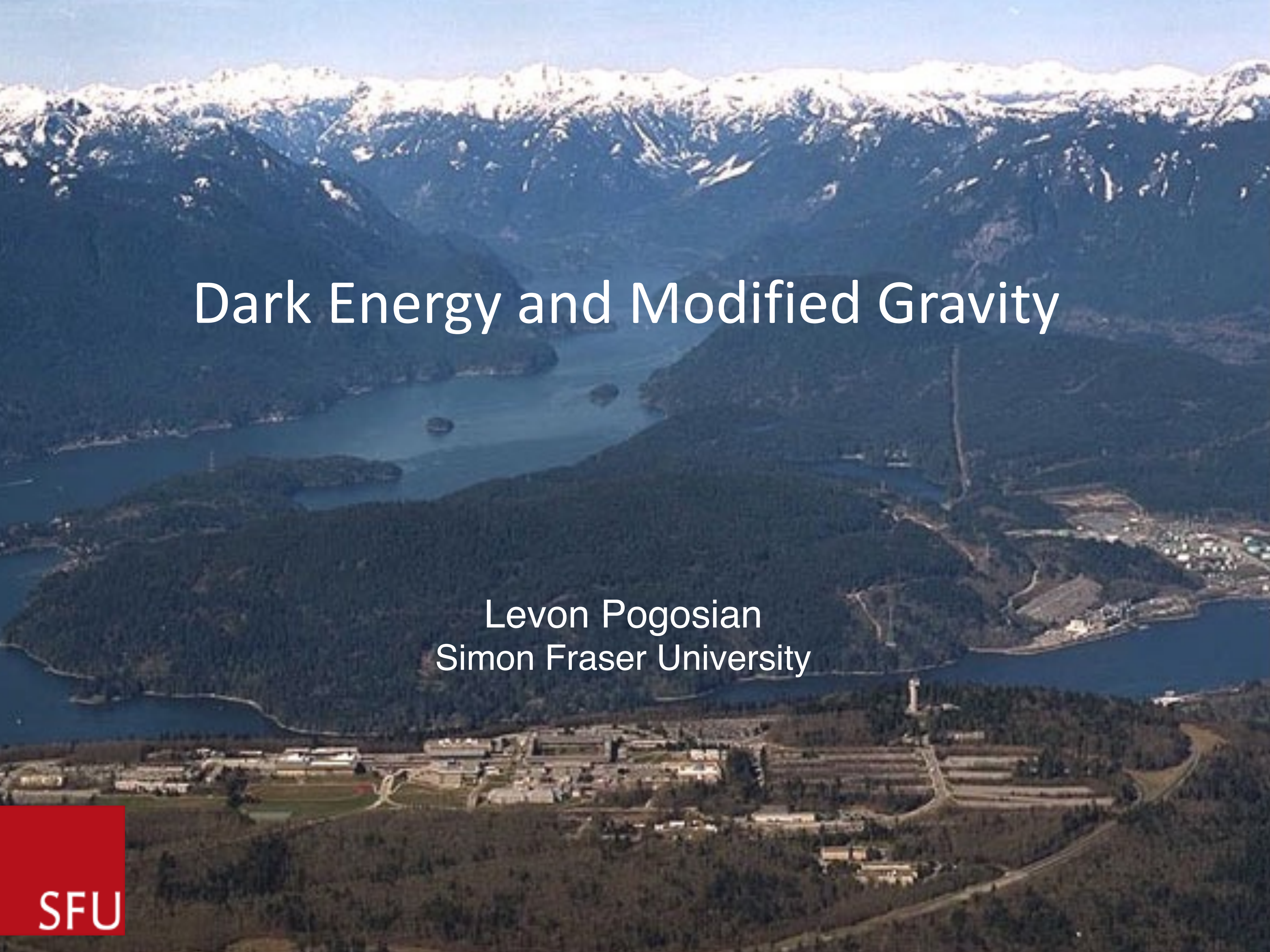
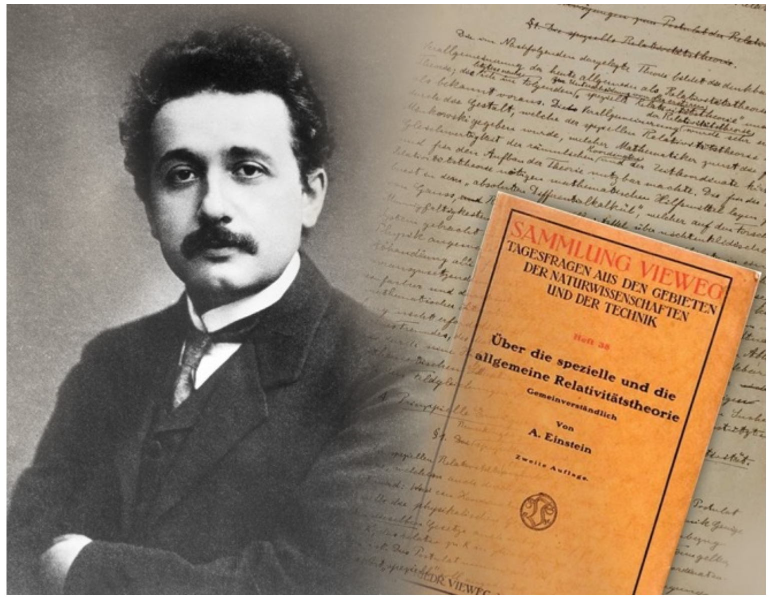


Dark Energy and Modified Gravity

Levon Pogosian
Simon Fraser University

SFU

A century+ of General Relativity



from

What is General Relativity telling us about Cosmology?

to

What is Cosmology telling us about Gravity?

Table of Content

Part I: What is wrong with Lambda?

Part II: Dark Energy and its equation of state

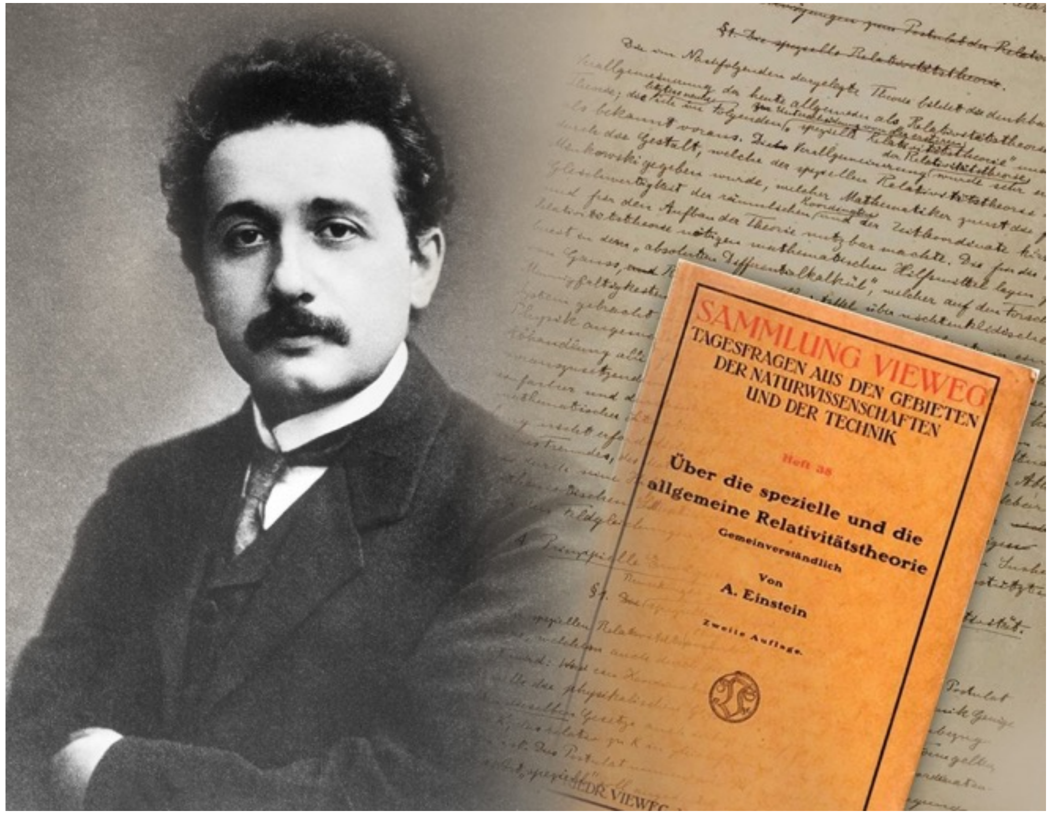
Part III: Modified gravity and its phenomenology

Part IV: What can Cosmology tell us about gravity?

WARNING: there will be some equations

Part I

What’s wrong with Lambda?



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}$$

Einstein's Perfect Universe

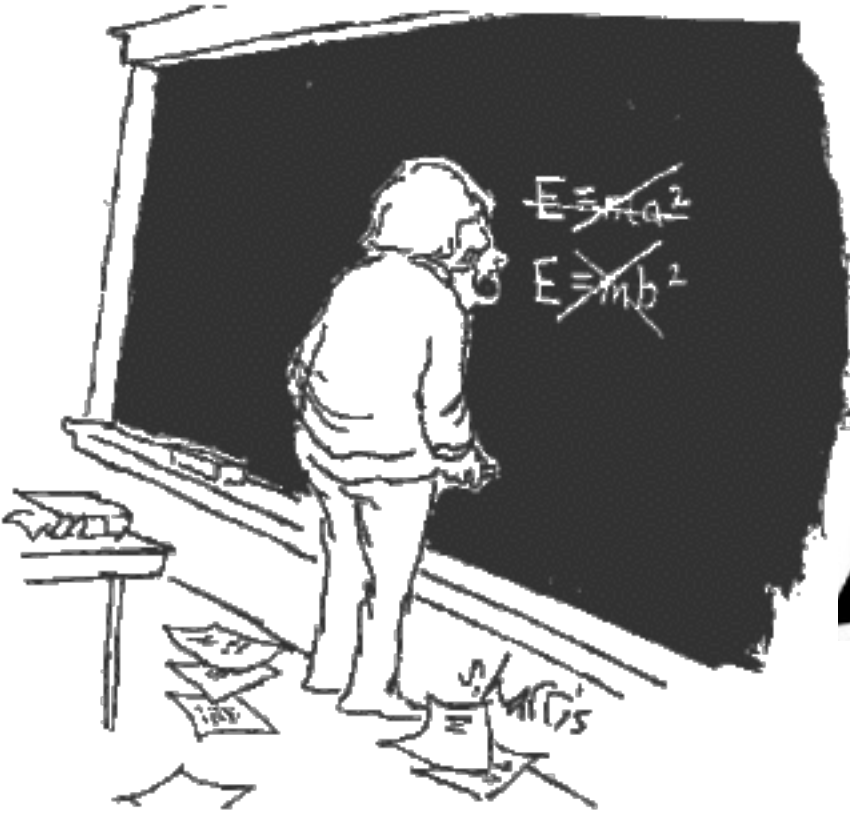
Same Everywhere

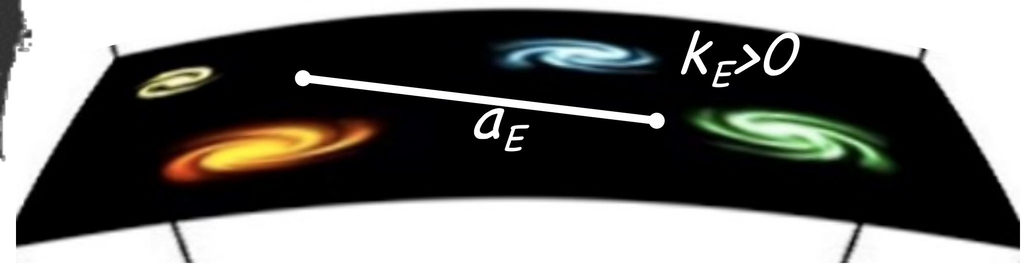
no evidence for this in 1917,
turned out to be correct

Ever the Same

he tried,
turned out to be wrong

Einstein's Static Universe

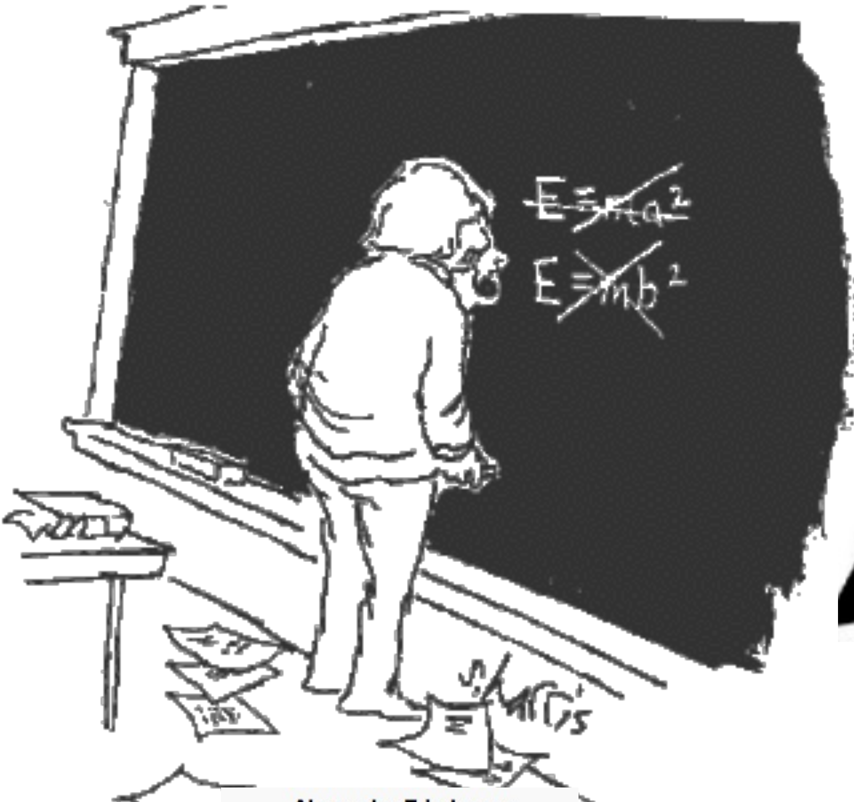




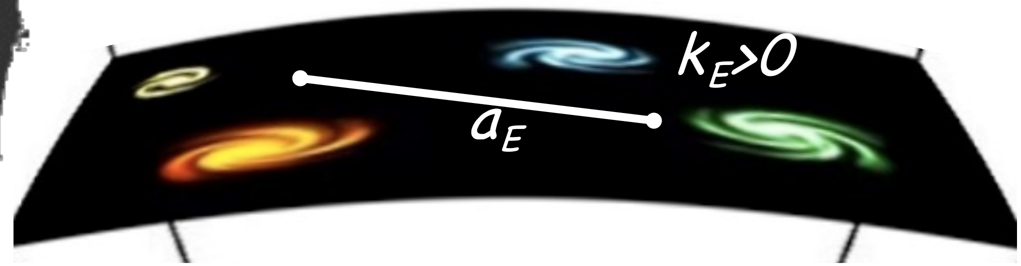
$$\left(\frac{\dot{a}}{a_E}\right)^2 = \frac{8\pi G \rho_M}{3} - \frac{k_E}{a_E^2} + \frac{\Lambda_E}{3} = 0$$

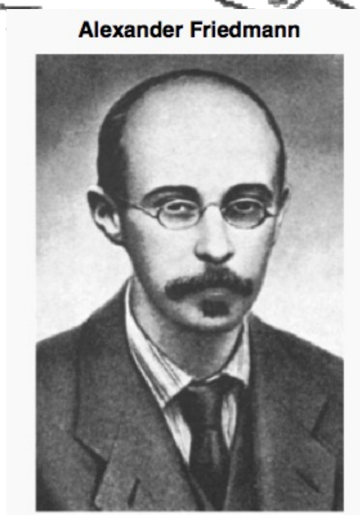
$$\frac{\ddot{a}}{a_E} = -\frac{4\pi G}{3}\rho_M + \frac{\Lambda_E}{3} = 0$$

$$\Lambda_E = 4\pi G \rho_M; \quad k_E = 4\pi G \rho_M a_E^2$$



Einstein's Static Universe





$$\left(\frac{\dot{a}}{a_E}\right)^2 = \frac{8\pi G\rho_M}{3} - \frac{k_E}{a_E^2} + \frac{\Lambda_E}{3} = 0$$

$$\frac{\ddot{a}}{a_E} = -\frac{4\pi G}{3}\rho_M + \frac{\Lambda_E}{3} = 0$$

$$\Lambda_E = 4\pi G\rho_M; \quad k_E = 4\pi G\rho_M a_E^2$$

A. Friedmann, “Über die Krümmung des Raumes”, Zeitschrift für Physik (1922)

A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY AMONG EXTRA-GALACTIC NEBULAE

