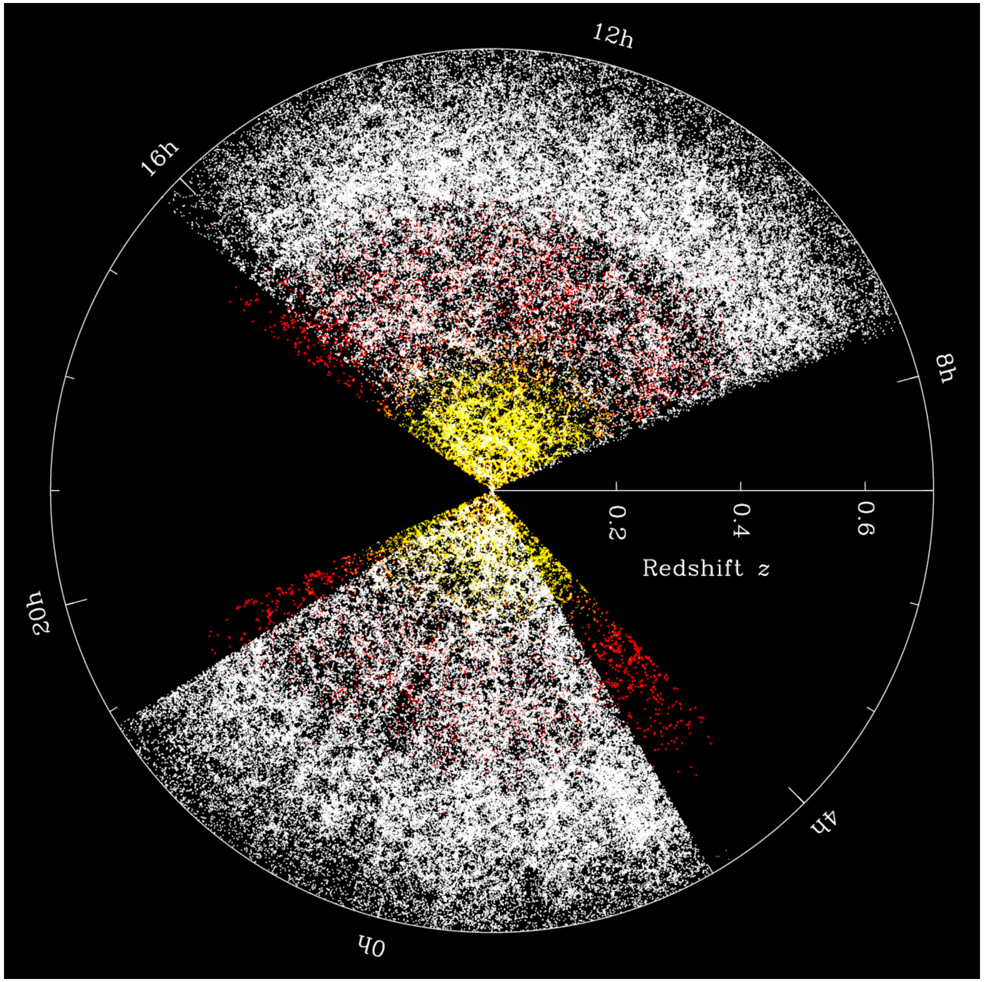


Planck

Galaxy Clustering

Redshift distortions
due to peculiar motion

$$V' + V = \frac{k}{aH}\Psi$$



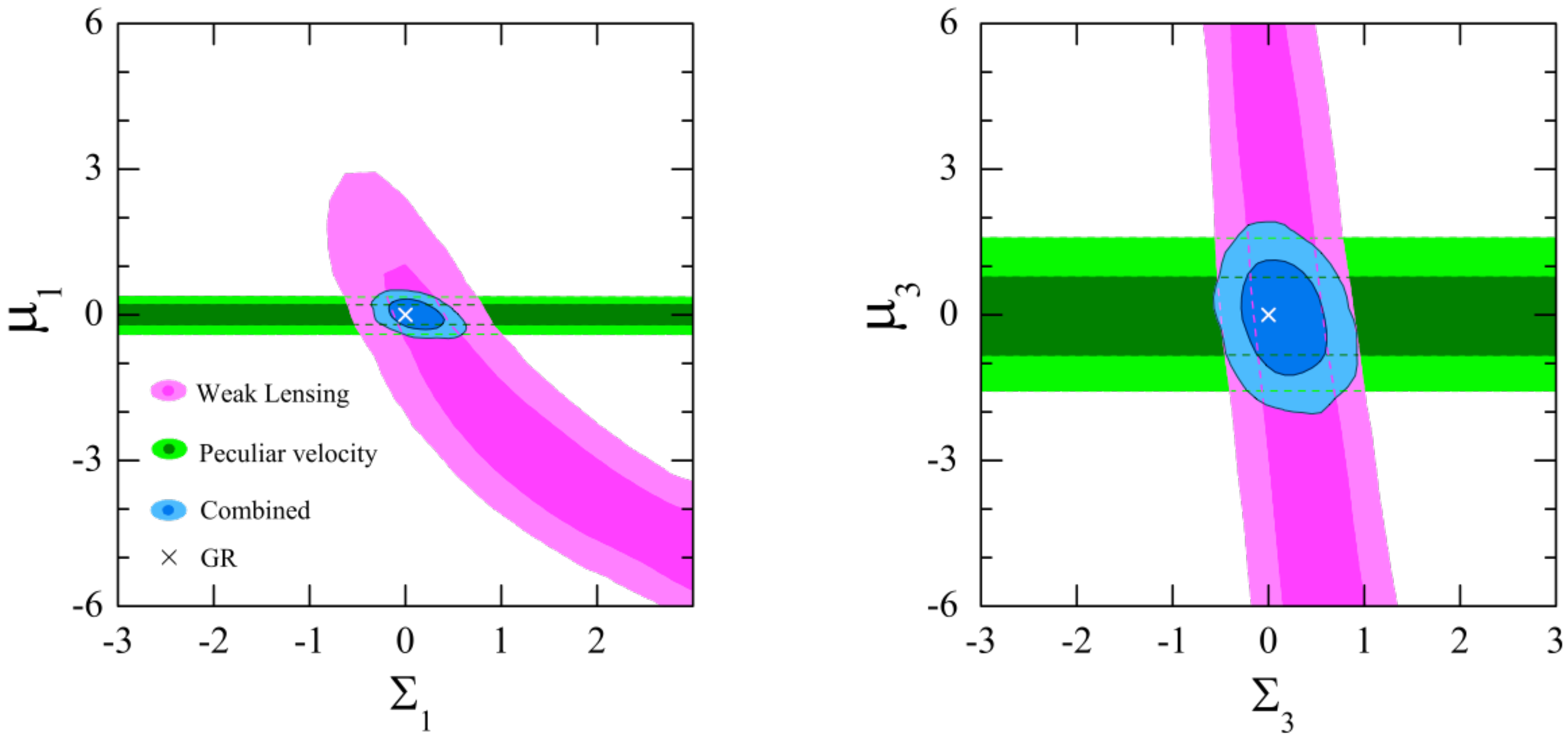
1011.2106, PRD'11

Complementarity of Weak Lensing and Peculiar Velocity Measurements in Testing General Relativity

Yong-Seon Song[1,2], Gong-Bo Zhao[2], David Bacon[2], Kazuya Koyama[2], Robert C Nichol[2], Levon Pogosian[3]
[1] *Korea Institute for Advanced Study, Dongdaemun-gu, Seoul 130-722, Korea*
[2] *Institute of Cosmology & Gravitation, University of Portsmouth, Dennis Sciama Building, Portsmouth, PO1 3FX, United Kingdom*
[3] *Department of Physics, Simon Fraser University, Burnaby, BC, V5A 1S6, Canada*

