Cosmic Microwave Background

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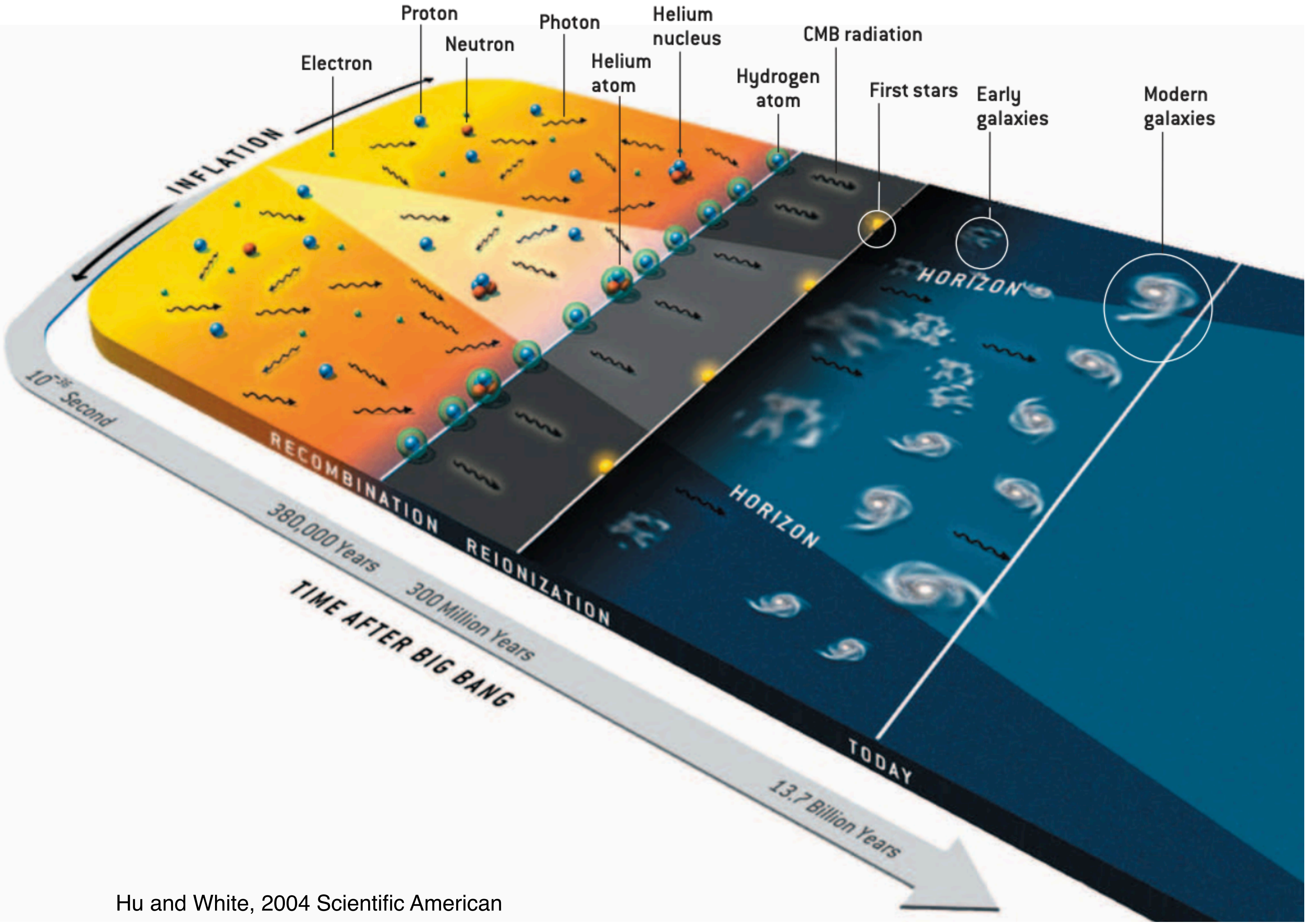
Outline

- overview of the cosmic microwave background fluctuations
  - ⋆ **lots to do with better CMB measurements**
- CMB at 2nd order:
  - ⋆ lensing of the cosmic microwave background by large scale structure for more cosmology
  - ⋆ Thomson scattering by moving objects (kinetic SZ) as new probe, e.g., cosmic birefringence