By Edwin Hubble

Mount Wilson Observatory, Carnegie Institution of Washington

Communicated January 17, 1929

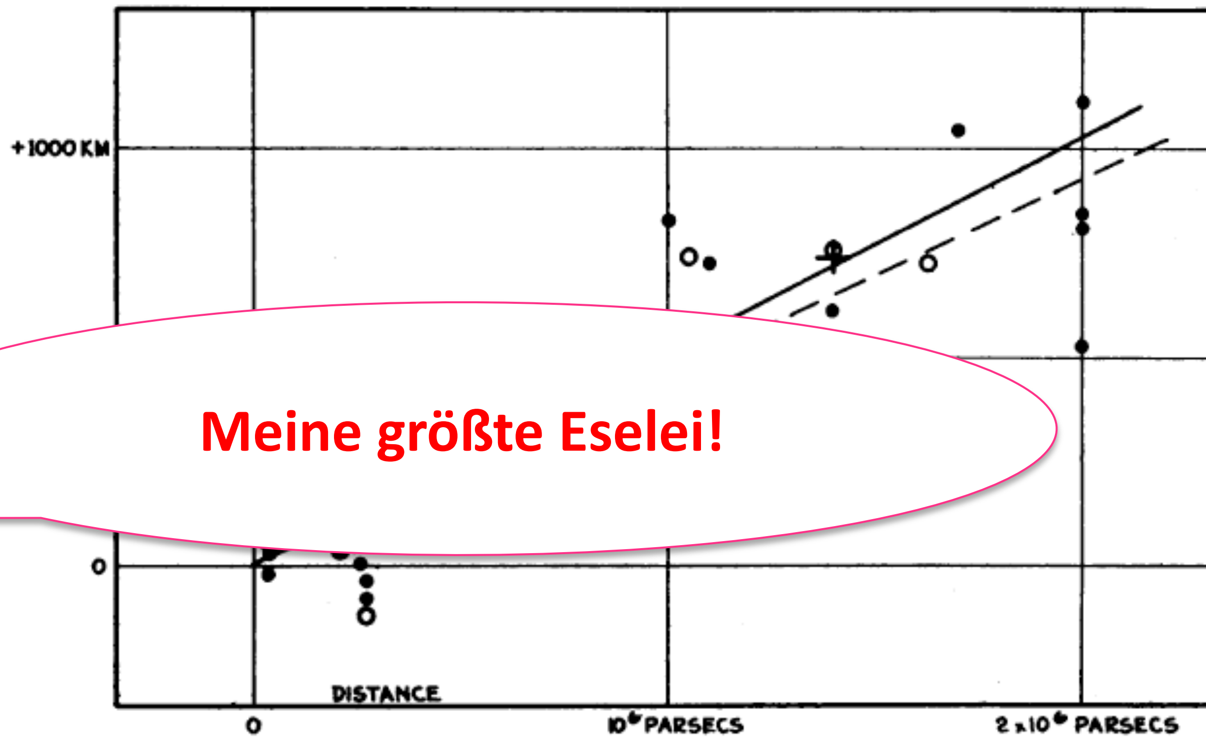
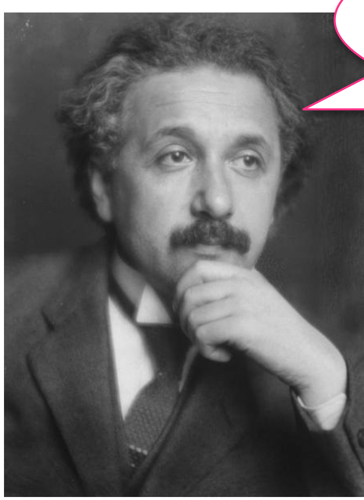


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.



Meine größte Eselei!

Vacuum energy and Lambda have the same Gravity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}$$

$$T_{\mu\nu}^{(\mathrm{vac})} = -g_{\mu\nu}\rho^{(\mathrm{vac})}$$

Each fundamental particle field contributes energy to the vacuum.

Vacuum energy and Lambda have the same Gravity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}$$

$$T_{\mu\nu}^{(\mathrm{vac})} = -g_{\mu\nu} \rho^{(\mathrm{vac})}$$

Each fundamental particle field contributes energy to the vacuum. For example, consider a scalar field *h*:

$$S = -\int \mathrm{d}^4 x \sqrt{-g} \left[V(h) + \frac{1}{2} g^{\mu\nu} \partial_\mu h \, \partial_\nu h \right]$$

Quantum field theory predicts an infinite contribution to the vacuum energy, unless one introduces a cutoff on the largest allowed momentum *k*

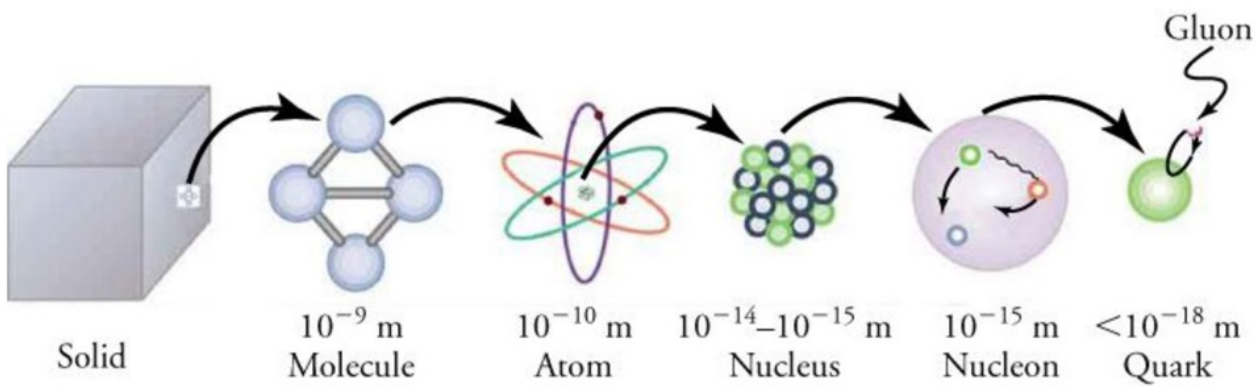
$$\rho_{\mathrm{vac}} = \frac{E_0}{\mathcal{V}} = V(\bar{h}) + \frac{1}{2\mathcal{V}} \sum_k \omega_k$$

Any reasonable choice of a cutoff results in a vacuum energy MUCH larger than the current energy density of the universe

So what? Can't we just adjust Lambda to cancel any vacuum energy?

The required fine-tuning is not just "too fine" for our comfort, it is "technically unnatural"*

All of the physical laws we know fit into the paradigm of effective field theories, where the details of short-distance physics do not matter much for the physics at much larger distance scales.



E.g. when dealing with atoms, we use an *effective theory* obtained after "integrating out" the momenta associated with energies much larger than the atomic scale. Our prediction of atomic levels is not sensitive to where exactly that cutoff is.

** The Cosmological Constant Problem: Why it's hard to get Dark Energy from Micro-physics, Cliff Burgess, 1309.4133, Les Houches Summer School (2013)*

Image credit: texasgateway.org

So what? Can’t we just adjust Lambda to cancel any vacuum energy?

Vacuum energy – a milli-eV phenomenon associated with the largest length-scales in the observable universe, is extremely sensitive to the cutoff.

As physicists, we do not expect phenomena taking place at milli-eV energies to care whether we cutoff at 100 MeV or 100 GeV, but vacuum energy does!

So, for a while, it was assumed that, for some yet to be discovered reasons, the vacuum contribution to Einstein’s equation must vanish.

** The Cosmological Constant Problem: Why it’s hard to get Dark Energy from Micro-physics, Cliff Burgess, Les Houches Summer School (2013)*

The SCDM model

Worked quite well, except for some “minor” problems:

- only 10%-50% of the energy density was accounted for
- there were stars in our galaxy older than the universe

THE COSMOLOGICAL CONSTANT IS BACK

Lawrence M. Krauss[1] and Michael S. Turner[2,3]

As we shall discuss, the observational case for a cosmological constant is so compelling today that it merits consideration in spite of its checkered history. On the theoretical side the value of the cosmological constant remains extremely puzzling, and it just could be that cosmology will provide a crucial clue. Fortunately, there are observations that should settle the issue sooner rather than later.

[3]*NASA/Fermilab Astrophysics Center*

Fermi National Accelerator Laboratory, Batavia, IL 60510-0500

(submitted to *Gravity Research Foundation Essay Competition*)

SUMMARY

A diverse set of observations now compellingly suggest that Universe possesses a nonzero cosmological constant. In the context of quantum-field theory a cosmological

1998: the expansion is accelerating!

THE ASTRONOMICAL JOURNAL, 116:1009–1038, 1998 September

OBSERVATIONAL EVIDENCE FROM SUPERNOVAE FOR AN ACCELERATING UNIVERSE AND A COSMOLOGICAL CONSTANT

ADAM G. RIESS,[1] ALEXEI V. FILIPPENKO,[1] PETER CHALLIS,[2] ALEJANDRO CLOCCHIATTI,[3] ALAN DIERCKS,[4] PETER M. GARNAVICH,[2] RON L. GILLILAND,[5] CRAIG J. HOGAN,[4] SAURABH JHA,[2] ROBERT P. KIRSHNER,[2] B. LEIBUNDGUT,[6] M. M. PHILLIPS,[7] DAVID REISS,[4] BRIAN P. SCHMIDT,[8,9] ROBERT A. SCHOMMER,[7] R. CHRIS SMITH,[7,10] J. SPYROMILIO,[6] CHRISTOPHER STUBBS,[4] NICHOLAS B. SUNTZEFF,[7] AND JOHN TONRY[11]

Received 1998 March 13; revised 1998 May 6

We present spectral and photometric o
range $0.16 \leq z \leq 0.62$. The luminosity dist
relations between SN Ia luminosity and
High-z Supernova Search Team and rece
supernovae and a set of 34 nearby super
logical parameters: the Hubble constant (
vacuum energy density, Ω_Λ), the decelerat
The distances of the high-redshift SNe Ia
density ($\Omega_M = 0.2$) universe without a cosı
subsamples, and prior constraints unanin
logical constant (i.e., $\Omega_\Lambda > 0$) and a curr
constraint on mass density other than Ω
consistent with $q_0 < 0$ at the 2.8 σ and 3
confidence levels, for two different fitting
0.2, results in the weakest detection, $\Omega_\Lambda >$
For a flat universe prior ($\Omega_M + \Omega_\Lambda = 1$),
and 9 σ formal statistical significance for t
matter (i.e., $\Omega_M = 1$) is formally ruled out
methods. We estimate the dynamical age
tainties in the current Cepheid distance s
atic error, including progenitor and m
perturbations in the expansion rate, gravi
these effects appear to reconcile the data

THE ASTROPHYSICAL JOURNAL, 517:565–586, 1999 June 1

MEASUREMENTS OF Ω AND Λ FROM 42 HIGH-REDSHIFT SUPERNOVAE

S. PERLMUTTER,[1] G. ALDERING, G. GOLDHABER,[1] R. A. KNOP, P. NUGENT, P. G. CASTRO,[2] S. DEUSTUA, S. FABBRO,[3] A. GOOBAR,[4] D. E. GROOM, I. M. HOOK,[5] A. G. KIM,[1,6] M. Y. KIM, J. C. LEE,[7] N. J. NUNES,[2] R. PAIN,[3] C. R. PENNYPACKER,[8] AND R. QUIMBY
Institute for Nuclear and Particle Astrophysics, E. O. Lawrence Berkeley National Laboratory, Berkeley, CA 94720

C. LIDMAN
European Southern Observatory, La Silla, Chile

R. S. ELLIS, M. IRWIN, AND R. G. MCMAHON
Institute of Astronomy, Cambridge, England, UK

P. RUIZ-LAPUENTE
Department of Astronomy, University of Barcelona, Barcelona, Spain

N. WALTON
Isaac Newton Group, La Palma, Spain

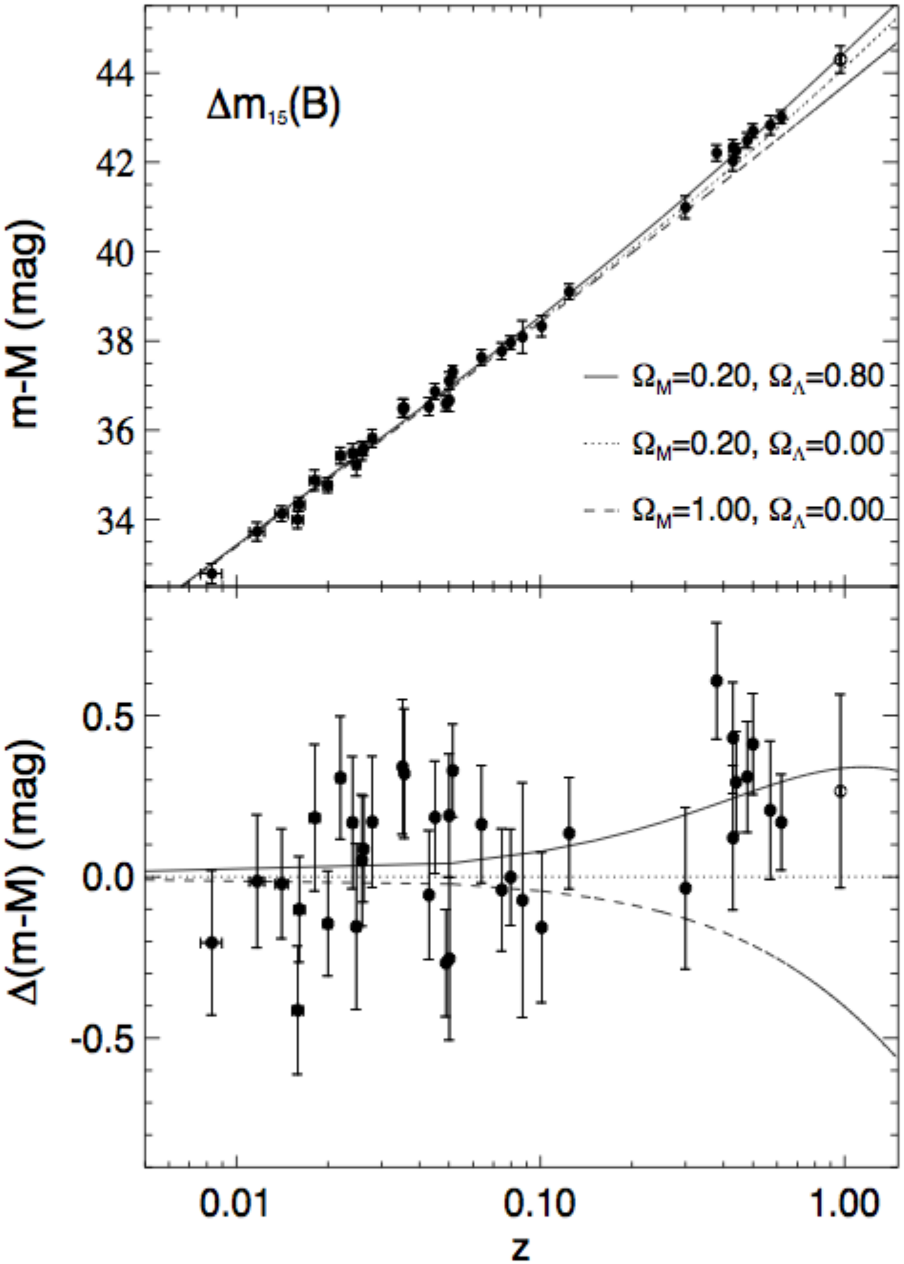
B. SCHAEFER
Department of Astronomy, Yale University, New Haven, CT