CFHTLS-Wide T003 (Fu et al, 2008), SDSS DR7

$$\Sigma = 1 + \Sigma_s a^s, \quad \mu = 1 + \mu_s a^s$$

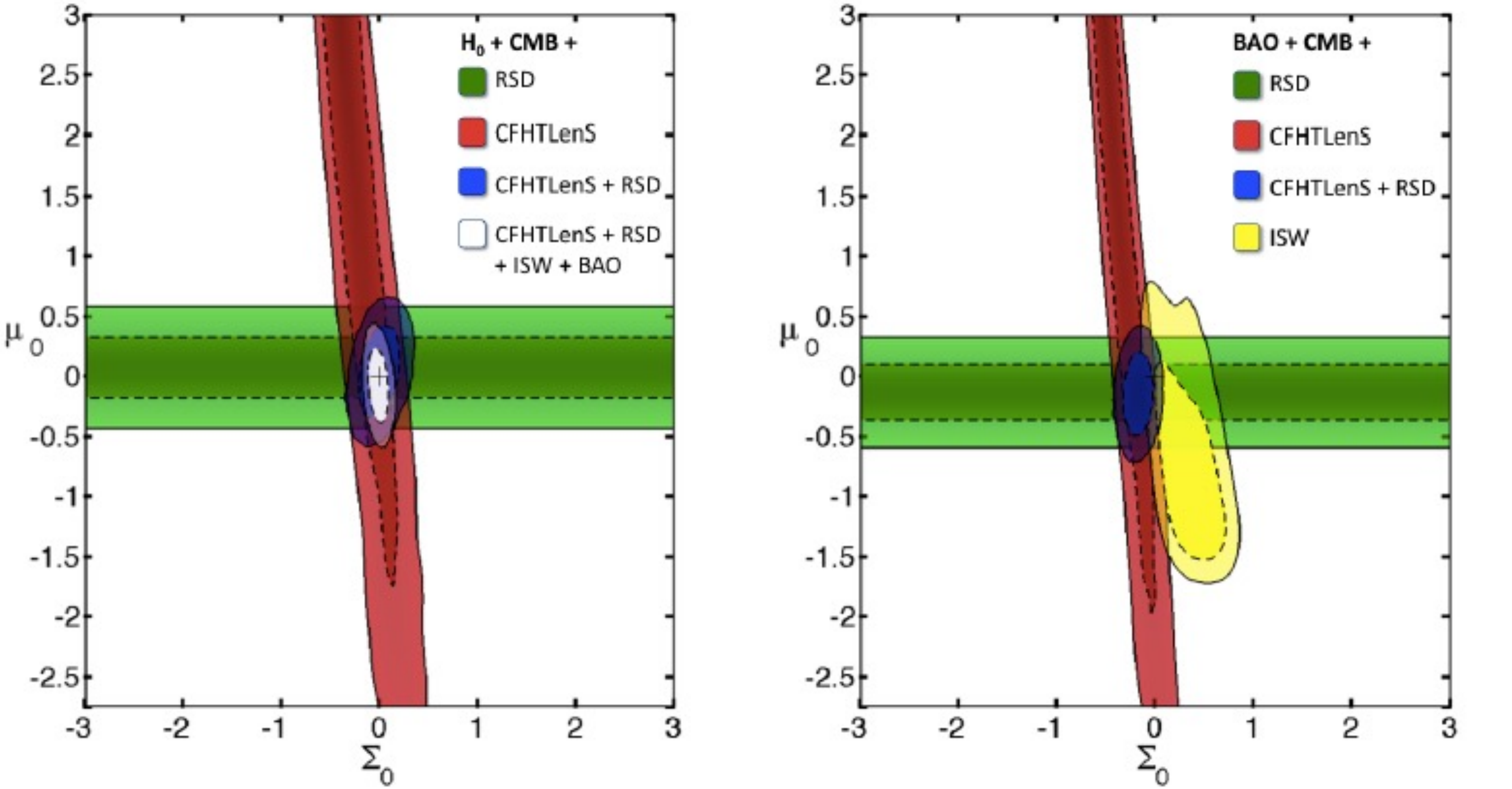


CFHTLenS: Testing the Laws of Gravity with Tomographic Weak Lensing and Redshift Space Distortions

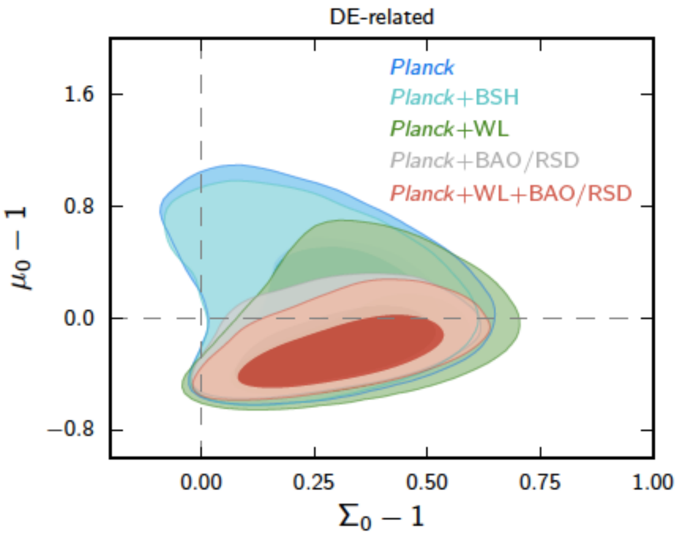
1212.3339, MNRAS'13

Fergus Simpson[1⋆], Catherine Heymans[1], David Parkinson[2], Chris Blake[3], Martin Kilbinger[4,5,6], Jonathan Benjamin[7], Thomas Erben[8], Hendrik Hildebrandt[7,8], Henk Hoekstra[9,10], Thomas D. Kitching[1], Yannick Mellier[11], Lance Miller[12], Ludovic Van Waerbeke[7], Jean Coupon[13], Liping Fu[14], Joachim Harnois-Déraps[15,16], Michael J. Hudson[17,18], Konrad Kuijken[9], Barnaby Rowe[19,20], Tim Schrabback[8,9,21], Elisabetta Semboloni[9], Sanaz Vafaei[7], Malin Velander[12,9].

$$\mu = 1 + \mu_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda} \ , \quad \Sigma = 1 + \Sigma_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$$

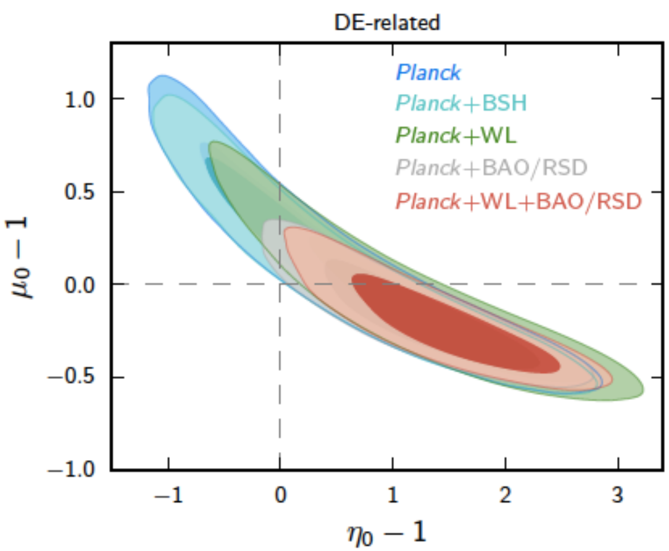


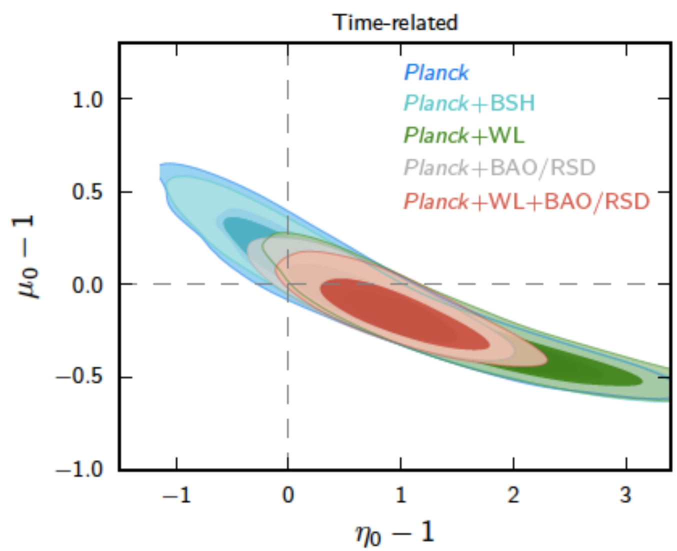
Planck 2015 results. XIV. Dark energy and modified gravity



$$\mu < 1$$
$$\gamma > 1$$
$$\Sigma > 1$$

What would this say about gravity?