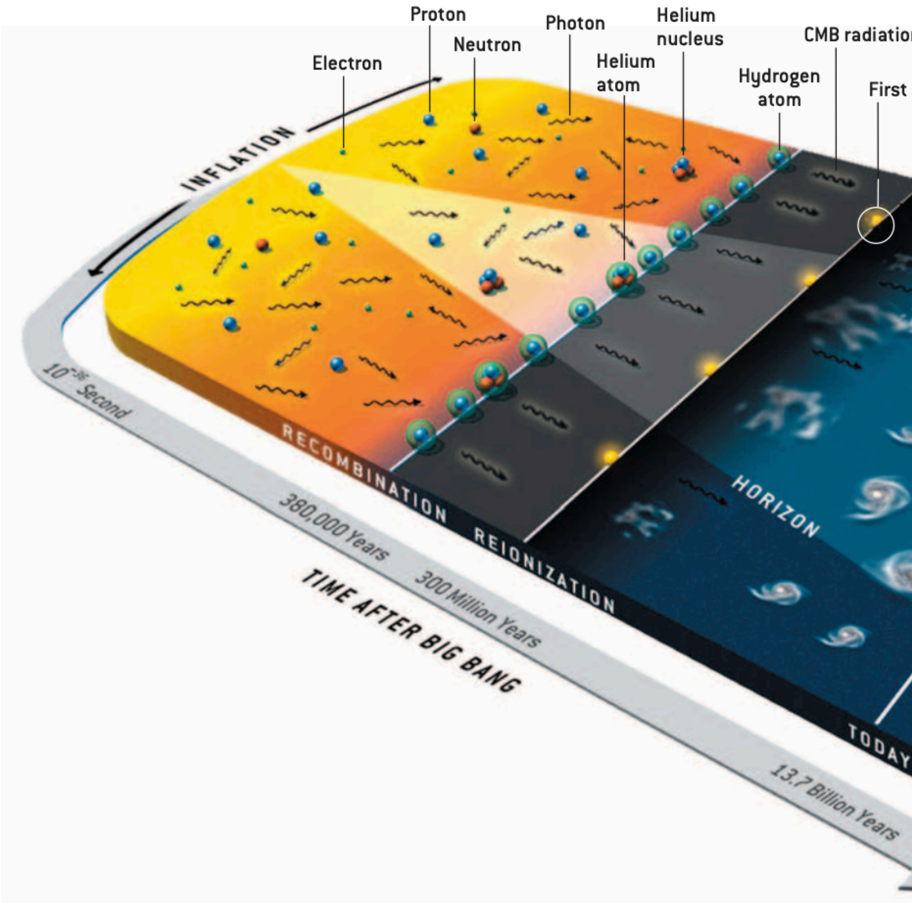
Hu and White, 2004 Scientific American

The photon-baryon plasma

Early universe is a plasma

Thomson scattering keeps photons and electrons tightly coupled

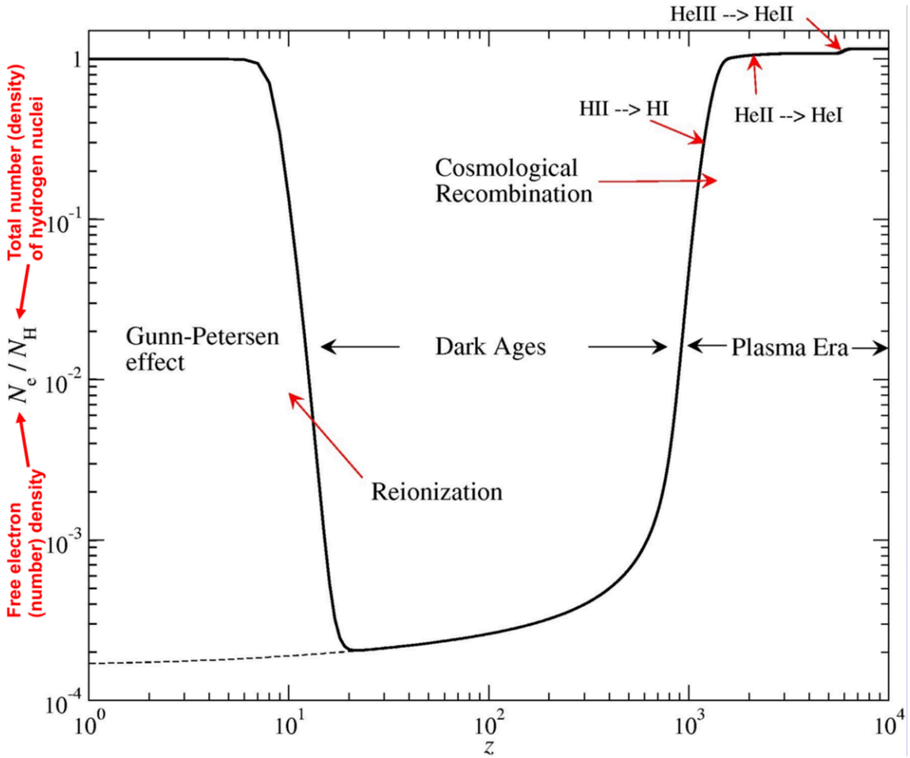
Coulomb scattering keeps electrons and nuclei tightly coupled