B. J. BOYLE
Anglo-Australian Observatory, Sydney, Australia

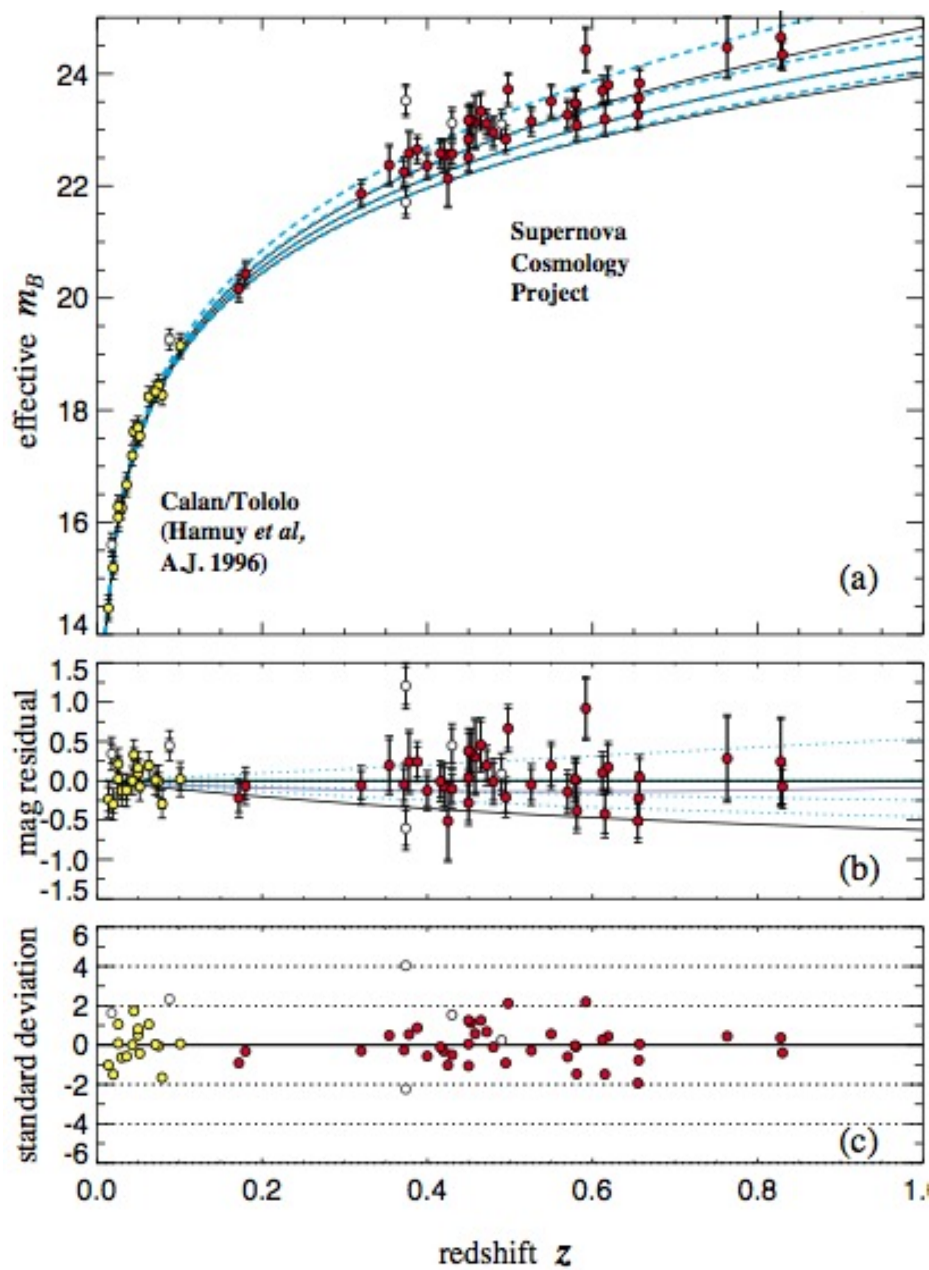
A. V FILIPPENKO AND T. MATHESON
Department of Astronomy, University of California, Berkeley, CA

A. S. FRUCHTER AND N. PANAGIA[9]
Space Telescope Science Institute, Baltimore, MD

H. J. M. NEWBERG

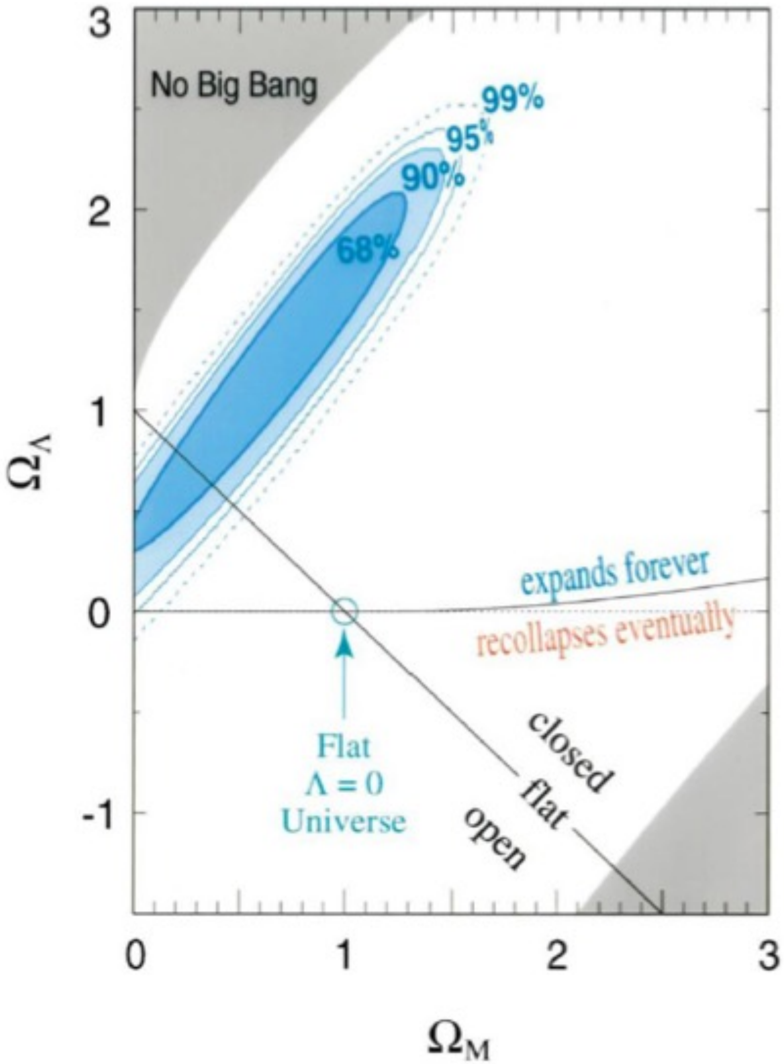


A. Riess et al, Astron.J.116:1009-1038 (1998)

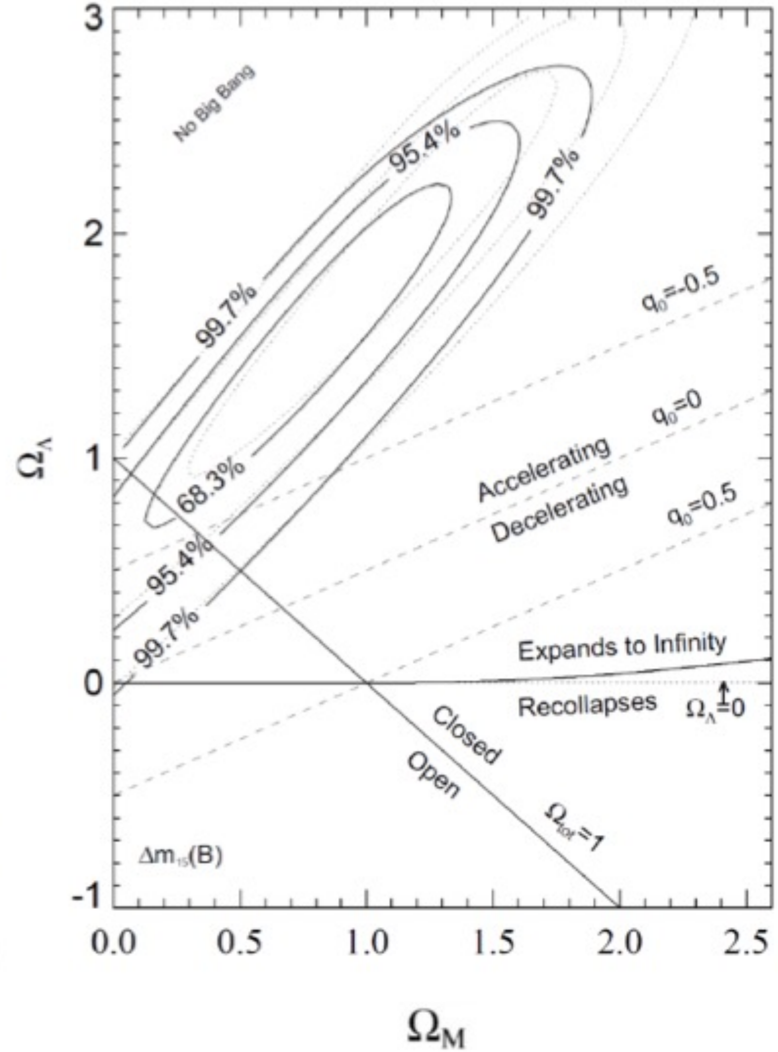


S. Perlmutter, Astrophys.J.517:565-586 (1999)

70% of the universe is Dark Energy!

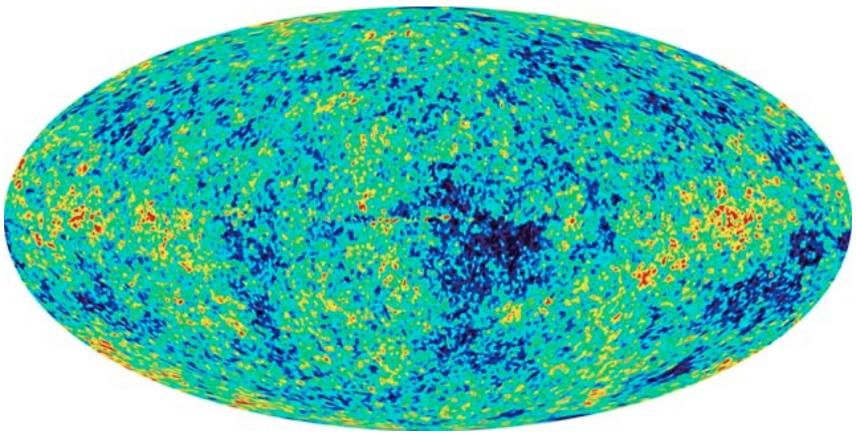


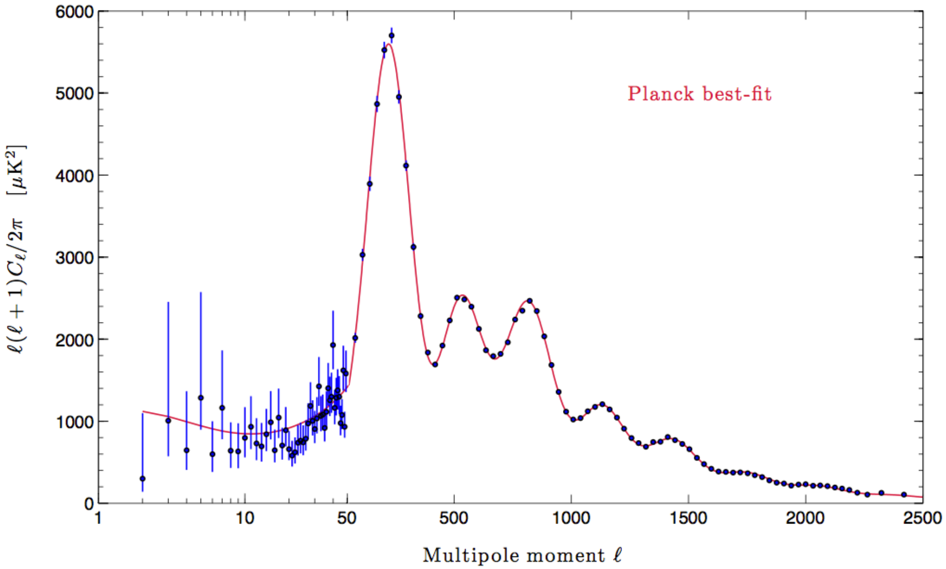
S. Perlmutter, Astrophys.J.517:565-586 (1999)



A. Riess et al, Astron.J.116:1009-1038 (1998)

Since 1998





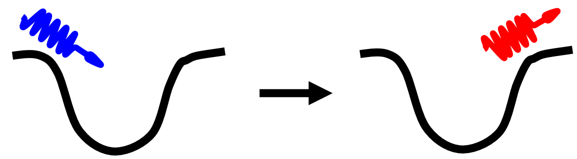
Spectacular CMB measurements by WMAP, Planck and other experiments

Millions of galaxy redshifts and shape distortions by gravitational lensing

1000+ supernovae, compared to 42+17 in 1998

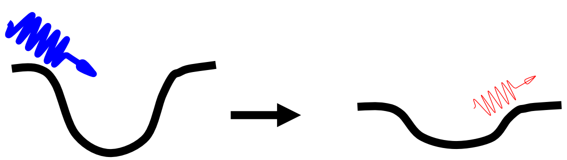
Integrated Sachs-Wolfe effect

$\Omega = \Omega_M$



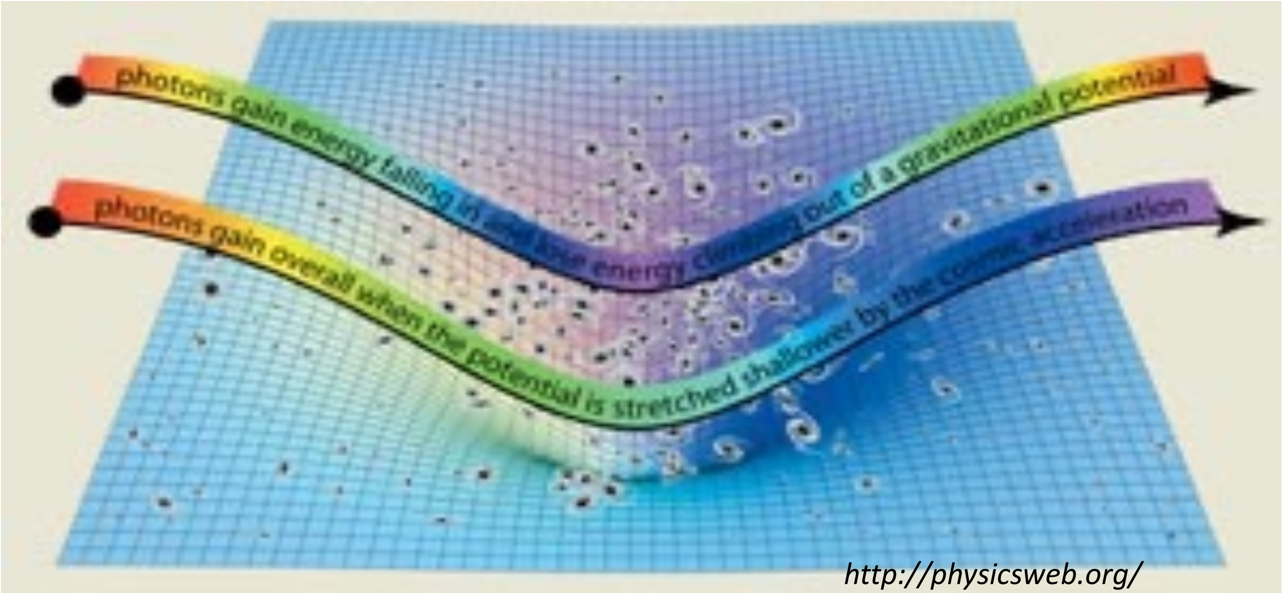
blueshift = redshift

$\Omega \neq \Omega_M$



blueshift ≠ redshift

Cosmic acceleration implies correlation between galaxies and ISW



R. Crittenden & N. Turok, astro-ph/9510072, Phys Rev Lett

"…the overall ISW signal is detected at the ~ 4.5 sigma level."
T. Giannantonio et al, arXiv:0801.4380, Phys Rev D

Acceleration is Beyond Reasonable Doubt

Photo: Ariel Zambelich, Copyright © Nobel Media AB

Saul Perlmutter

Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

Photo: Homewood Photography

Adam G. Riess

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*.

Why does the constant energy density result in accelerated expansion?

A "mathy" explanation:

- Vacuum energy has negative pressure, specifically, $P = -\rho$
- In General Relativity, both pressure and energy gravitate

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) = -\frac{4\pi G}{3}(1 + 3w)\rho$$

$$\boxed{\ddot{a} > 0 \quad \text{if} \quad w \equiv \frac{p}{\rho} < -\frac{1}{3}}$$

- For Lambda (vacuum), the repulsive gravity of negative pressure overcomes the attractive gravity positive energy

Why does the constant energy density result in accelerated expansion?