Cosmological phenomenology of generalized Brans-Dicke

$$
\begin{aligned}
-k^2\Psi &= \mu(k,a) \quad 4\pi G a^2 \rho\Delta \\
\Phi &= \gamma(k,a) \quad \Psi \\
-k^2(\Psi+\Phi) &= \Sigma(a,k) \quad 8\pi G\, a^2\rho\Delta
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\mu &= A^2(\phi)[1+\epsilon(k,a)] \geq 1 \\
\gamma &= \frac{1-\epsilon(k,a)}{1+\epsilon(k,a)} \leq 1 \\
\Sigma &= A^2(\phi) \approx 1
\end{aligned}
}
$$

$$
\epsilon(k,a) = \frac{2\beta^2(a)}{1+m^2(a)a^2/k^2}
$$

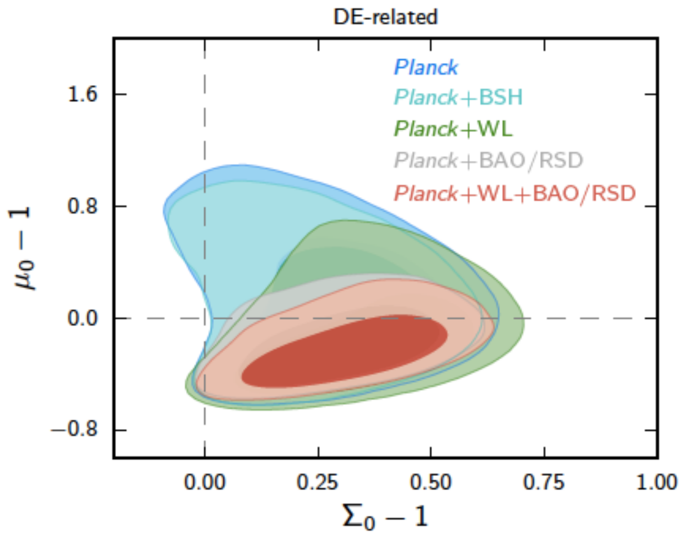
* In quasi-static approximation

The mass of the scalar field: $m^2 = \dfrac{d^2 V_{\mathrm{eff}}}{d\phi^2}$

The coupling strength: $\beta = m_{\mathrm{Pl}} \dfrac{d\ln A}{d\phi}$

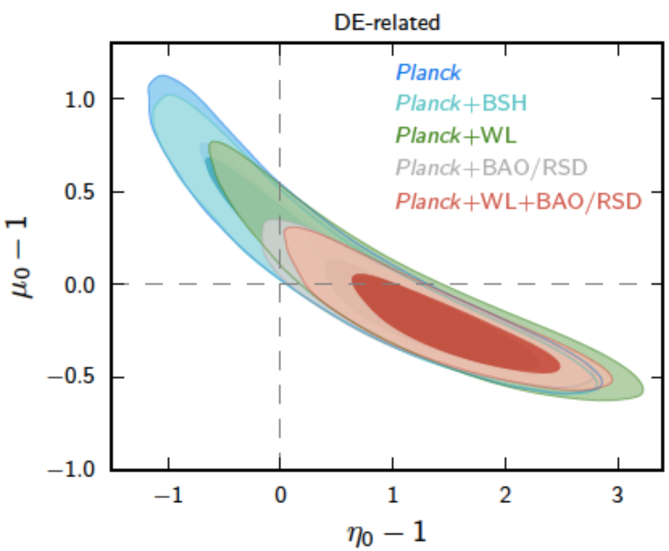
Brax et al, 1203.4812

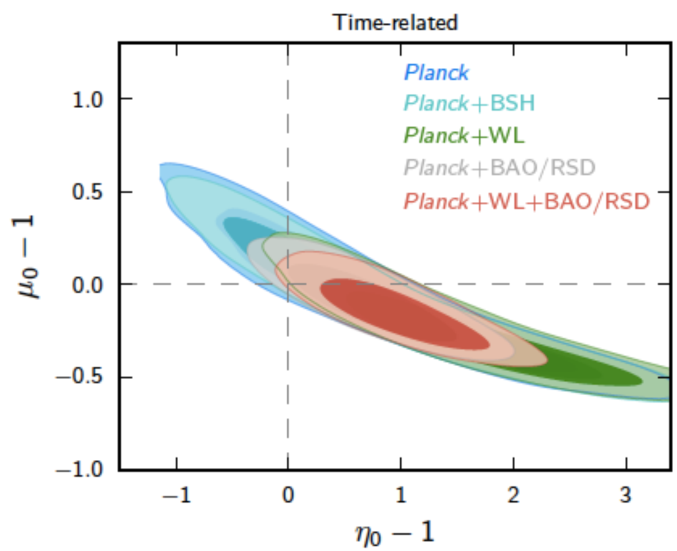
Planck 2015 results. XIV. Dark energy and modified gravity



$$\mu < 1$$
$$\gamma > 1$$
$$\Sigma > 1$$

would rule out all GBD models





What to expect in general Scalar-Tensor theories?

Gregory Horndeski, *Talking About Gravity*

G. W. Horndeski, Int. J. Theor. Phys (1974)
C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, PRD (2011)

The Horndeski Lagrangian

$$S = \int d^4x \sqrt{-g} \left[\sum_{i=2}^{5} \mathcal{L}_i + \mathcal{L}_{\mathrm{m}}[g_{\mu\nu}] \right]$$

$$\mathcal{L}_2 = K(\phi, X), \qquad X = -\phi^{;\mu}\phi_{;\mu}/2$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi,$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]$$

“Effective Theory” or “Unified” approach to Horndeski

“Original EFT”:

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big\{\frac{m_0^2}{2}\Omega(t)R + \Lambda(t) - c(t)\delta g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2$$
$$- \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K^i{}_i - \frac{\bar{M}_2^2(t)}{2}\left(\delta K^i{}_i{}^2 - \delta K^i{}_j\delta K^j{}_i + 2\delta g^{00}\delta R^{(3)}\right)\Big\}$$

Gubitosi et al 1210.0201; Bloomeld et al 1211.7054, 1304.6712; EFTCAMB (Hu et al) 1312.5742

“Unifying description”, with a given H(t):

$$S_{\mathrm{g}}^{(2)} = \int d^3x dt\, a^3\frac{M^2}{2}\Big[\delta K^i_j\delta K^j_i - \delta K^2 + R\delta N + (1+\alpha_{\mathrm{T}})\,\delta_2\left(\sqrt{h}R/a^3\right)$$
$$+ \alpha_{\mathrm{K}}H^2\delta N^2 + 4\alpha_{\mathrm{B}}H\delta K\delta N\Big],$$

Bellini & Sawicki 1404.3713; Gleyzes et al 1411.3712; HI_CLASS (Zumalacarregui et al) 1605.06102)

Interpretation of "effective" coefficients

- $\alpha_T = c_T^2 - 1$ is the excess speed of gravity waves, and is non-zero whenever there is a non-linear derivative coupling of the scalar field to the metric. The same non-linearity is responsible for a non-zero anisotropic stress component in the scalar field energy-momentum tensor.