Ionization non-equilibrium

Hubble expansion causes recombinations to "freeze out" as e- and p+ can't find each other in the dilute universe

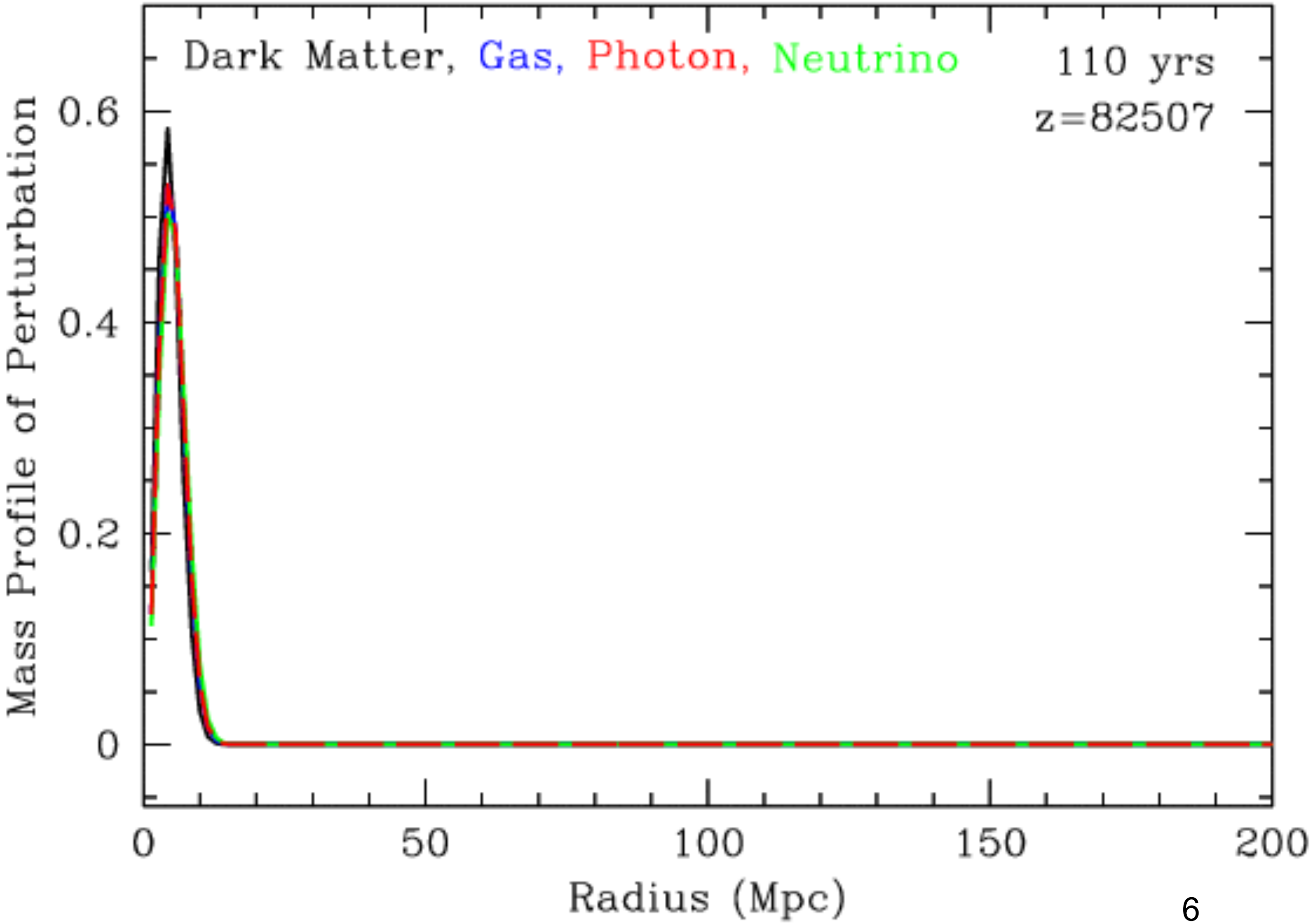
small residual ionization keeps gas and CMB thermally coupled for a surprisingly long time



Sunyaev & Chluba 2009

Sound waves in the early universe

evolution of an initial overdensity



Re-normalized to be equal at beginning

“Adiabatic” initial fluctuations actually mean equal perturbations in number density Δn/n

Daniel Eisenstein: https://lweb.cfa.harvard.edu/~deisenst/acousticpeak/acoustic_anim.html