An attempt at a more intuitive explanation:

We are used to the notion of curvature causing acceleration

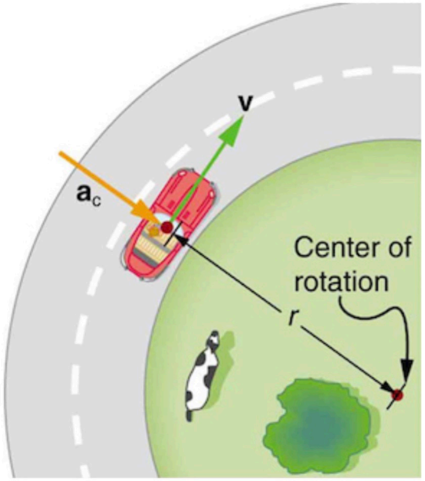


Image credit: Openstax College Physics

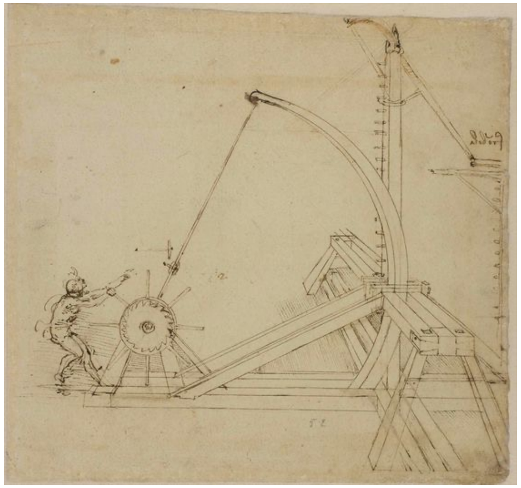


Image credit: Leonardo da Vinci

Constant vacuum energy density => constant scalar curvature ($R=2\Lambda$)

The universe is trying to "straighten" itself out by expanding, but the curvature stays constant

How much energy is in the vacuum?

We observe the sum: $\rho^{(\mathrm{vac}+\Lambda)} \sim 10^3\ \mathrm{eV/cm}^3$

$$\mathrm{Mass}\ \ 1\ \mathrm{eV}/c^2 \quad \approx \quad 1.8 \times 10^{-36}\ \mathrm{kg}$$

$$\frac{\mathrm{Mass}}{\mathrm{Volume}} \sim 10^{-30}\ \mathrm{g/cm}^3$$

Theory predicts $\rho^{(\mathrm{vac})}_{\mathrm{theory}} \sim 10^{120} \rho^{(\mathrm{vac}+\Lambda)}_{\mathrm{obs}}$

Requires a technically unnatural tuning of Lambda

The Two Cosmological Constant Problems

The old problem: *What is the vacuum energy and how does it gravitate?*

The new problem (Dark Energy): *What sets the observed value of Lambda?*

A possible “solution” to both problems is *anthropic:*

- IF the microscopic theory has an enormous number of candidate vacua, with the cosmological constant differing from vacuum to vacuum (e.g. the string theory landscape)

- IF the microscopic theory includes a mechanism to sample many of these vacua somewhere in space at some time over the history of the universe (e.g. eternal inflation)

- THEN, observers like us would only exist in the parts of the universe where the vacuum energy is comparable to the observed Dark Energy density

Weinberg 1989, Vilenkin 1995

Is there a reason for cosmologists to think beyond Lambda?

A popular viewpoint	A different viewpoint
General Relativity works great in our solar system and is appealing for its uniqueness and elegance	We know that we will need to extend General Relativity to make it compatible with quantum theory
The LCDM model works well for explaining observations	We have another dark component that we had to invent to make it all work
There are no compelling alternative models at this time	The universe surprised us before… Perhaps a reason to keep an open mind
Let's work with LCDM and let the theorists work out what the vacuum energy is and how it gravitates	The data allows us to measure more than just the LCDM parameters. Why not look for physics beyond LCDM?

What does Cosmology test?

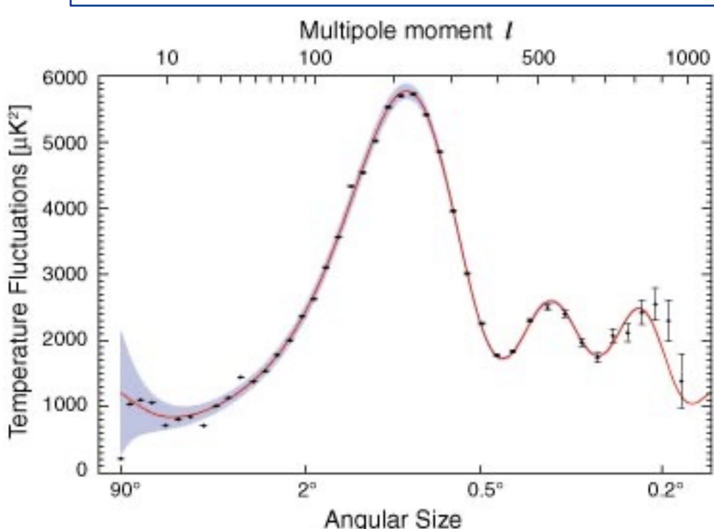
FRW Metric + Initial conditions (Inflation) + Content: CDM, baryons, photons, neutrinos, DE

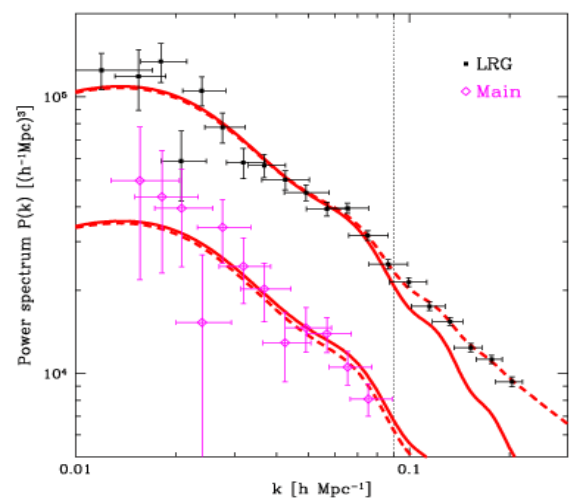
↓

Conservation Equations
Einstein's Equation
for the background and small perturbations

↓

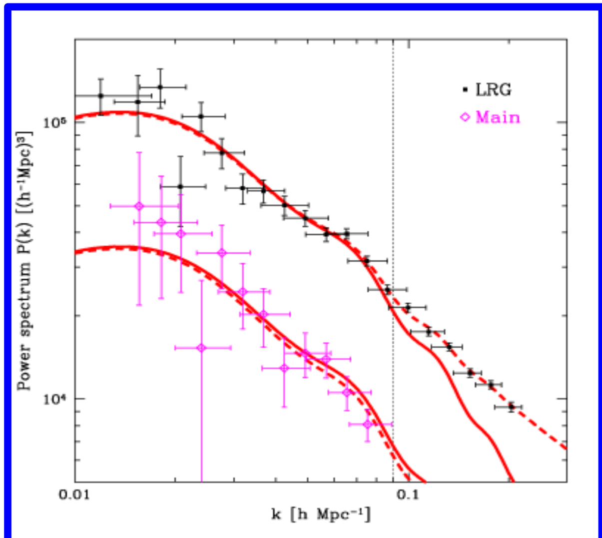
Observables: statistics of CMB, galaxy distribution, …

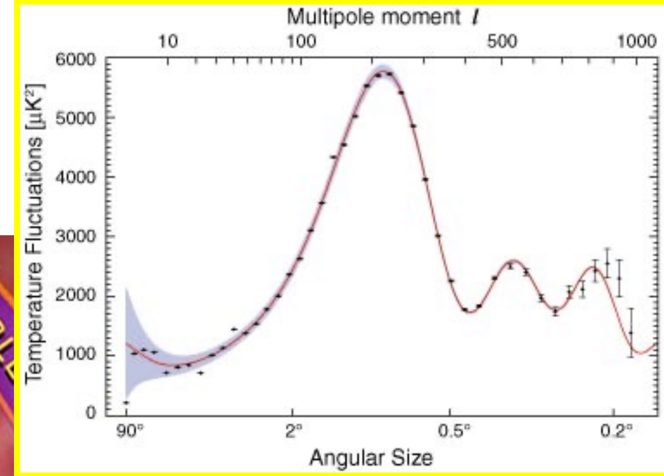


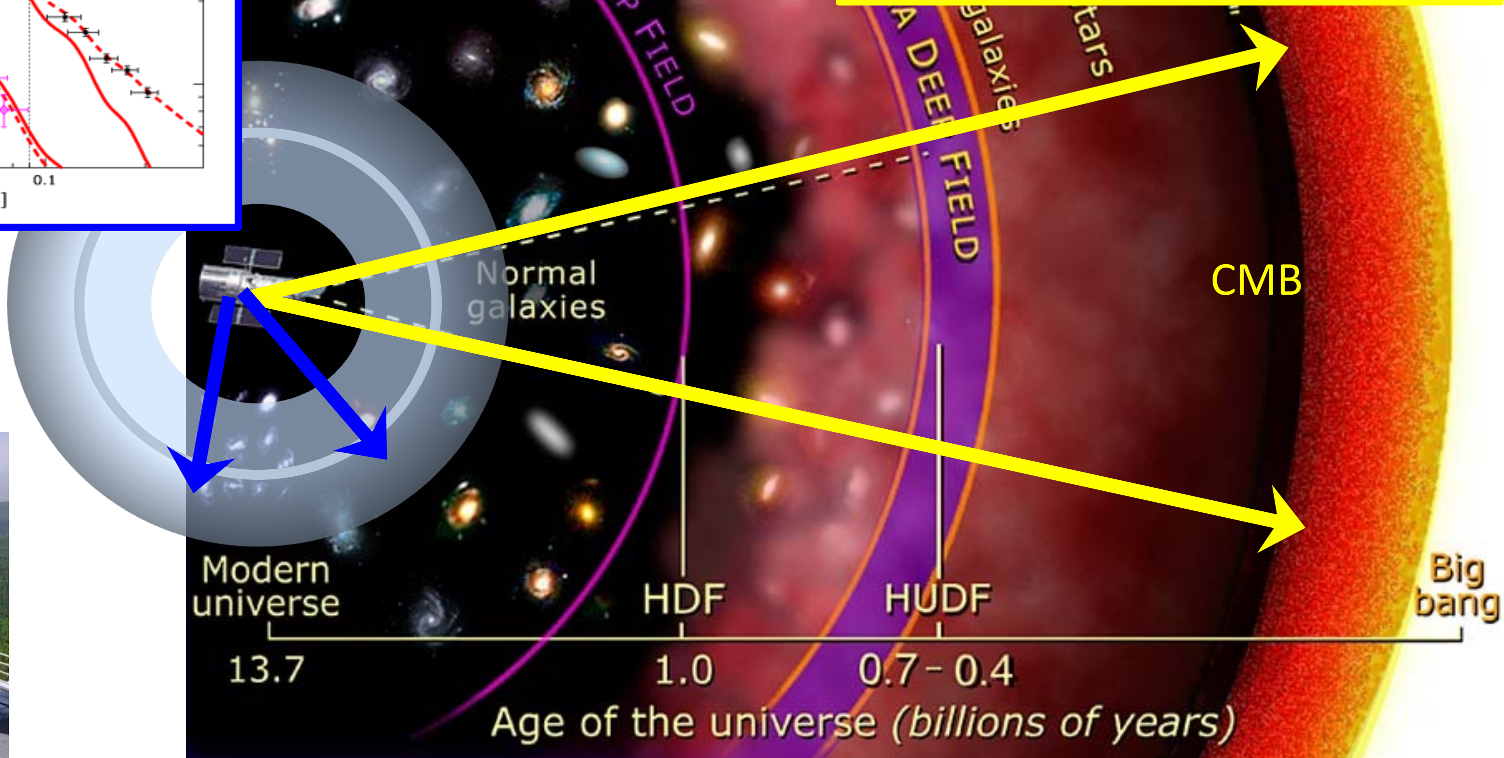


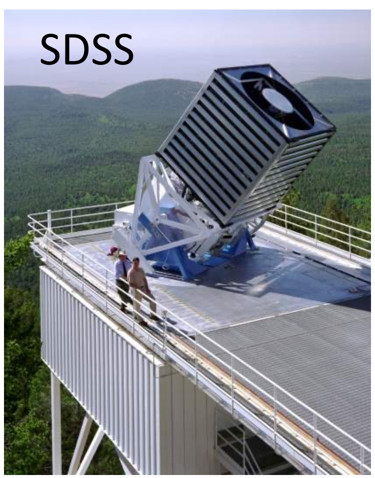
Yesterday

Matter Spectrum (SDSS)



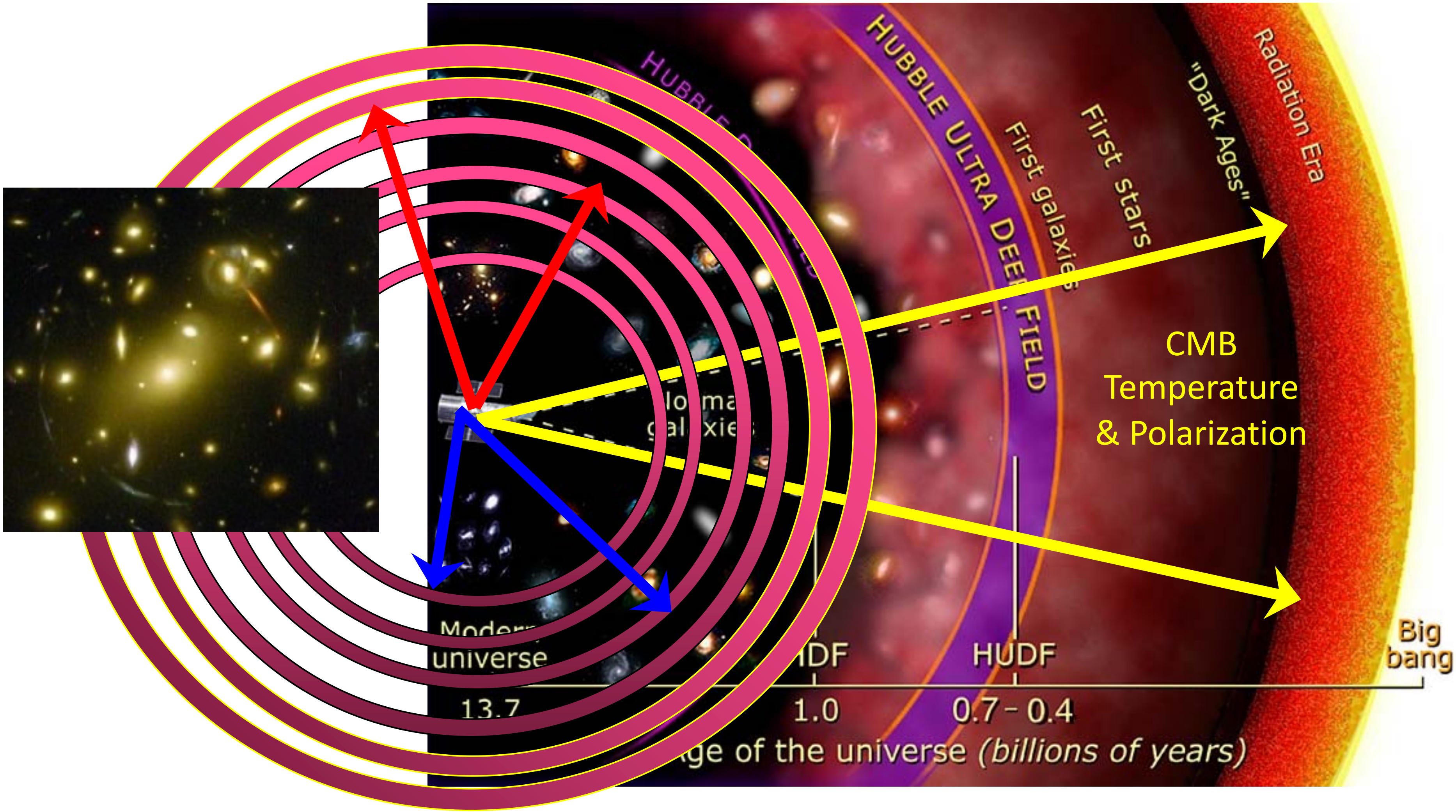






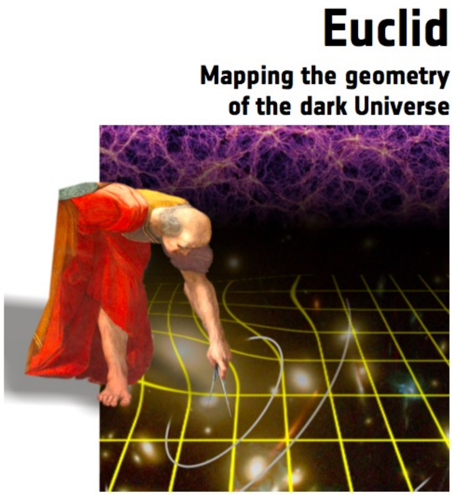
Today and tomorrow

Weak gravitational lensing of galaxies



Galaxy counts and redshifts: evolution of structures through several epochs

Questions we could ask



Is data consistent with Lambda?