- α_K quantifies the "independent" dynamics of the scalar field, stemming from the existence of a kinetic energy term in the scalar field Lagrangian. For example, $\alpha_K \neq 0$ in minimally coupled scalar fields, such as quintessence and k-essence, while $f(R)$ models have $\alpha_K = 0$. In the latter case, the scalar field is df/dR, and is completely determined by the dynamics of the Ricci scalar.

- α_B signifies a coupling between the metric and the scalar field degrees of freedom. It is zero for minimally coupled models, such as quintessence and k-essence, and non-zero for all known modified gravity models, *i.e.* all models with a fifth force.

- The running of the Planck mass, α_M, is also generated by a non-minimal coupling, but of a more restricted type. All known models with $\alpha_M \neq 0$,

μ, Σ, γ phenomenology of Horndeski

$$\mu = \frac{m_0^2}{M_*^2} \frac{1 + M^2 a^2/k^2}{f_3/2f_1 M_*^2 + M^2(1+\alpha_T)^{-1} a^2/k^2},$$

$$\gamma = \frac{f_5/f_1 + M^2(1+\alpha_T)^{-1}\, a^2/k^2}{1 + M^2 a^2/k^2},$$

$$\Sigma = \frac{m_0^2}{2M_*^2} \frac{1 + f_5/f_1 + M^2[1 + (1+\alpha_T)^{-1}] a^2/k^2}{f_3/2f_1 M_*^2 + M^2(1+\alpha_T)^{-1} a^2/k^2}$$

Transition scale set by the Compton mass M

Fifth force mediated by the scalar field for k/a >> M

LP & Silvestri, arXiv:1606.05339

Phenomenology of Horndeski theories: $\boldsymbol{\Sigma}$-$\boldsymbol{\mu}$

The Super-Compton Limit: k/a << M

$$\Sigma_0 = \frac{m_{\rm Pl}^2}{M_*^2}\left(1+\frac{\alpha_T}{2}\right)$$

$$\gamma_0 = \frac{1}{1+\alpha_T}$$

$$\mu_0 = \frac{m_{\rm Pl}^2}{M_*^2}(1+\alpha_T)$$

$\boldsymbol{\Sigma} \neq \boldsymbol{\mu}$ on super-Compton scales would signal a modified speed of GW

LP & Silvestri, arXiv:1606.05339, PRD

Phenomenology of Horndeski theories: Σ-μ

The Sub-Compton Limit: k/a >> M

$$\mu_\infty = \frac{m_0^2}{M_*^2}(1 + \alpha_T + \beta_\xi^2)$$

$$\Sigma_\infty = \frac{m_0^2}{M_*^2}\left(1 + \frac{\alpha_T}{2} + \frac{\beta_\xi^2 + \beta_B\beta_\xi}{2}\right)$$

Fifth force

Expect Σ-1 and μ-1 to be of the same sign

LP & Silvestri, arXiv:1606.05339, PRD

Priors for cosmological tests of gravity

Phenomenological functions $\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$ offer a promising way to look for new gravitational physics

In any specific theory, $\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$ depend on the same functions of the Lagrangian

Treating $\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$ as completely independent is unphysical and opens the possibility of false detections (e.g. caused by systematics)

$\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$ can also be correlated with ρ_{DE}

<u>Our approach:</u> derive the joint prior covariance between μ, Σ and ρ_{DE} in general scalar-tensor theories and use it to <u>jointly reconstruct them from data</u>

Generating priors from Horndeski

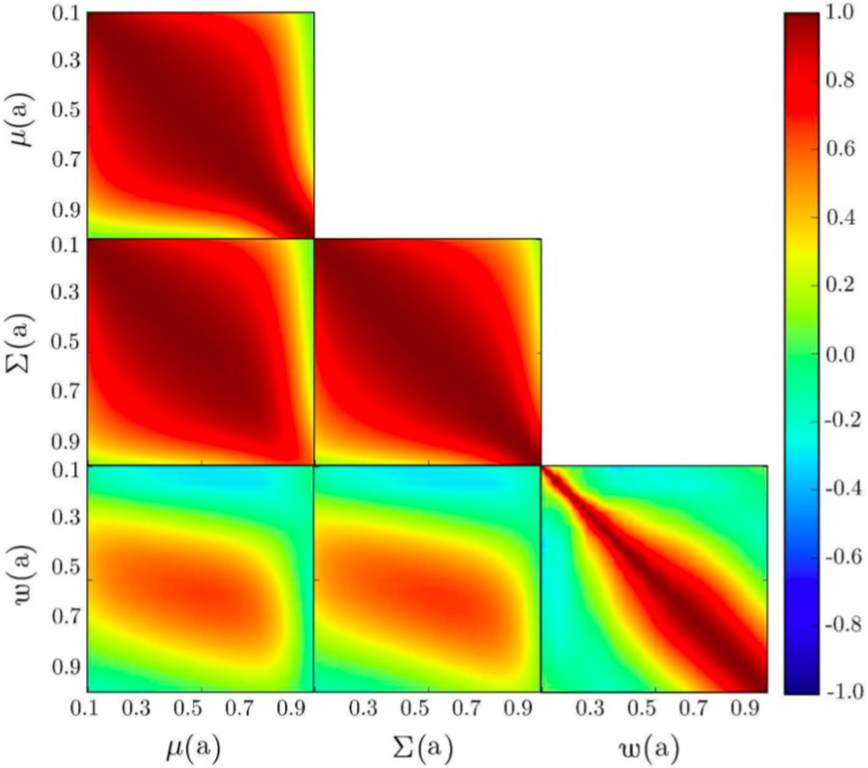
$$S = \int d^4x\sqrt{-g}\left\{\frac{m_0^2}{2}\Omega(t)R + \Lambda(t) - c(t)\delta g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K^i{}_i - \frac{\bar{M}_2^2(t)}{2}\left(\delta K^i{}_i{}^2 - \delta K^i{}_j\delta K^j{}_i + 2\delta g^{00}\delta R^{(3)}\right)\right\} + S_{matter}[g_{\mu\nu}]$$

- Generate an ensemble of EFT functions
  - Parameterize the EFT functions as Pade polynomials (9th order)
  - Sample the coefficients, filter out unphysical solutions
- Filter out models with
  - unacceptable background expansion histories
  - unacceptable gravitational wave speed
  - unacceptable variations of the Newton's constant