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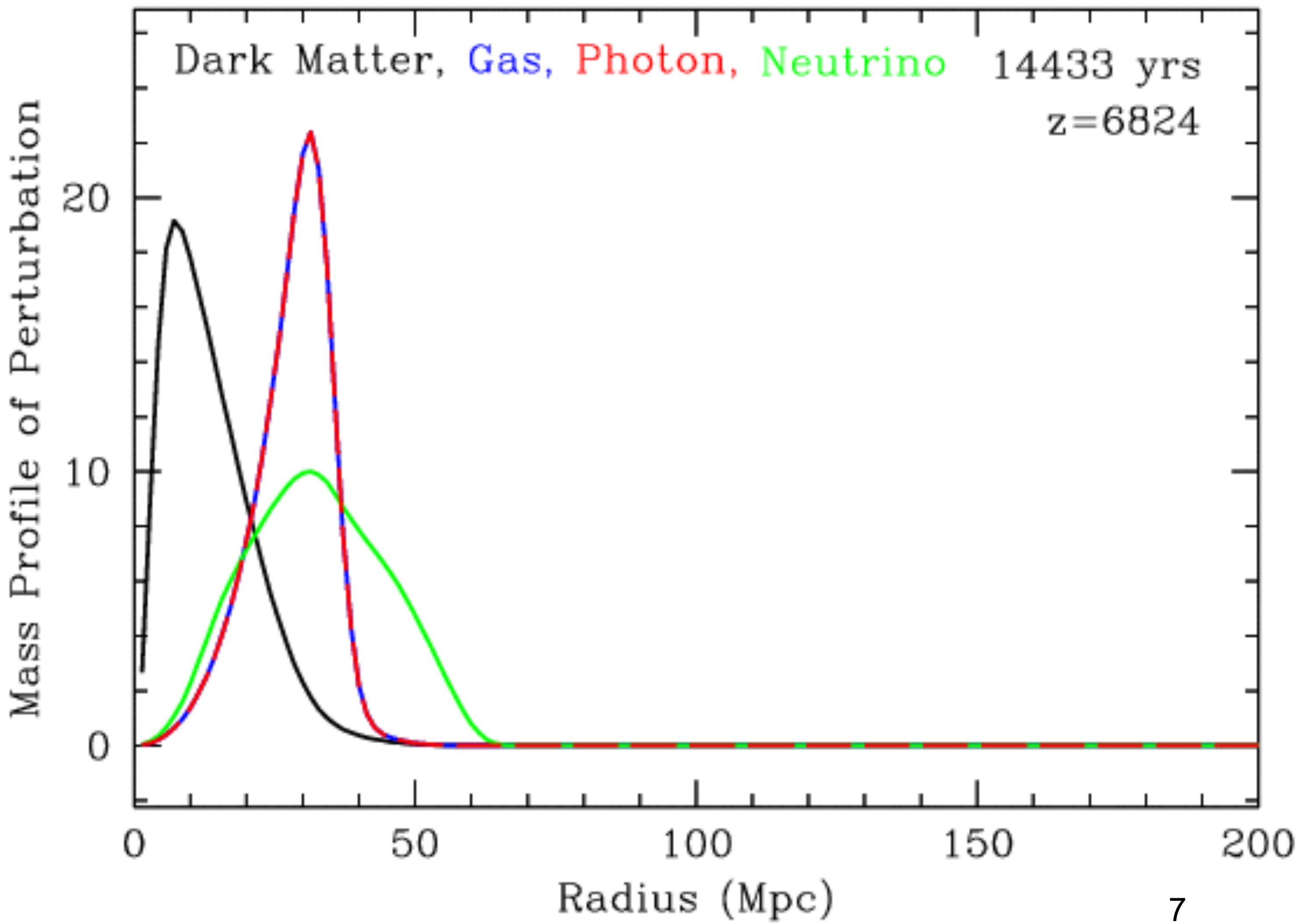