Assuming Dark Energy is dynamical, what are its properties?

Is the evolution of cosmic structure consistent with General Relativity?

What are constraints on alternative theories of gravity?

Part II

Dark Energy and its equation of state

A quick refresher on some tensor math

The FRW metric $$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) d\mathbf{x}^2$$

Raising indices $$T^\mu_\nu = g^{\mu\alpha} T_{\alpha\nu}$$

Energy-momentum tensor $$T^0_0 = -\rho \ , \quad T^0_i = -T^i_0 = p_i \ , \quad T^i_j = P\delta^j_i + \Sigma^j_i$$

For a homogeneous universe $$T^\mu_\nu = \mathrm{diag}[-\rho, P, P, P]$$

For vacuum energy, $P = -\rho$

Conservation of EMT $$\nabla^\mu T_{\mu\nu} = 0$$

$$\frac{\partial \rho}{\partial t} + 3\frac{\dot{a}}{a}[\rho + P] = 0$$

for a constant w $$\rho = \rho_0 \ a^{-3(1+w)}$$

Dark Energy as a cosmological fluid

For a conserved, homogeneous fluid,

$$\dot{\rho}_{\rm DE} + 3H(\rho_{\rm DE} + p_{\rm DE}) = 0$$

$$w(a) = \frac{p_{\rm DE}(a)}{\rho_{\rm DE}(a)}$$

$$\rho_{\rm DE}(a) = \rho_0 \exp\left[\int_a^1 3(1 + w(a'))\frac{da'}{a'}\right]$$

w(a) fully specifies the background dynamics of DE

Note that this assumes that DE energy density does not change its sign (Why would one worry about that?)

Dark Energy perturbations

Because the metric and the matter have inhomogeneities, a dynamical Dark Energy is necessarily inhomogeneous

Stress-energy perturbations of a general fluid can be described by fluctuations of its *density, momentum, pressure and shear*

Conservation of energy-momentum provides two equations, so one needs two additional state functions (in addition to w) to describe perturbations: e.g the effective speed of sound (c_S^2) and viscosity (c_{vis}^2):

c_S^2 ~ pressure perturbation / density perturbation
c_{vis}^2 ~ shear / velocity

c_S^2 sets the length scale below which DE is smooth

(for quintessence, c_S^2=1, implying smoothness on sub-horizon scales)

Structure Formation with Generalized Dark Matter, Wayne Hu, astro-ph/9801234

Scalar field Dark Energy

General Relativity with a minimally coupled scalar field AKA "quintessence"

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} \{ R - \partial^\mu \phi \partial_\mu \phi - 2V(\phi) \} + \mathcal{L}_M(g_{\mu\nu}, \psi) \right]$$

Compare this to GR+Lambda: $S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} \{ R - 2\Lambda \} + \mathcal{L}_M(g_{\mu\nu}, \psi) \right]$

Quintessence energy-momentum:

$$T^\phi_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi + V(\phi) \right]$$

Quintessence equation of state:

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \geq -1$$

A physicist’s view on scalar field Dark Energy

General Relativity with a minimally coupled scalar field AKA ”quintessence”

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}\left\{R - \partial^\mu\phi\partial_\mu\phi - 2V(\phi)\right\} + \mathcal{L}_M(g_{\mu\nu},\psi)\right]$$

In order to be Dark Energy, the scalar field must be very light

A light scalar field is likely to couple non-minimally to ordinary matter and mediate a long-range force, which is strongly constrained

If we observe w(a)=-1, we can expect to find other effects, such as fifth forces or birefringence

Quintessence and the Rest of the World, Sean Carroll, astro-ph/9806099, Phys Rev Lett

Measuring the equation of state

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left\{ \frac{\Omega_{\rm r}}{a^4} + \frac{\Omega_{\rm M}}{a^3} + \frac{\rho_{\rm DE}(a)}{\rho_c} \right\}$$

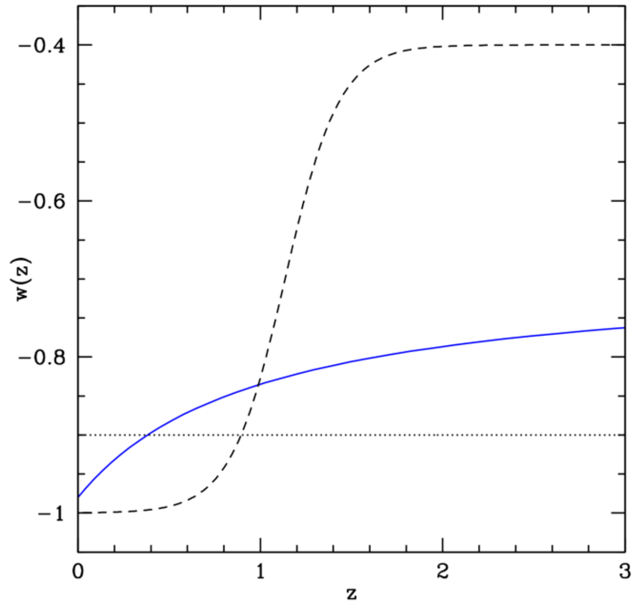
$$\rho_{\rm DE}(a) = \rho_0 \exp\left[\int_a^1 3(1 + w(a'))\frac{da'}{a'}\right]$$

Try a constant *w*

$$\rho \propto a^{-3(1+w)}$$

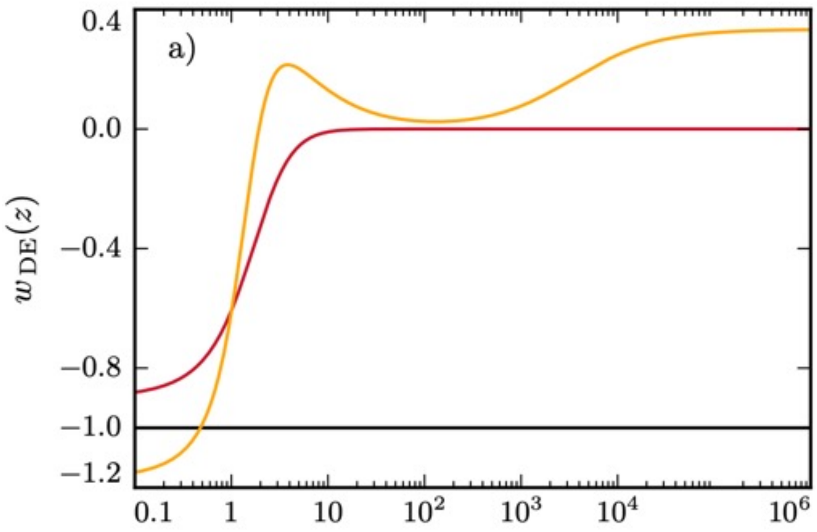
Fitting a constant parameter *w* measures a certain weighted average

$$\langle w \rangle \equiv \frac{\int_{a_{\rm LS}}^1 da\ w(a)\ \Omega_D(a)}{\int_{a_{\rm LS}}^1 da\ \Omega_D(a)}$$



Varying equation of state

Simple parameterizations, such as $w(a)=w_0+(1-a)w_a$, provide a reasonable approximation for slowly evolving quintessence, but may fail to capture signatures of rapid transitions or modified gravity



M. Raveri, P. Bull, A. Silvestri, LP, arXiv:1703.05297, PRD

We will consider "non-parametric", "model-agnostic" methods to reconstruct w(a) from the data

Can one have w < -1?

Quintessence scalar field Dark Energy has w ≥ -1

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \geq -1$$

One can get w < -1 if the *kinetic energy is negative*, or kinetic energy has an unconventional form (*k-essence*)

R. Caldwell, astro-ph/9908168
Armendariz-Picon, Mukhanov, Steinhardt, astro-ph/0004134

Negative kinetic energy means you can produce particles out of nothing, hence the name: ”ghost” AKA ”phantom”

A fluid with w < -1 is really weird – its density *increases* with the expansion!

$$\rho \propto a^{-3(1+w)}$$

The “phantom divide”

Perturbations of a single fluid are unstable when w(a) evolves across w = - 1, which makes observational studies of w challenging

It helps to be able to sample all values of w

Modern cosmological Boltzmann codes, such as CAMB and CLASS, evolve DE perturbations using the Parameterized Post-Friedmann (PPF) method that allows for the phantom crossing

Parameterized Post-Friedmann Signatures of Acceleration in the CMB, Wayne Hu, arXiv:0801.2433
Crossing the Phantom Divide with Parameterized Post-Friedmann Dark Energy, W. Fang, W. Hu, A. Lewis, arXiv:0808.3125

Reconstructing *w(a)* from the data

Step 1: discretize $w(a)$ into N values at a_i, so that $w_i = w(a_i)$

Step 2: treat w_i as model parameters and fit to the data

Practical implementation requires making decisions:

- How do you turn w_i into a continuous function $w(a)$?
  - Cubic spline?
  - Step-like functions, such as tanh?

  Ideally, you want your results to be independent of the choice

- How big should N be?
  - if N is too small, we may miss a feature of the function
  - if N is too large, there is a large degeneracy and none of w_i's is constrained

One cannot proceed without having some idea in mind of what $w(a)$ could be, i.e. we need a *theoretical prior*

What do we mean when we say "prior"?

Bayes' theorem:

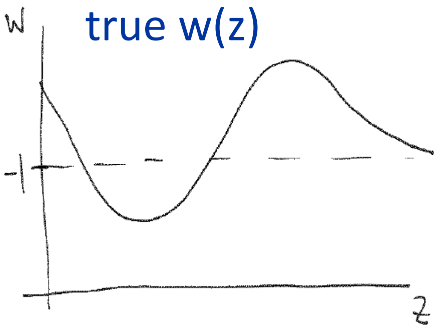
$$P(A \mid B) = \frac{P(B \mid A) P(A)}{P(B)}$$

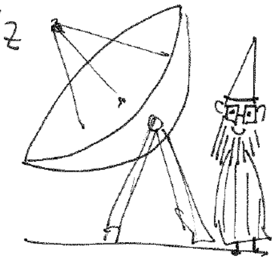
Posterior probability for w_i:

$$\mathcal{P}(\mathbf{w}|\mathrm{data}) = \mathcal{P}(\mathrm{data}|\mathbf{w}) \times \mathcal{P}_{\mathrm{prior}}(\mathbf{w})$$

E.g. a Gaussian prior:

$$\chi^2_{\mathrm{prior}} = -2 \ln \mathcal{P}_{\mathrm{prior}} = (\mathbf{w} - \mathbf{w}^{\mathrm{fid}})^T \mathbf{C}^{-1} (\mathbf{w} - \mathbf{w}^{\mathrm{fid}})$$

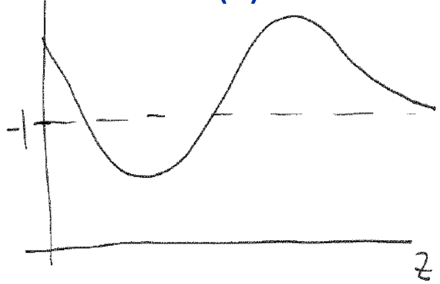


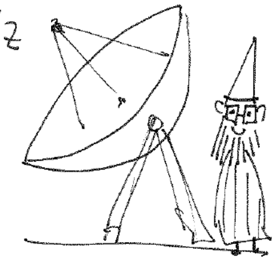


MCMC fit
using many w-bins

no prior

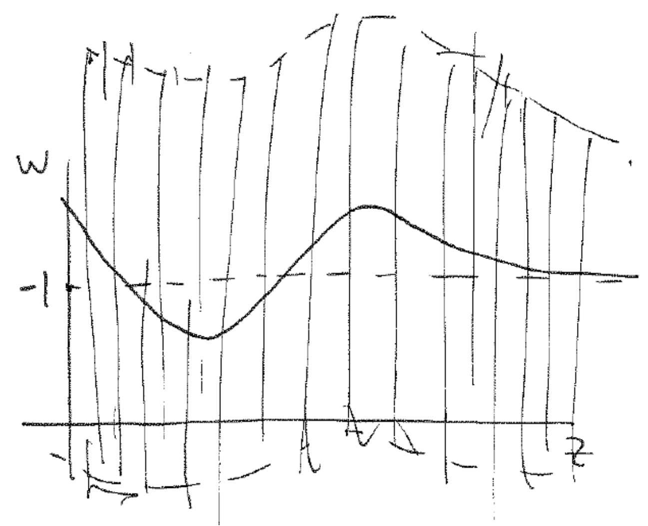
true w(z)



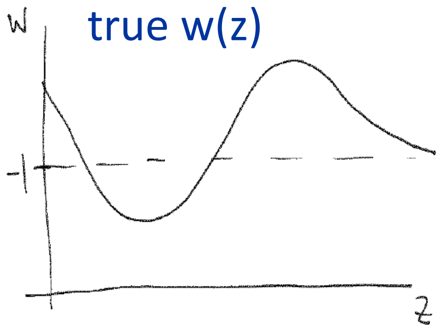


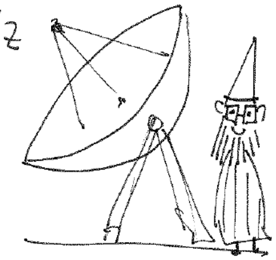
MCMC fit
using many w-bins

reconstructed w(z)



- large variance
- zero bias

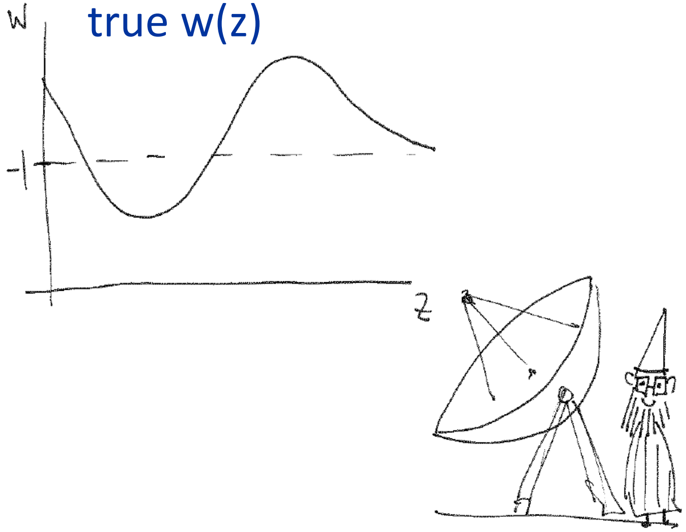




$$\chi^2_{\rm prior} = -2\ln\mathcal{P}_{\rm prior} = (\mathbf{w}-\mathbf{w}^{\rm fid})^T\mathbf{C}^{-1}(\mathbf{w}-\mathbf{w}^{\rm fid})$$

MCMC fit
using many w-bins

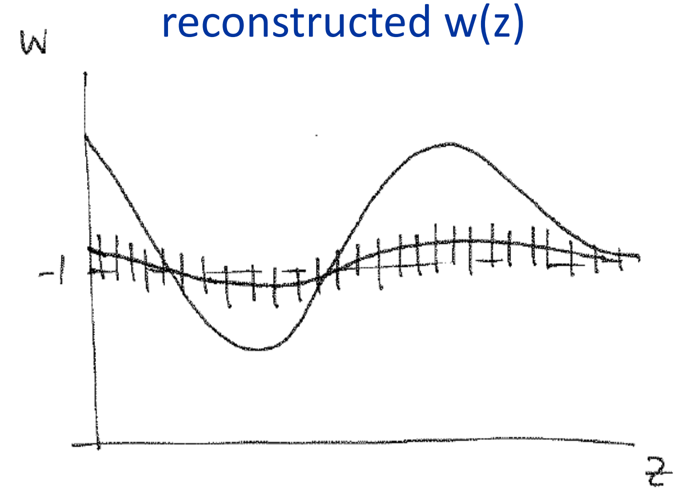
true w(z)