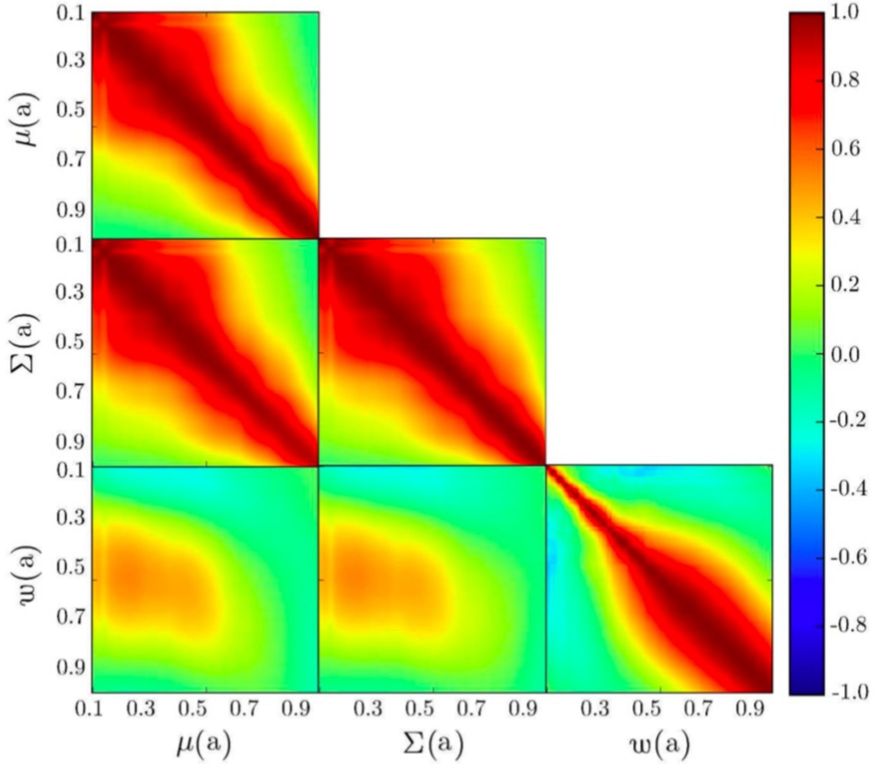
J. Espejo, S. Peirone, M. Raveri, LP, A. Silvestri, K. Koyama, arXiv:1809.01121, PRD

Prior correlations in theories with c_{GW}=1

GBD

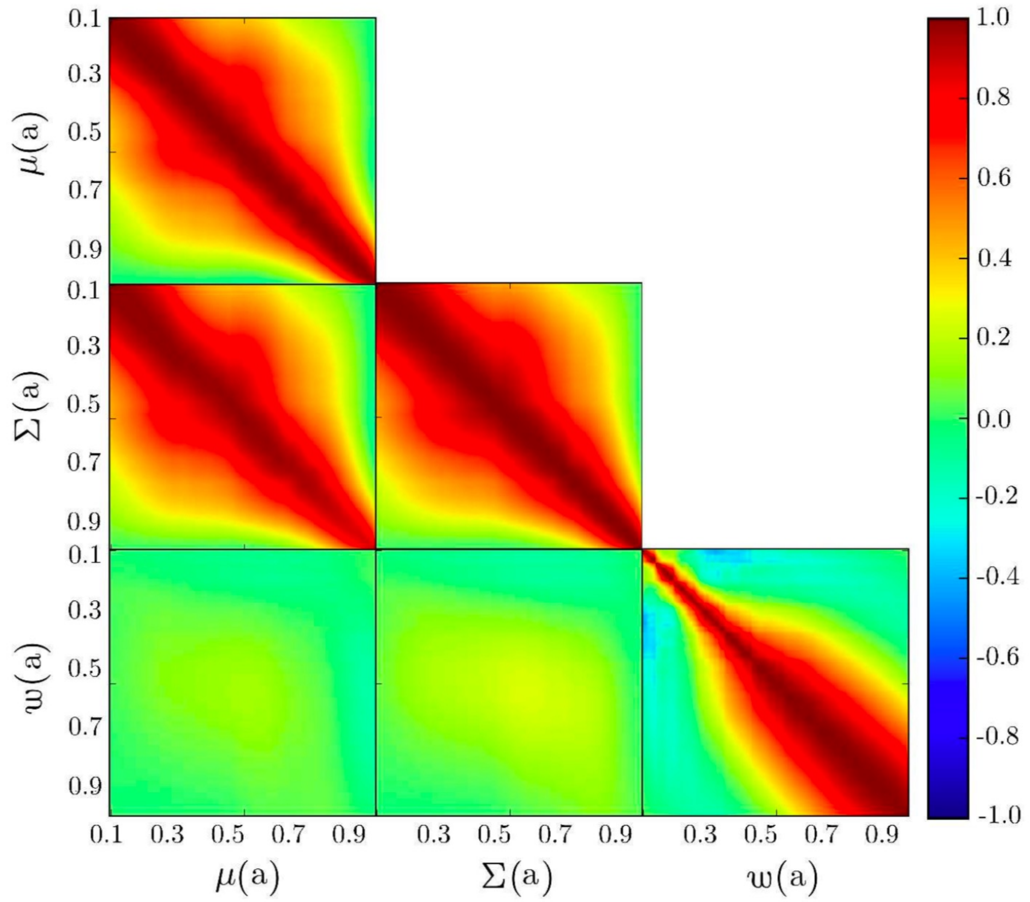


H_S



J. Espejo, S. Peirone, M. Raveri, LP, A. Silvestri, K. Koyama, arXiv:1809.01121, PRD

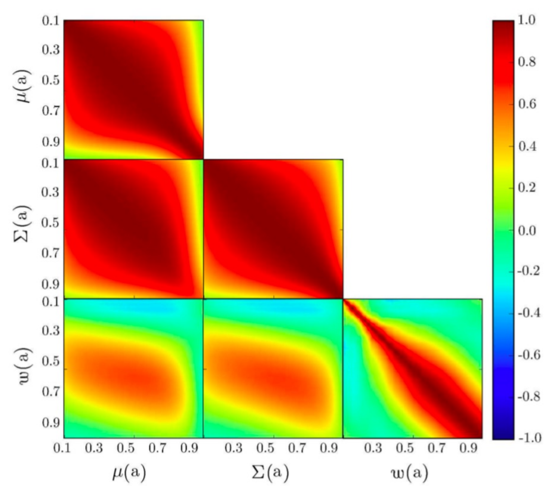
Prior correlations in theories with c_{GW}=1 today

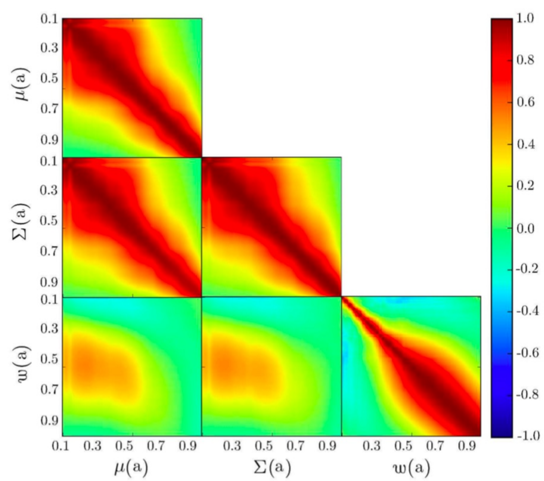


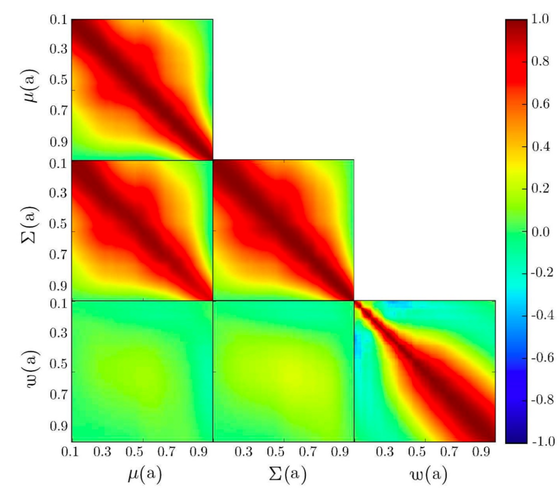
J. Espejo, S. Peirone, M. Raveri, LP, A. Silvestri, K. Koyama, arXiv:1809.01121, PRD

General inferences

- $\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$ are strongly correlated in scalar-tensor theories
- Background expansion is correlated with $\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$ in theories with c_{GW}=1