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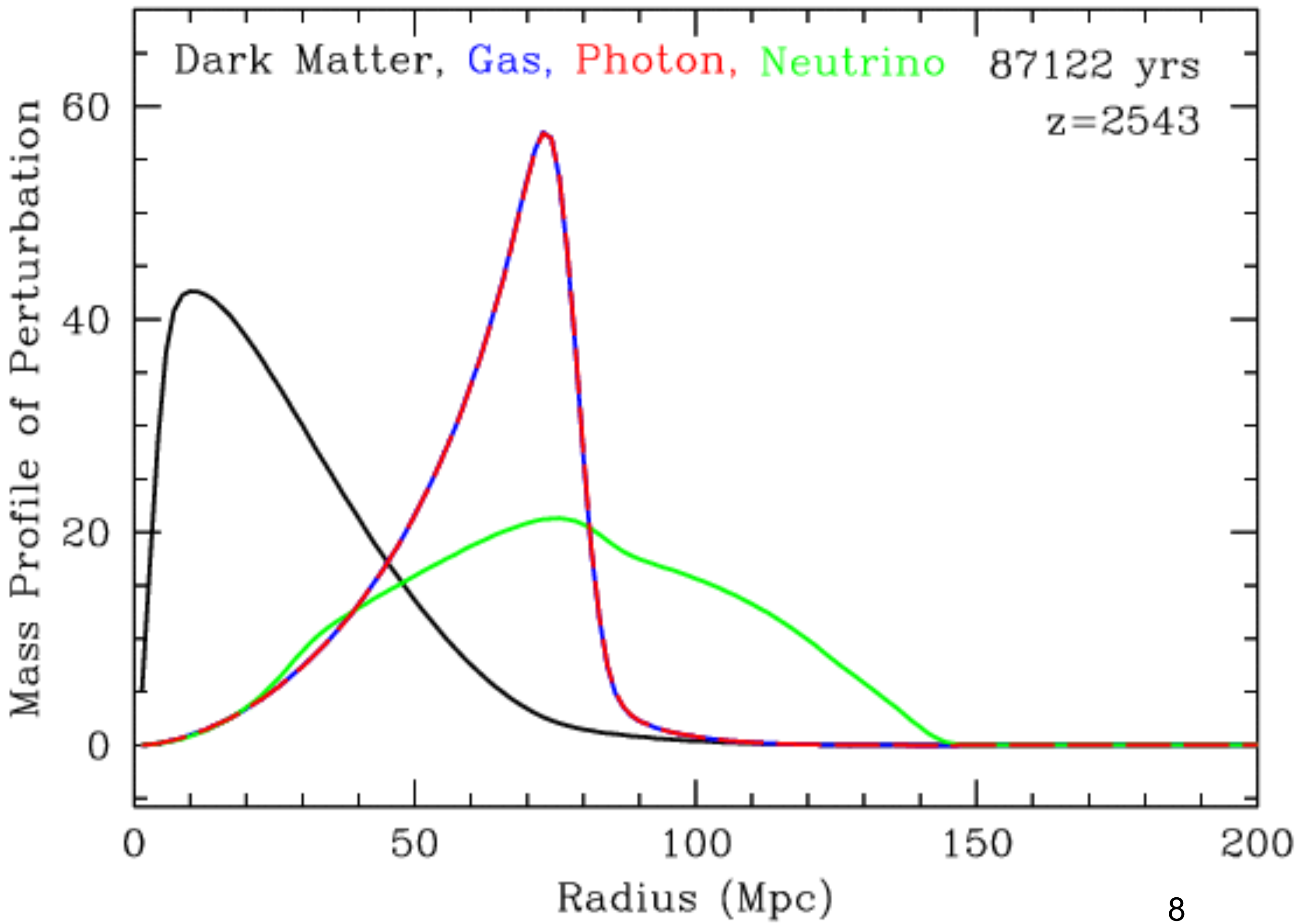

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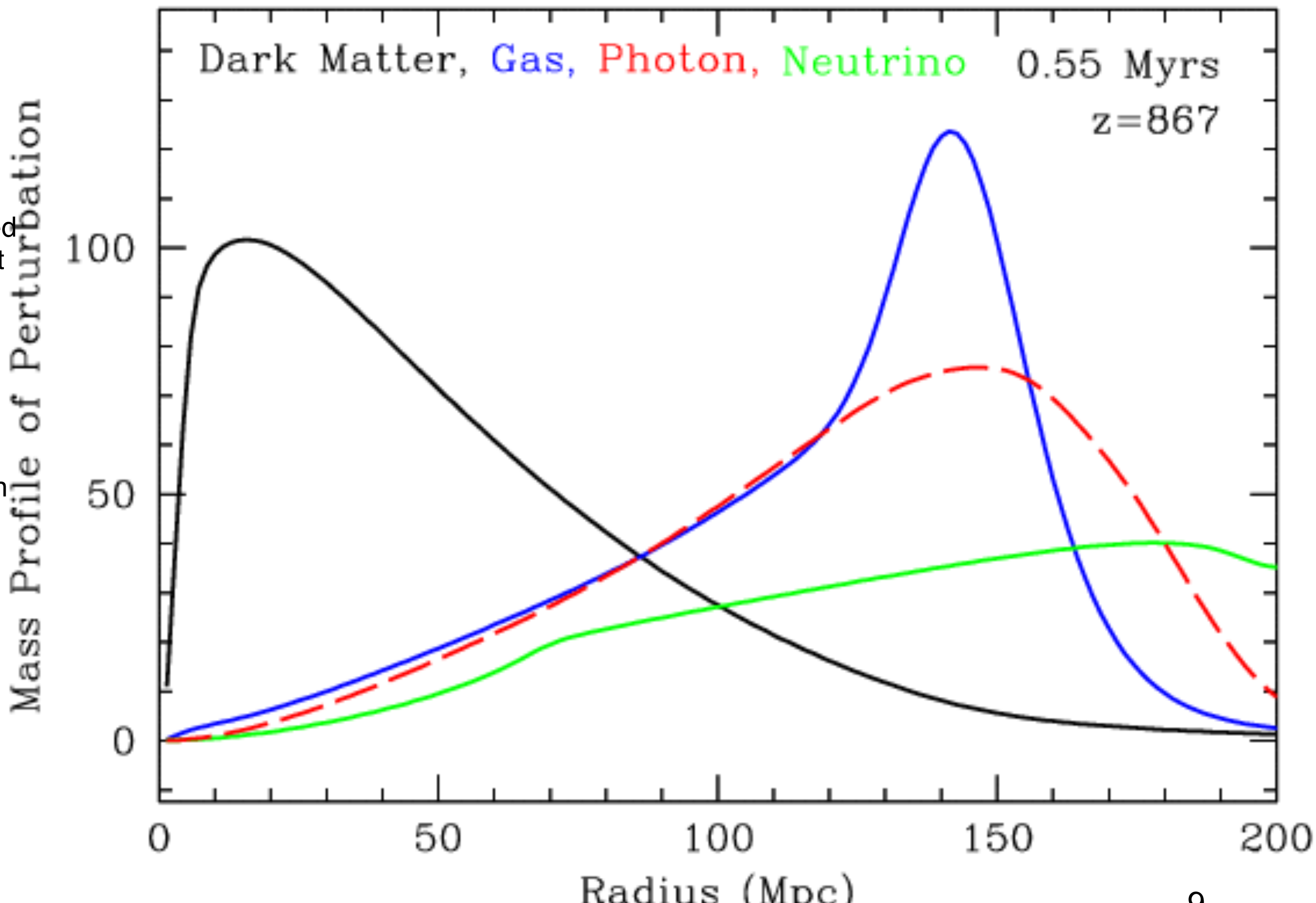