Excessively strong prior

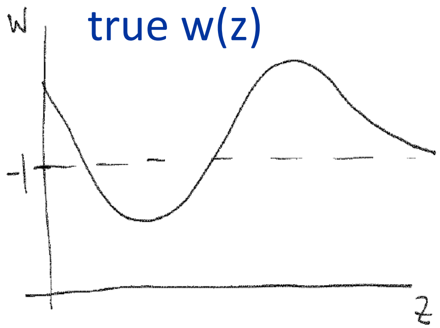
MCMC fit
using many w-bins

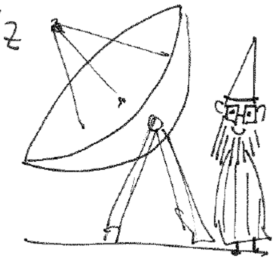
reconstructed w(z)



- ○ tiny error bars (small variance)
- ○ large bias

true w(z)

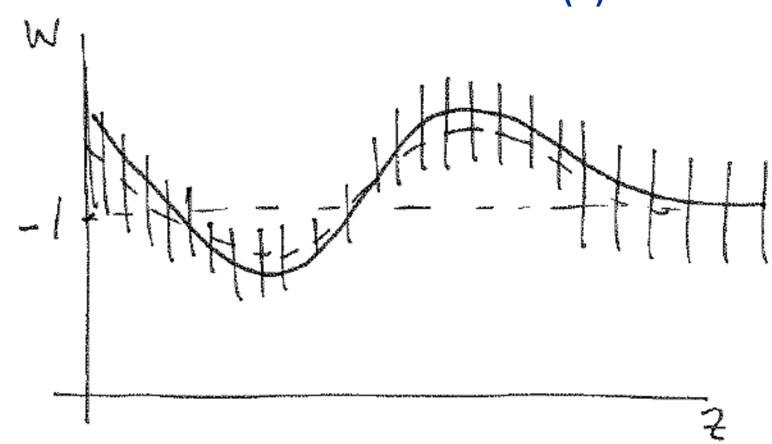




reasonable prior

MCMC fit
using many w-bins

reconstructed w(z)



- moderate variance
- insignificant bias, i.e. the bias is smaller than the variance

Reconstructing w(a) with a smoothness prior

Smooth features are well constrained by the data, not biased by the prior

Noisy features are poorly constrained by the data, determined by the prior

Fables of Reconstruction, Crittenden, Zhao, LP, Samushia, Zhang, 1112.1693, JCAP

Approach 1: adopt a ”reasonable” analytical form for the w(a) correlation function, apply it to reconstruct of w(a) from the data, use Principal Component Analysis (PCA) and Bayesian evidence to interpret results

Approach 2: perform simulations to generate many Dark Energy histories, deduce the expected covariance of *w(a)* at different *a*, use it as your theoretical prior in the reconstruction

The prior covariance matrix

- The correlation function:

$$\xi_w(|a-a'|) \equiv \left\langle [w(a) - w^{\rm fid}(a)][w(a') - w^{\rm fid}(a')] \right\rangle$$

- The functional form:

$$\xi_w(\delta a) = \frac{\xi_w(0)}{1 + (\delta a/a_c)^2} \qquad \delta a \equiv |a - a'|$$

- Build a covariance matrix from the correlation function

$$C_{ij} \equiv \langle \delta w_i \delta w_j \rangle = \frac{1}{\Delta^2} \int_{a_i}^{a_i+\Delta} da \int_{a_j}^{a_j+\Delta} da' \xi_w(|a-a'|).$$

- Build a Gaussian prior from the covariance matrix

$$\chi^2_{\rm prior} = -2 \ln \mathcal{P}_{\rm prior} = (\mathbf{w} - \mathbf{w}^{\rm fid})^T \mathbf{C}^{-1} (\mathbf{w} - \mathbf{w}^{\rm fid})$$

Fables of Reconstruction, Crittenden, Zhao, LP, Samushia, Zhang, 1112.1693, JCAP

- Use MCMC to fit a large number of w_i bins to data

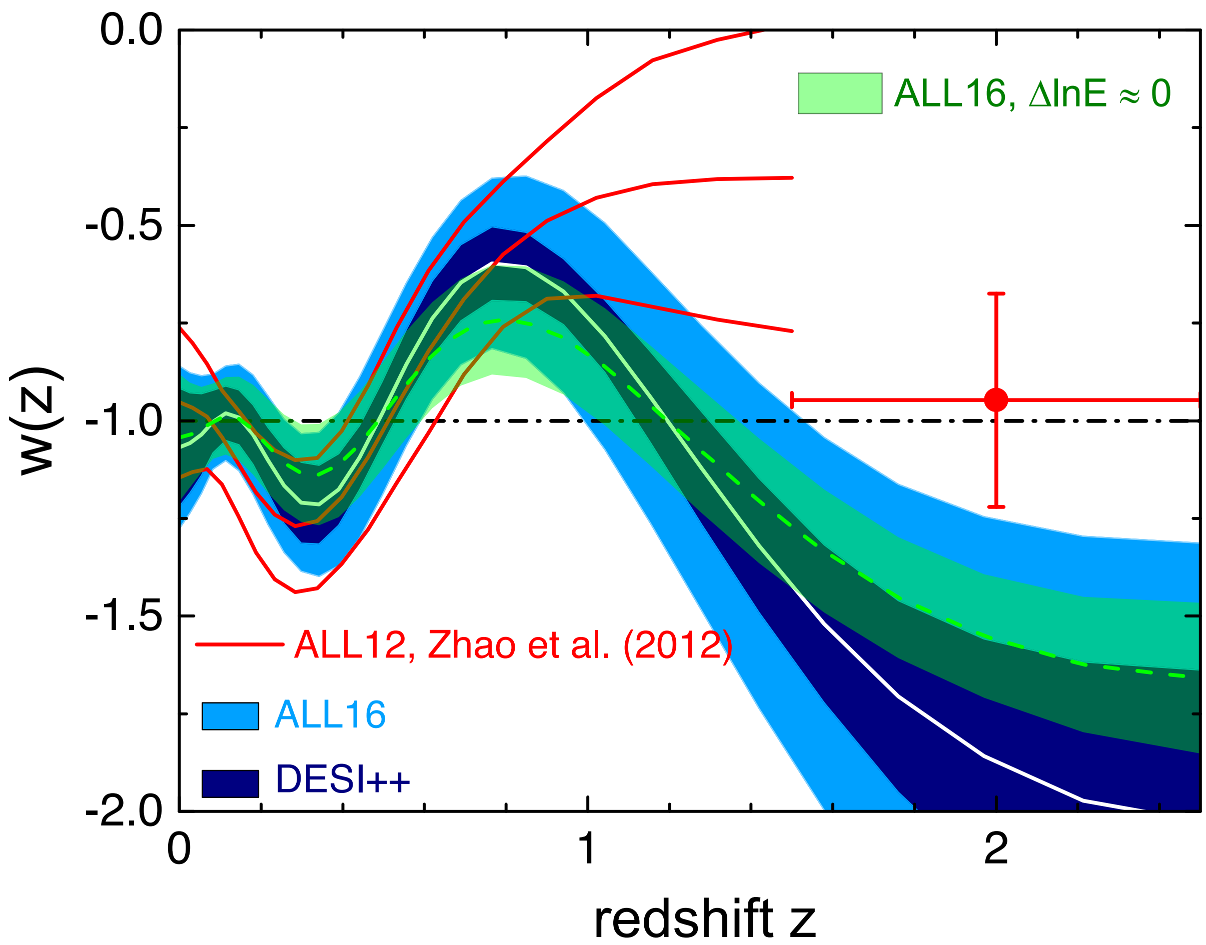
$$\chi^2 = \chi^2_{\rm data} + \chi^2_{\rm prior}$$

- Two prior parameters are chosen and kept fixed

  - “correlation scale” a_c
  - the variance in the mean w

$$\sigma^2_{\bar{w}} \equiv \int_{a_{\rm min}}^{1} \int_{a_{\rm min}}^{1} \frac{da\ da'\ \xi_w(a-a')}{(1-a_{\rm min})^2} \simeq \frac{\pi \xi(0) a_c}{1-a_{\rm min}}$$

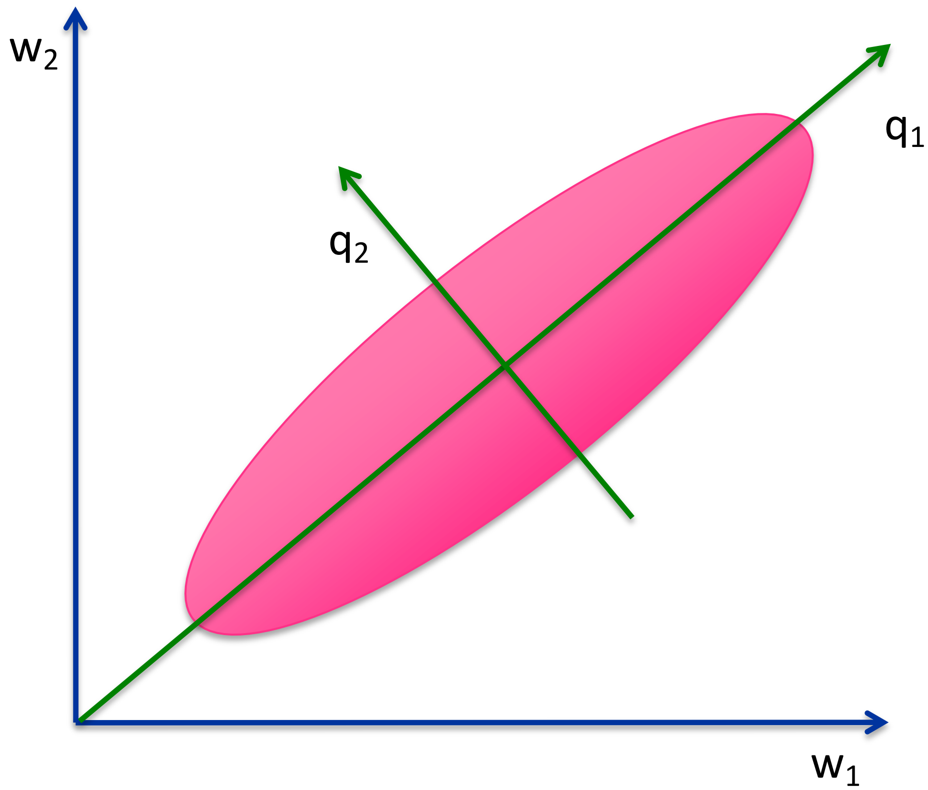
Dynamical dark energy in light of the latest observations



G.-B. Zhao et al, arXiv:1701.08165, Nature Astronomy

Principal Component Analysis:

decorrelating correlated parameters



Principal Component Analysis of w_i

D. Huterer and G. Starkman, astro-ph/0207517
R. Crittenden, L.P., G.-B. Zhao, astro-ph/0510293

Start with the covariance matrix:

$$C_{ij} \equiv \langle (w_i - \bar{w}_i)(w_j - \bar{w}_j) \rangle \neq 0 \quad \text{for } i \neq j$$

Then, decorrelate to define the "rotated" parameters q_i:

$$C = W^T \Lambda W \;\; ; \quad \Lambda_{ij} = \lambda_i \delta_{ij}$$

$$q_i = \sum_{j=1}^{N} W_{ij} w_j$$

$$\langle (q_i - \bar{q}_i)(q_j - \bar{q}_j) \rangle = \lambda_i \delta_{ij}$$

and their uncertainties:

$$\lambda_i = \sigma^2(q_i)$$

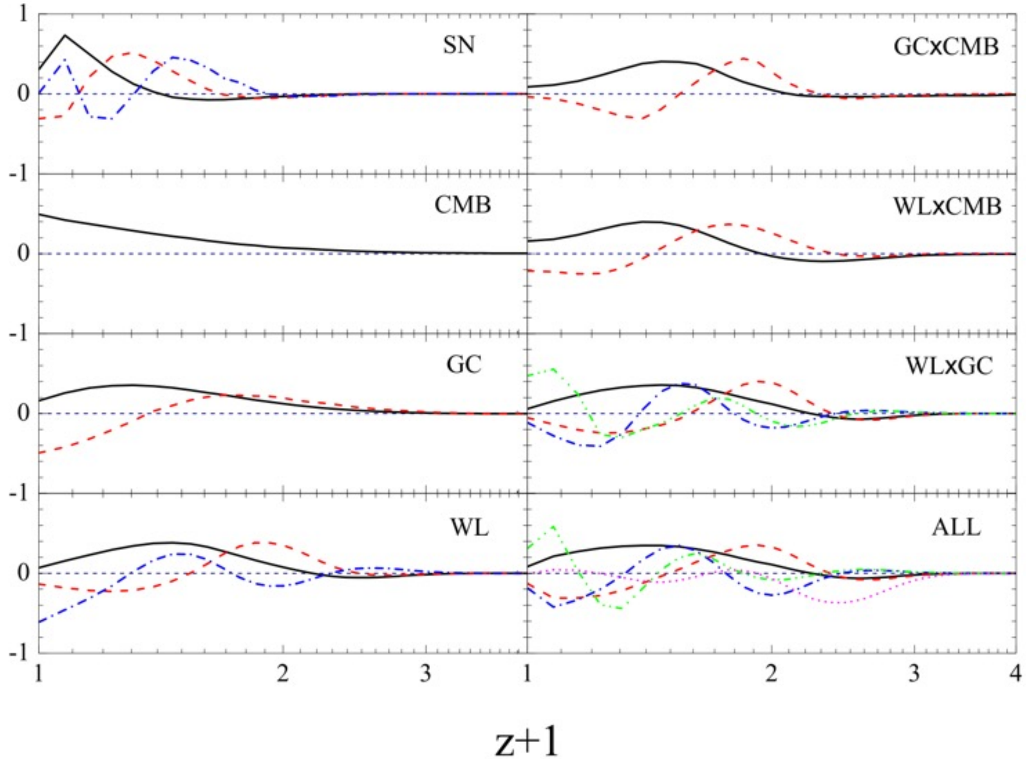
Define the eigenmodes $e(z_i)$:

$$w_i \equiv 1 + w(z_i) = \sum_{j=1}^{N} W_{ij}^T q_j \equiv \sum_{j=1}^{N} e_j(z_i) q_j$$

$$_{N \to \infty} \rightarrow 1 + w(z) = \sum_{j=1}^{N} e_j(z) q_j$$

The eigenmodes of w(z)

For each dataset, plot the best constrained eigenmodes, i.e. those with smallest λ_i