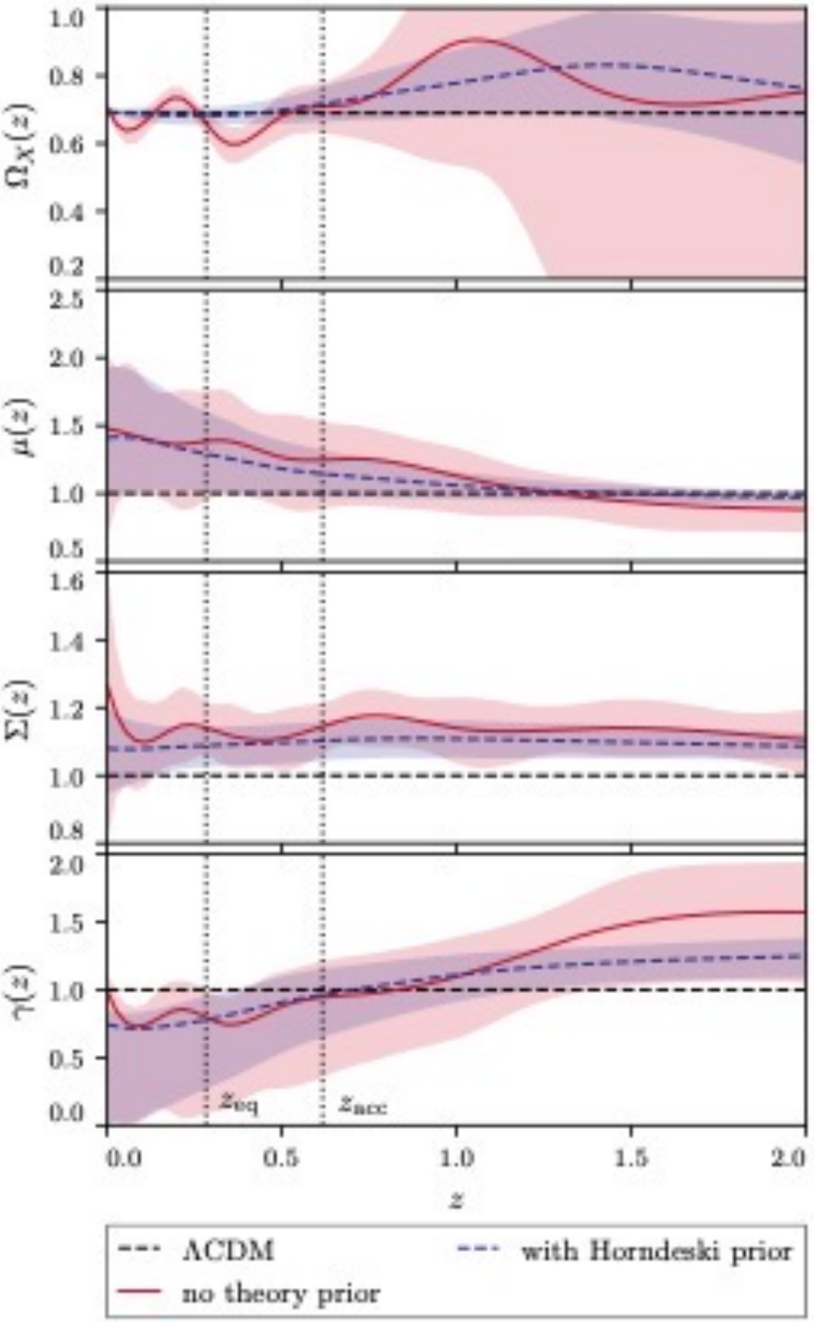


J. Espejo, S. Peirone, M. Raveri, LP, A. Silvestri, K. Koyama, arXiv:1809.01121, PRD

$\Omega_X(a)$, $\mu(a)$, $\Sigma(a)$ ($\gamma(a)$) reconstructed from Planck+DES-Y1+RSD+BAO+SN

With and without a Horndeski prior: a way to separate features consistent with theory from potential systematics

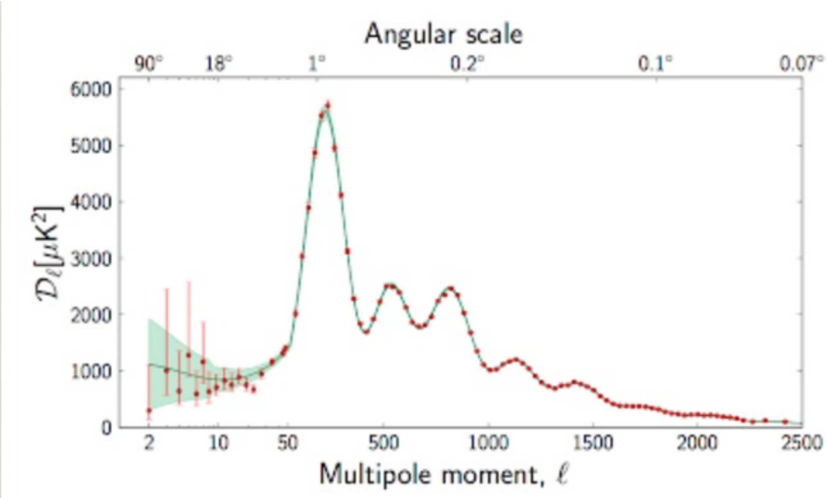
Current data can constrain 15 combined eigenmodes of *$\Omega_X(a)$, $\mu(a)$, $\Sigma(a)$* relative to the Horndeski prior

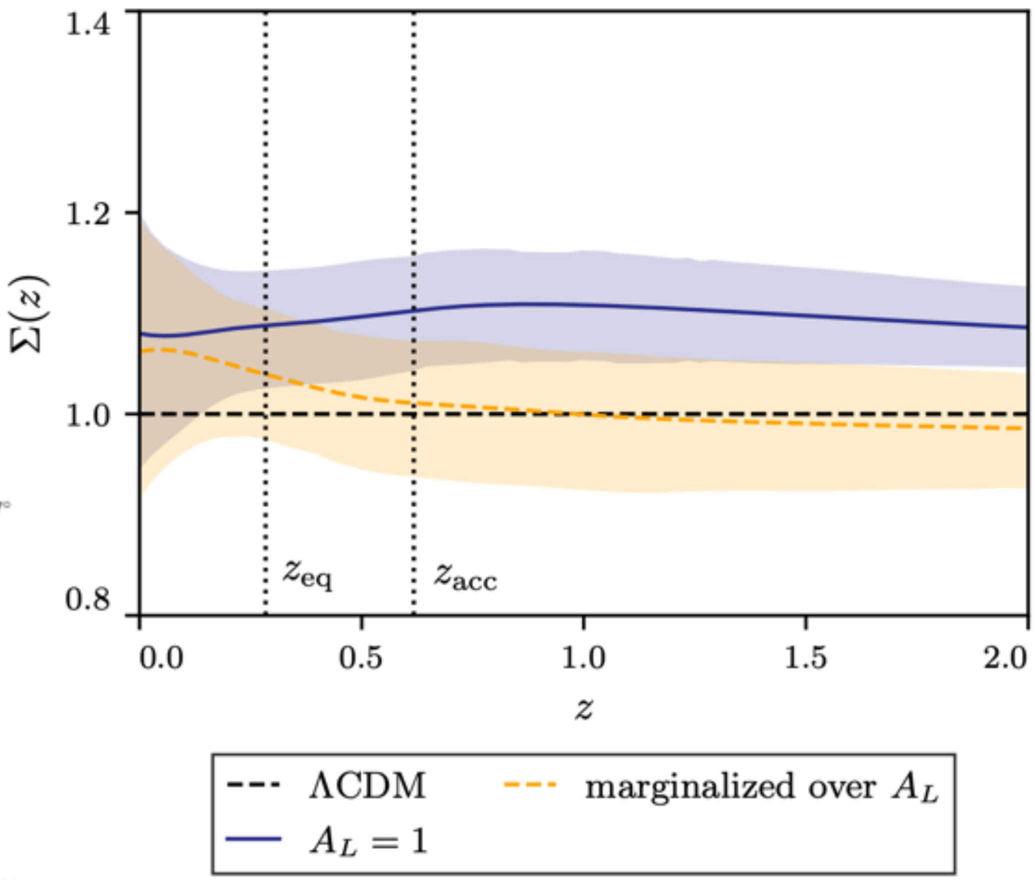


LP, M. Raveri, et al, arXiv:2107.12992, Nature Astronomy (2022)
M. Raveri, LP, et al, arXiv:2107.12990, JCAP