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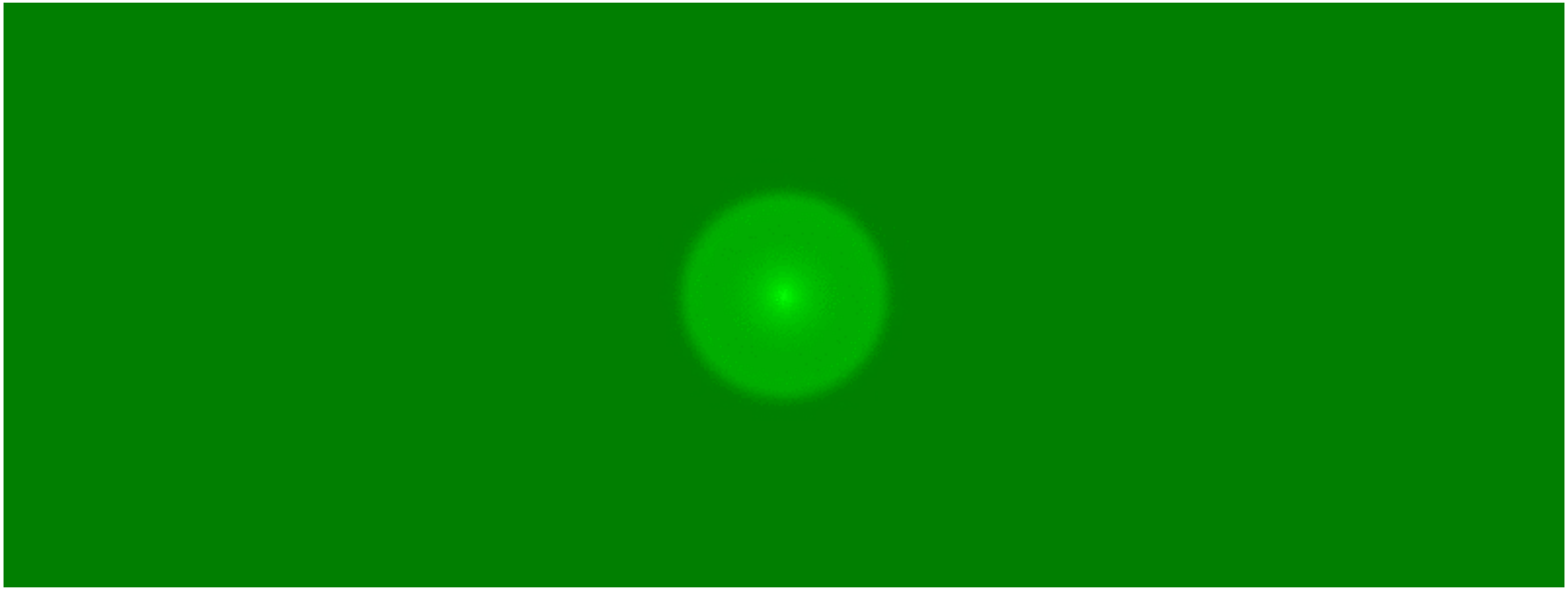