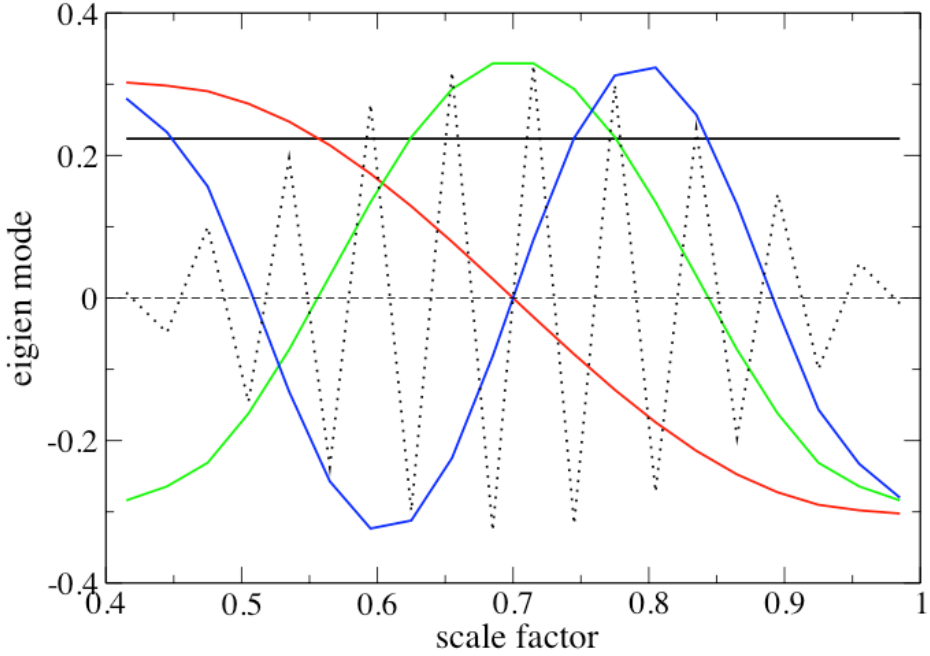


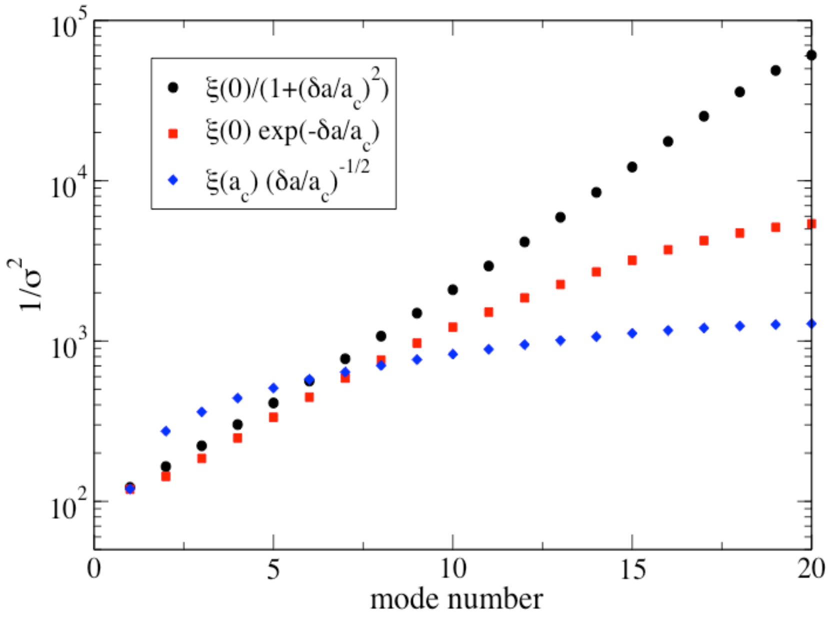
This tells you what features of w(z) a given dataset can probe

R. Crittenden, L.P., G.-B. Zhao, astro-ph/0510293

Do the same for the prior

Perform PCA to find the eigenmodes and eigenvalues of the prior covariance matrix

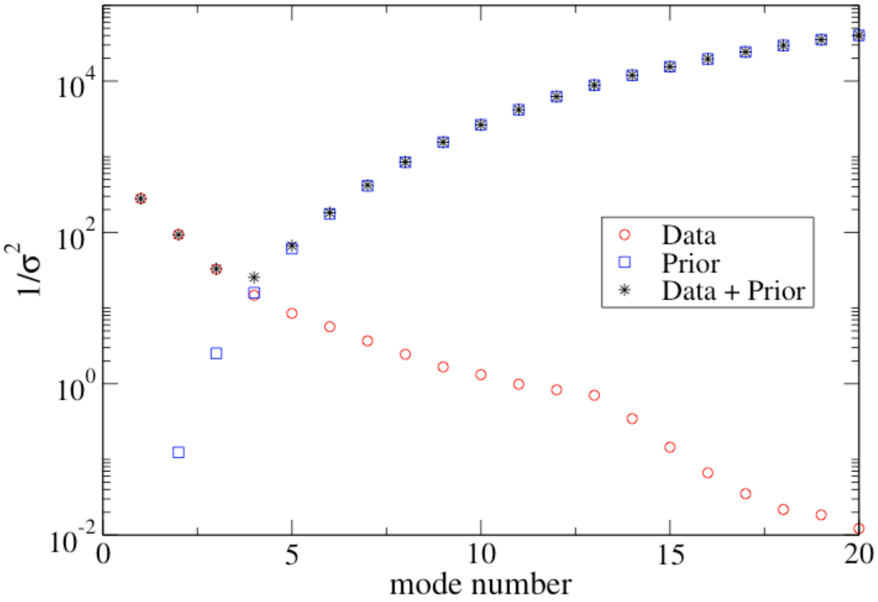


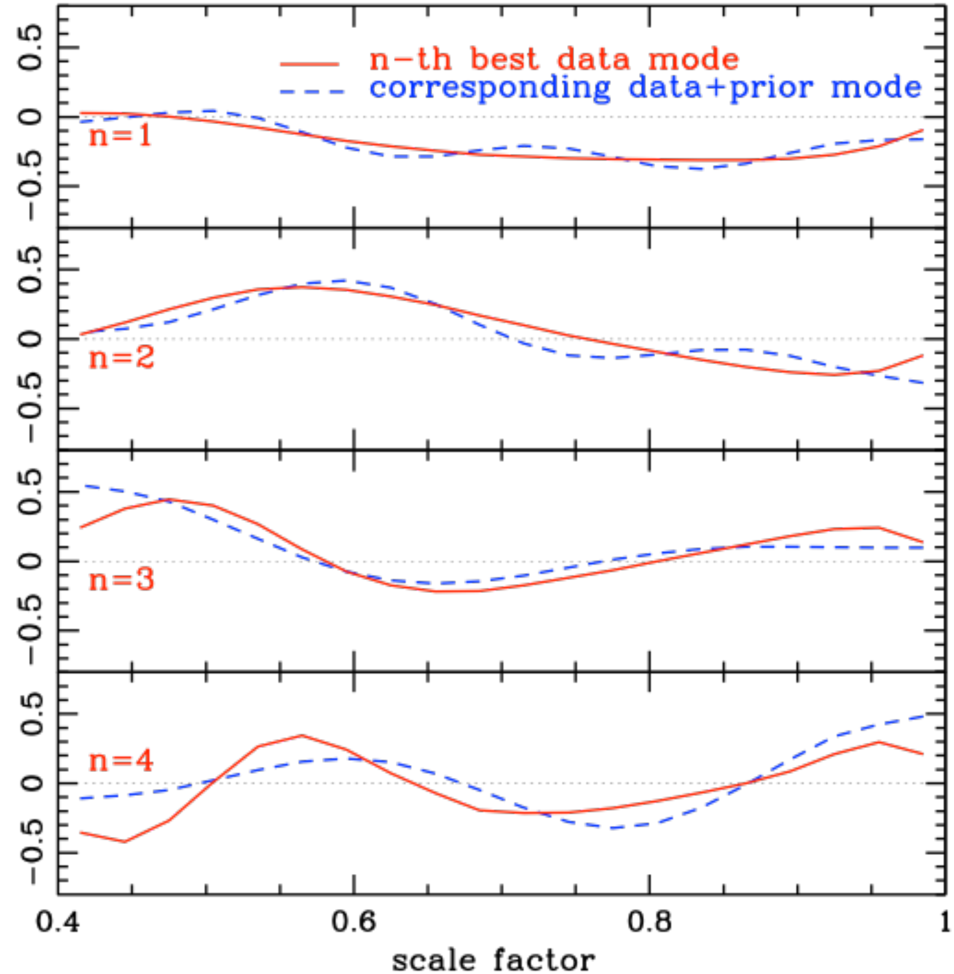


Fables of Reconstruction, Crittenden, Zhao, LP, Samushia, Zhang, 1112.1693, JCAP

Now do the same for the posterior

Find the eigenmodes and eigenvalues of the data+prior covariance

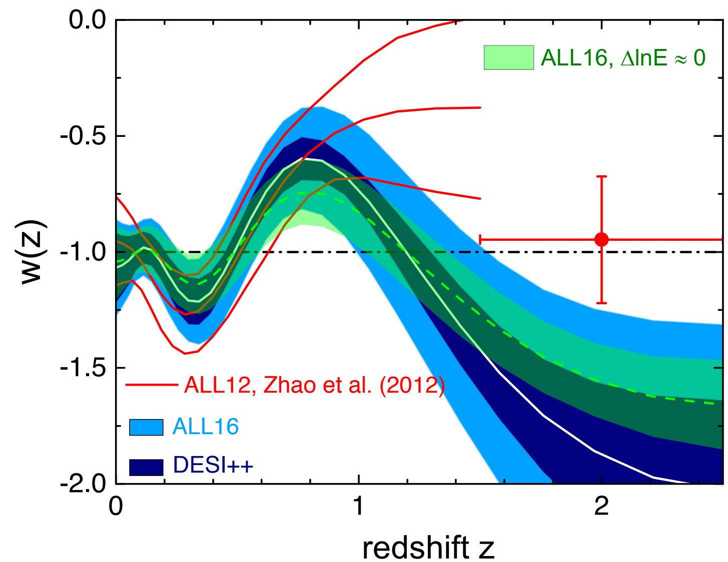


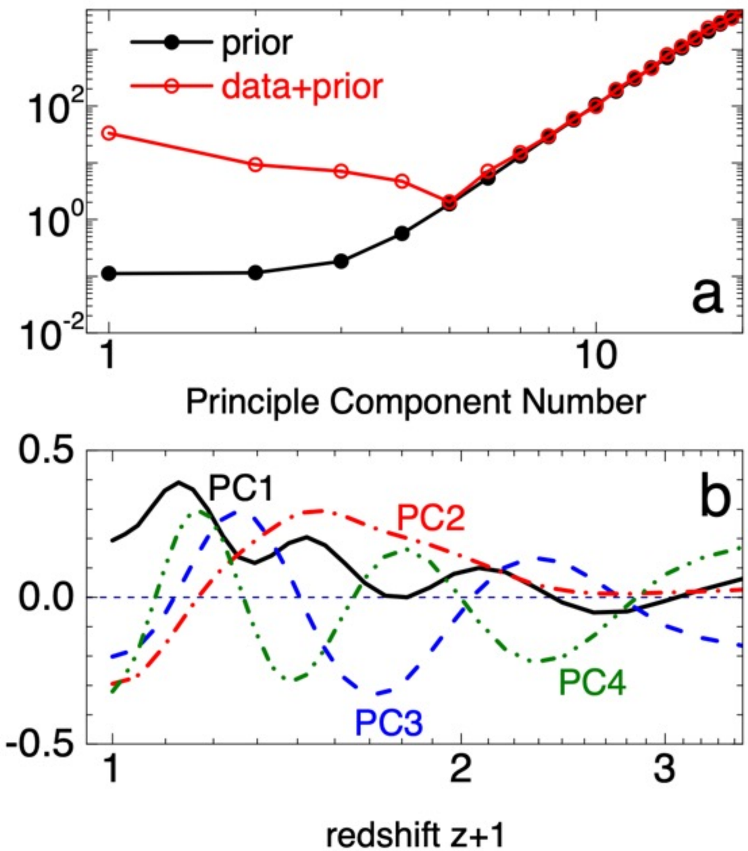


this dataset constrains 4 eigenmodes of w(a) relative to this prior

Interpreting the reconstruction: surviving eigenmodes

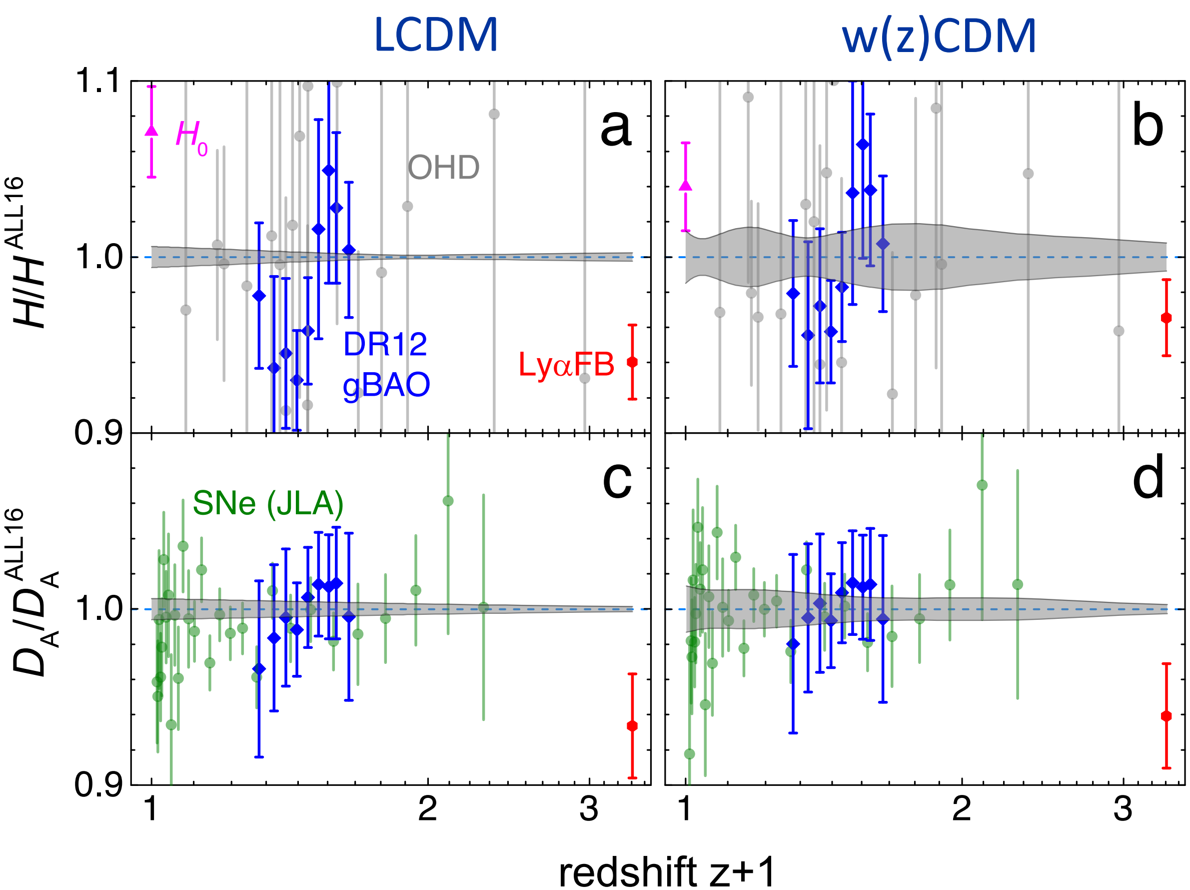
Perform a PCA of the posterior (prior+data), and a PCA of the prior covariance, to determine how many eigenmodes "survived" the prior





This tells you the extent to which the prior erases the fine features of w(a)

Interpreting the reconstruction: look at the data



Interpreting the reconstruction: look at the evidence

What is the Bayesian evidence for a varying w(z)?

$$E \equiv \int d\theta \, \mathcal{L}(\mathbf{D}|\theta) \, P(\theta)$$

How does it compare to the evidence for LCDM?

Interpreting the reconstruction: look at the evidence

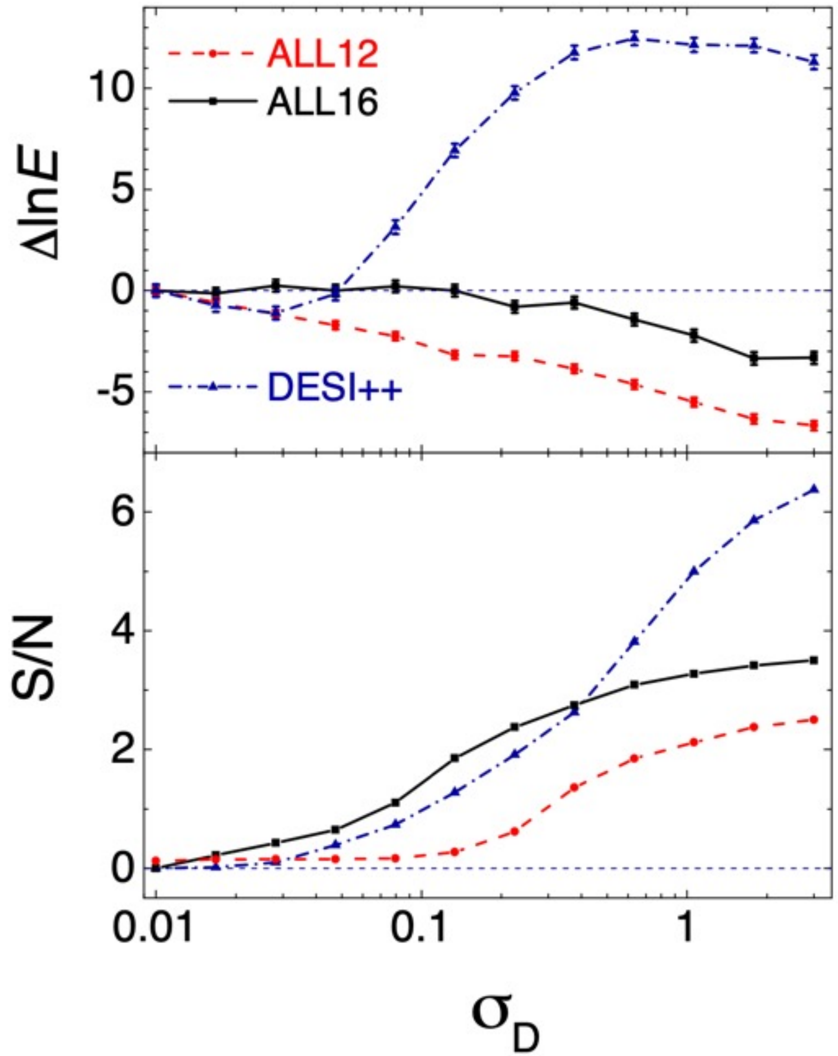
What is the Bayesian evidence for a varying w(z)?

$$E \equiv \int d\theta \, \mathcal{L}(\mathbf{D}|\theta) \, P(\theta)$$

How does it compare to the evidence for LCDM?