The Planck "lensing anomaly"

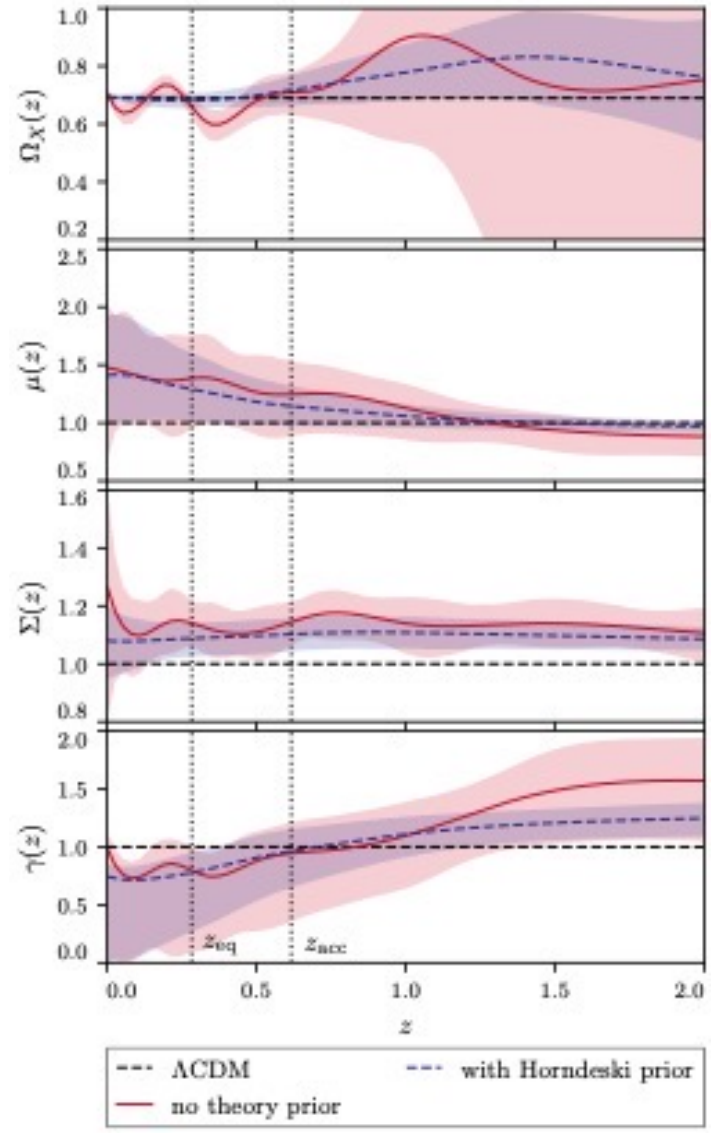
$$
\begin{aligned}
-k^2\Psi &= 4\pi\ \mu(a,k)G\ a^2\delta\rho \\
\Phi &= \gamma(a,k)\ \Psi \\
-k^2\left(\frac{\Phi+\Psi}{2}\right) &= 4\pi\ \Sigma(a,k)G\ a^2\delta\rho
\end{aligned}
$$





Reconstructing gravity from Planck+DES-Y1+RSD+BAO+SN

- First simultaneous reconstruction of $\mu(a)$, $\Sigma(a)$ and $\Omega_X(a)$
- With and without a Horndeski prior: a way to separate features consistent with theory from potential systematics
- Current data can constrain 15 eigenmodes
- Late-time modified gravity is unlikely to resolve the tensions
- Implications for scalar-tensor theories



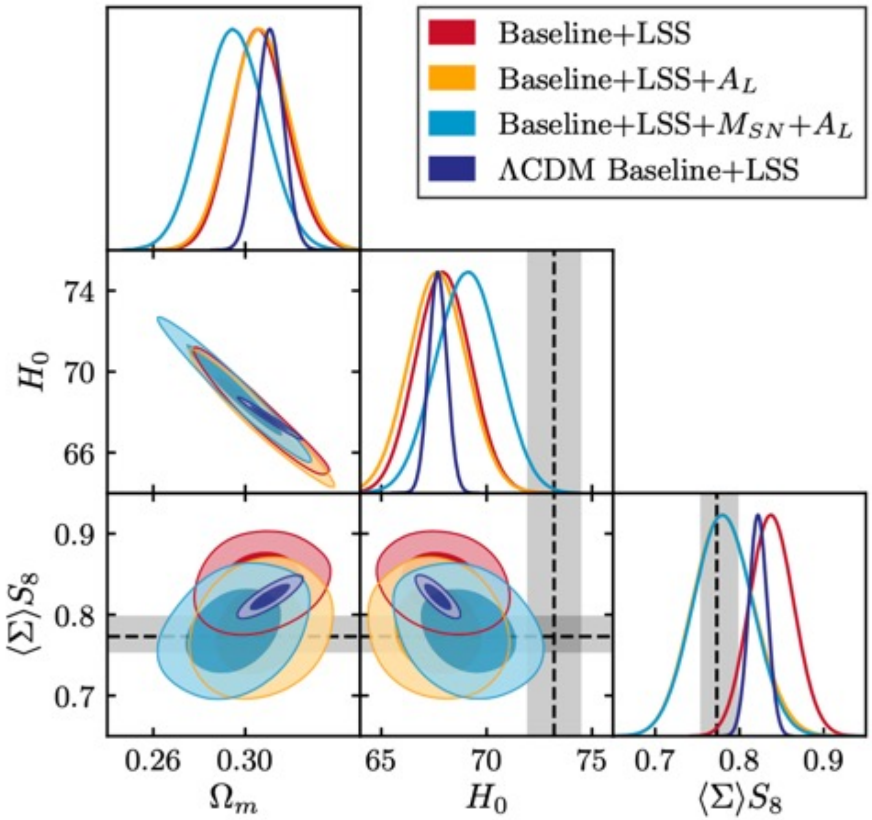
LP, M. Raveri, K. Koyama, M. Martinelli, A. Silvestri, G.-B. Zhao, J. Li, S. Peirone, A. Zucca, arXiv:2107.12992, Nature Astronomy (2022)
M. Raveri, LP, K. Koyama, M. Martinelli, A. Silvestri, G.-B. Zhao, arXiv:2107.12990, JCAP