superposition of multiple shells

Adam Hincks: https://adh-sj.info/bao_cmb.php

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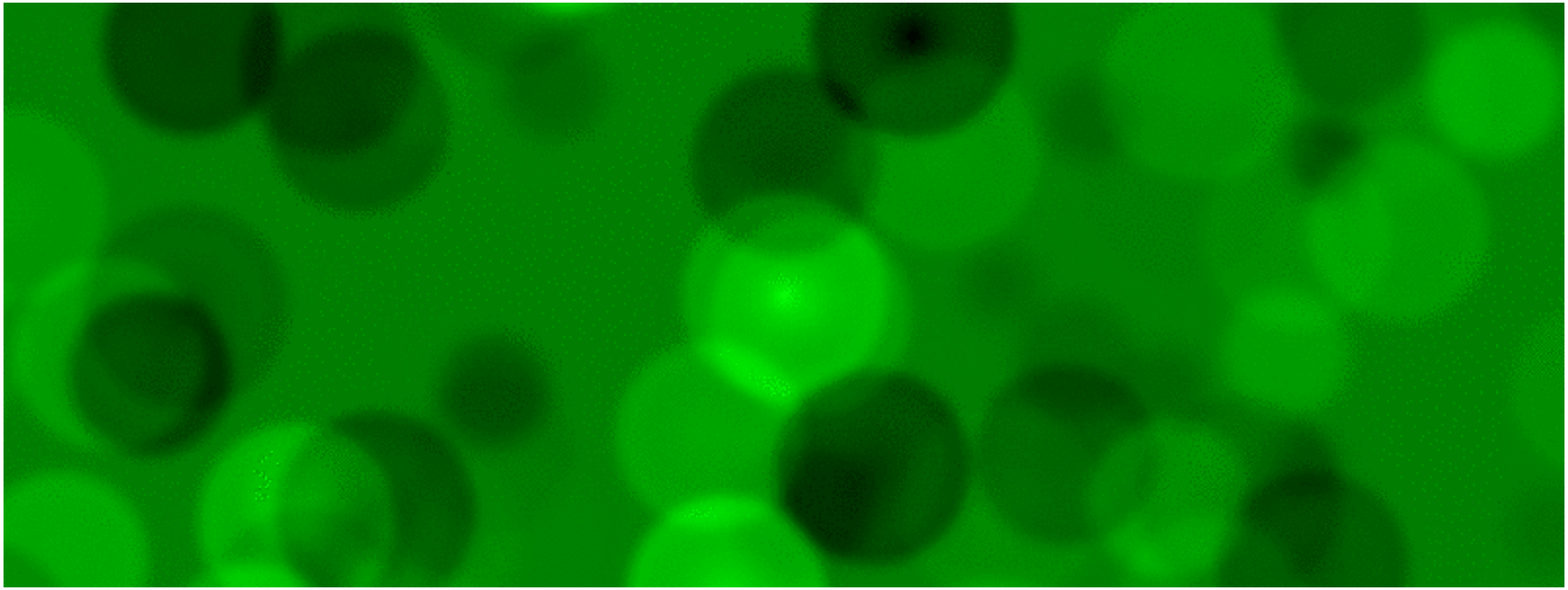