How does the ratio of evidences, AKA the Bayes factor, change as we vary the strength of the prior?

No preference for varying w(z) if the Bayes factor is small and if it depends strongly on the strength of the prior



Another approach – derive the prior covariance from theory

arXiv.org > astro-ph > arXiv:1703.05297

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Astrophysics > Cosmology and Nongalactic Astrophysics

Priors on the effective Dark Energy equation of state in scalar-tensor theories

Marco Raveri, Philip Bull, Alessandra Silvestri, Levon Pogosian

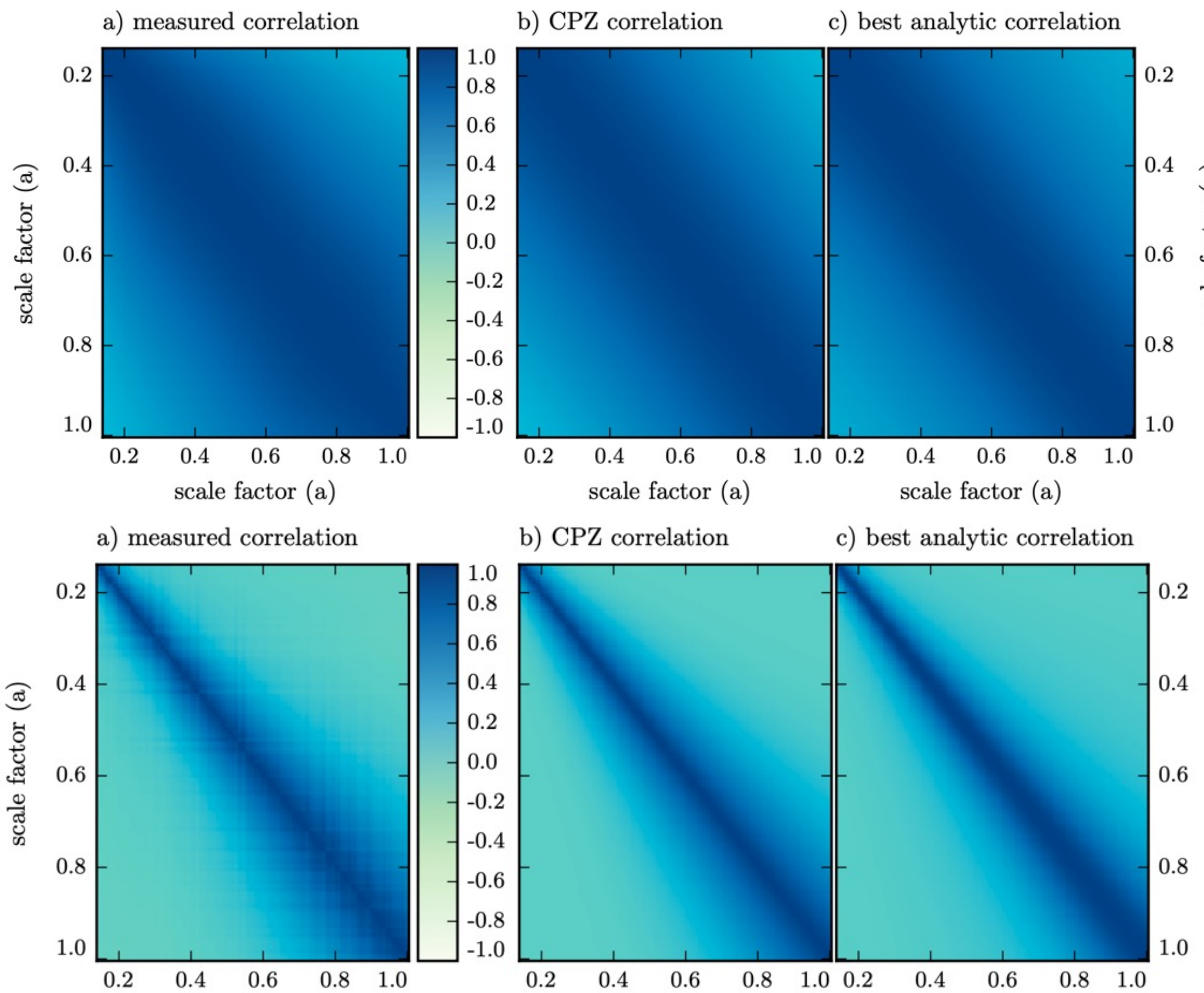
(Submitted on 15 Mar 2017)

Constraining the Dark Energy (DE) equation of state, w, is one of the primary science goals of ongoing and future cosmological surveys. In practice, with imperfect data and incomplete redshift coverage, this requires making assumptions about the evolution of w with redshift z. These assumptions can be manifested in a choice of a specific parametric form, which can potentially bias the outcome, or else one can reconstruct w(z) non-parametrically, by specifying a prior covariance matrix that correlates values of w at different redshifts. In this work, we derive the theoretical prior covariance for the effective DE equation of state predicted by general scalar-tensor theories with second order equations of motion (Horndeski theories). This is achieved by generating a large ensemble of possible scalar-tensor theories using a Monte Carlo methodology, including the application of physical viability conditions. We also separately consider the special sub-case of the minimally coupled scalar field, or quintessence. The prior shows a preference for tracking behaviors in the most general case. Given the covariance matrix, theoretical priors on parameters of any specific parametrization of w(z) can also be readily derived by projection.

Correlation of w(a)

Quintessence

Horndeski



Correlation of w(a)

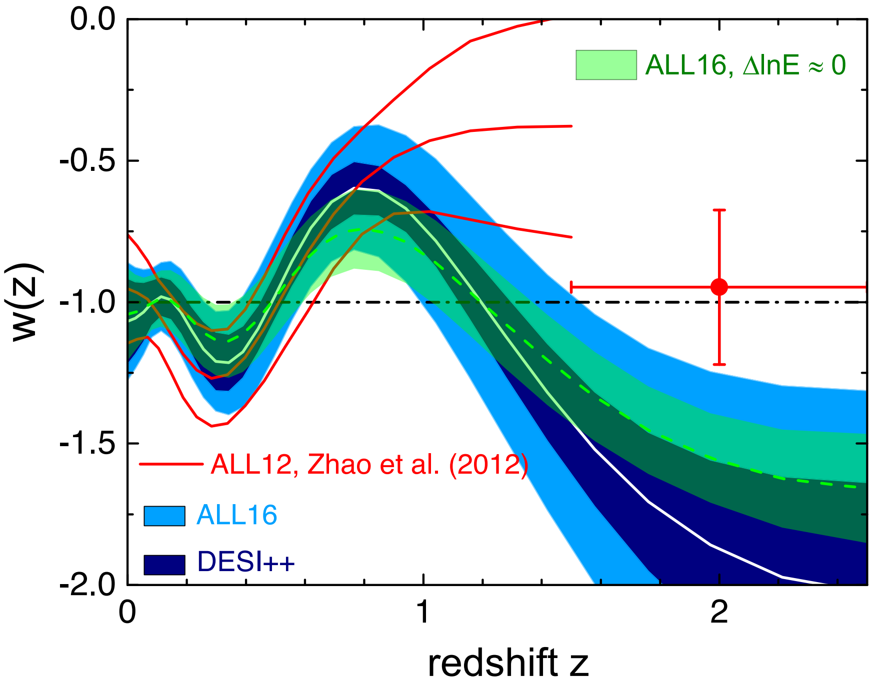
	Best-fit auto-corr	residuals
Quintessence	$0.03 + 0.3\ \exp[6.5\,(a-1)]$	0.01
GBD	$0.05 + 0.8\ \exp[1.8\ \ln a]$	0.007
Horndeski	$0.05 + 0.8\ \exp[2\ \ln a]$	0.007

	Best-fit corr	residuals
Quintessence	$\exp[-(\lvert\delta a\rvert/0.7)^{1.8}]$	9
GBD	$\exp[-(\lvert\delta \ln a\rvert/0.3)^{1.3}]$	6
Horndeski	$\exp[-(\lvert\delta \ln a\rvert/0.3)^{1.2}]$	6

	Best-fit corr (fixed CPZ)	residuals
Quintessence	$(1 + (\lvert\delta a\rvert/0.6)^2)^{-1}$	11
GBD	$(1 + (\lvert\delta \ln a\rvert/0.2)^2)^{-1}$	12
Horndeski	$(1 + (\lvert\delta \ln a\rvert/0.2)^2)^{-1}$	13

Raveri, Bull, Silvestri, LP, 1703.05297

If this was real, what could it be?



Definitely not quintessence

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \geq -1$$

Modified gravity: a scalar-tensor theory

$$S = \int d^4x\sqrt{-g}\Big[\frac{F(\phi)R}{16\pi G} - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) + \mathcal{L}_M\Big]$$

$$\begin{aligned} G_{\mu\nu} &= 8\pi G F^{-1}\left\{T^M_{\mu\nu} + T^\phi_{\mu\nu} + \nabla_\mu\nabla_\nu F - g_{\mu\nu}\Box F\right\} \\ &= 8\pi G\left\{T^M_{\mu\nu} + (T^{\rm eff}_{\rm DE})_{\mu\nu}\right\} \ , \end{aligned}$$

Effective dark energy density:

$$\rho^{\rm eff}_{\rm DE} = F^{-1}\left\{\frac{1}{2}\dot\phi^2 + V(\phi) - 3H\dot F + (1-F)\rho_M\right\}$$

Effective dark energy equation of state:

$$w^{\rm eff}_{\rm DE} = \frac{\dot\phi^2/2 - V(\phi) + 2H\dot F + \ddot F}{\dot\phi^2/2 + V(\phi) - 3H\dot F + (1-F)\rho_M}$$

Is working with $w_{\rm eff}$ justified when probing modified gravity?

Effective dark energy density is conserved (by construction)

$$\dot{\rho}_{\rm DE}^{\rm eff} + 3H(\rho_{\rm DE}^{\rm eff} + p_{\rm DE}^{\rm eff}) = 0$$

Working with $w_{\rm eff}$ assumes that the effective density doesn't change sign

$$\dot{\rho}_{\rm DE}^{\rm eff} + 3H\rho_{\rm DE}^{\rm eff}(1 + w_{\rm DE}^{\rm eff}) = 0$$

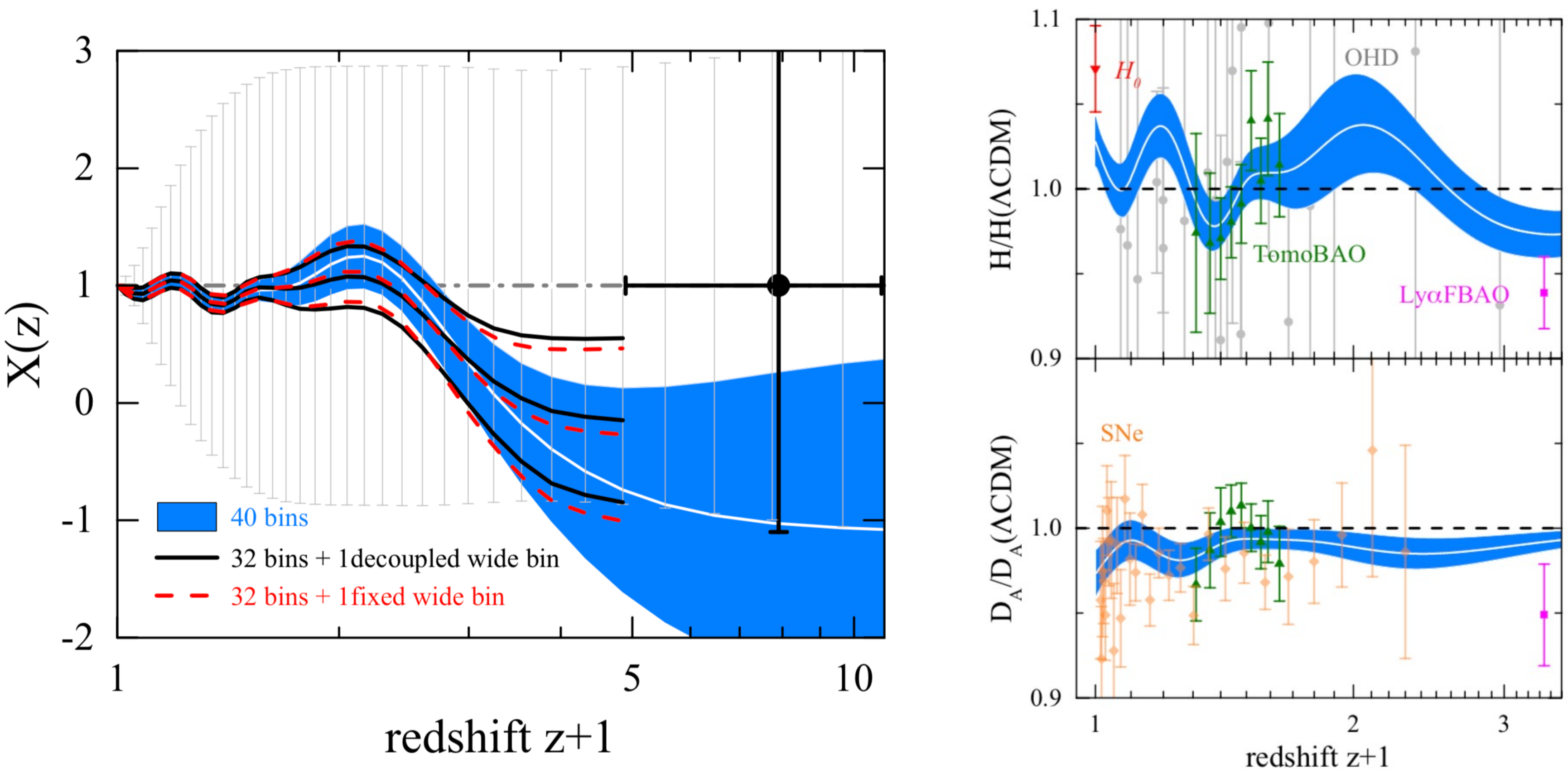
$$\boxed{w_{\rm DE}^{\rm eff} = p_{\rm DE}^{\rm eff}/\rho_{\rm DE}^{\rm eff}}\ ?$$

$$w_{\rm DE}^{\rm eff} = \frac{\dot{\phi}^2/2 - V(\phi) + 2H\dot{F} + \ddot{F}}{\dot{\phi}^2/2 + V(\phi) - 3H\dot{F} + (1-F)\rho_M}$$

Working with $w_{\rm eff}$ can bias studies of modified gravity. It's safer to work directly with $\boldsymbol{\rho}_{\rm eff}$:

$$\frac{H^2}{H_0^2} = \frac{\Omega_{\rm r}}{a^4} + \frac{\Omega_{\rm M}}{a^3} + \Omega_{\rm DE}X(a)$$

Reconstructed Dark Energy Density



Y. Wang, G.-B. Zhao and LP, 1807.03772, Ap J Lett

Summary of Parts I and II

Lambda, despite the problems, is still the best motivated Dark Energy candidate we have

Today's and tomorrow's data is good enough to allow reconstructions of w(z). No need to limit ourselves to constant w or w_0, w_a

If we find evidence for w(a) ≠ -1, if our theoretical expectations are correct, there are likely to be other signatures, such as fifth forces or birefringence