The S_8 tension

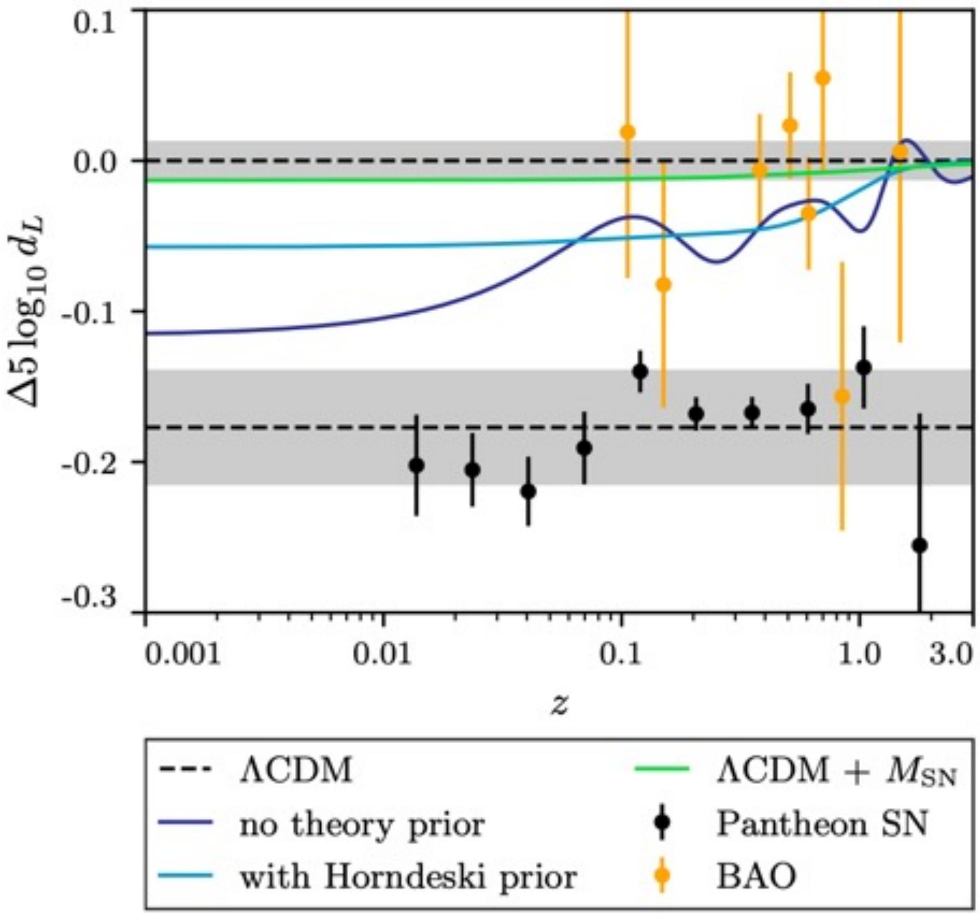
$$-k^2\Psi = 4\pi\ \mu(a,k)G\ a^2\delta\rho$$

$$\Phi = \gamma(a,k)\ \Psi$$

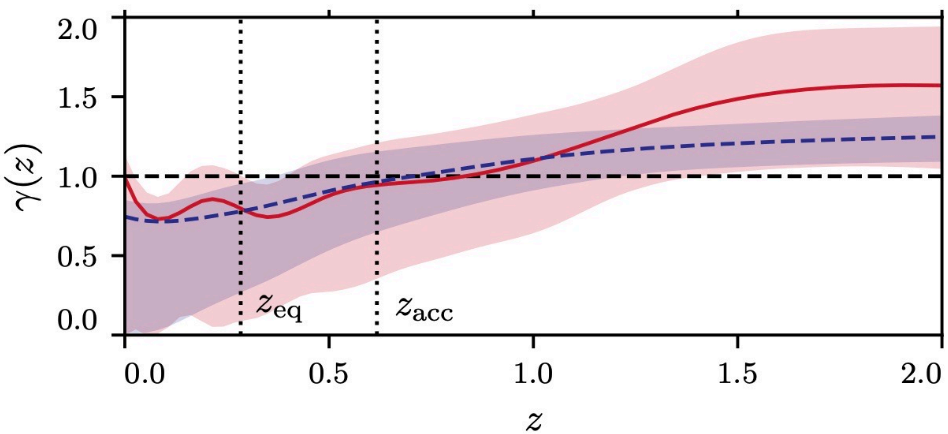
$$-k^2\left(\frac{\Phi+\Psi}{2}\right) = 4\pi\ \Sigma(a,k)G\ a^2\delta\rho$$



The H_0 tension

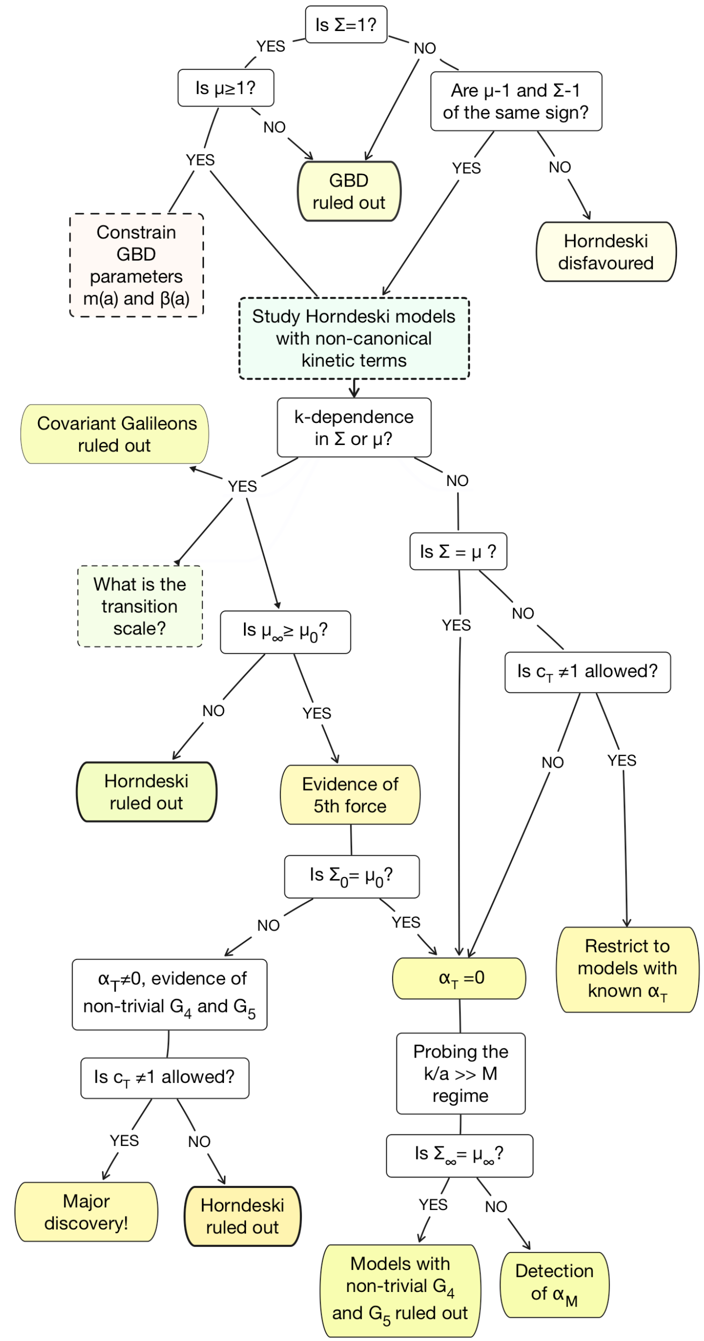


What can cosmology tell us about gravity? Constraining Horndeski with Σ and μ



Hints from the reconstruction:

- LCDM is under some tension (but we knew that already)
- $\gamma > 1$ would rule out Brans-Dicke theories
- $\Sigma \neq \mu$, or $\gamma \neq 1$, can only be due to $c_{GW} \neq 1$ or a fifth force
- No violation of $(\Sigma - 1)(\mu - 1) \geq 0$



LP & Silvestri, arXiv:1606.05339, PRD

Summary

Lambda, despite the problems, is still the best motivated Dark Energy candidate we have

We developed general theoretical and phenomenological frameworks for systematic searches for departures from Lambda

Today's and tomorrow's data is good enough to allow reconstruction of key phenomenological functions to learn how gravity works on cosmological scales

No need to limit ourselves to w_0, w_a, Σ_0, μ_0

If we find evidence for $w(a) \neq -1$, if our theoretical expectations are correct, there are likely to be other signatures, such as fifth forces or birefringence