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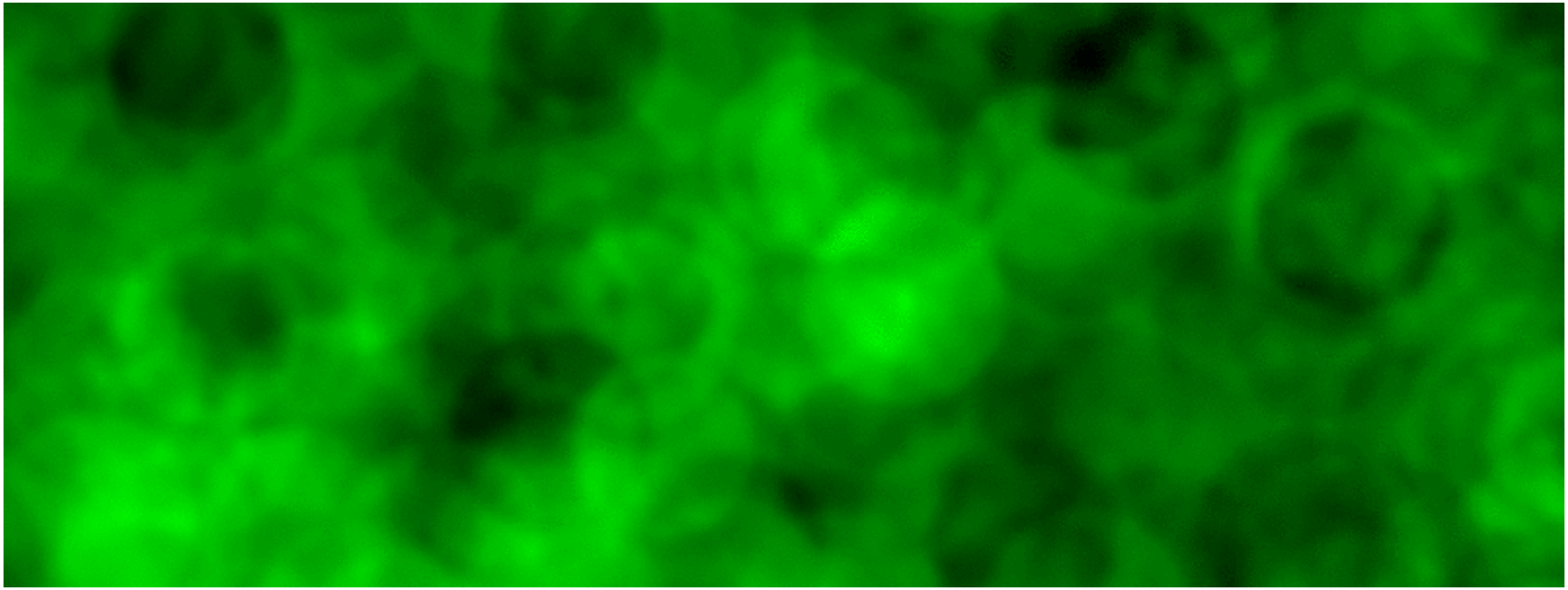

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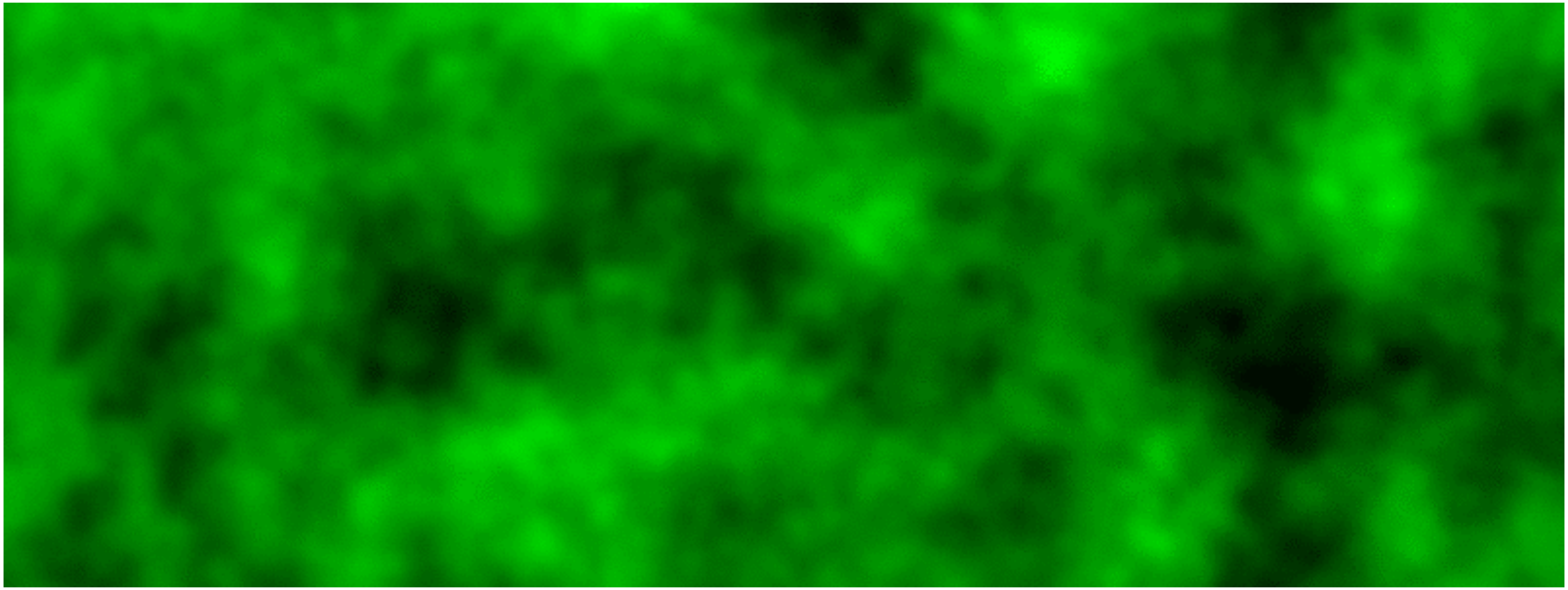

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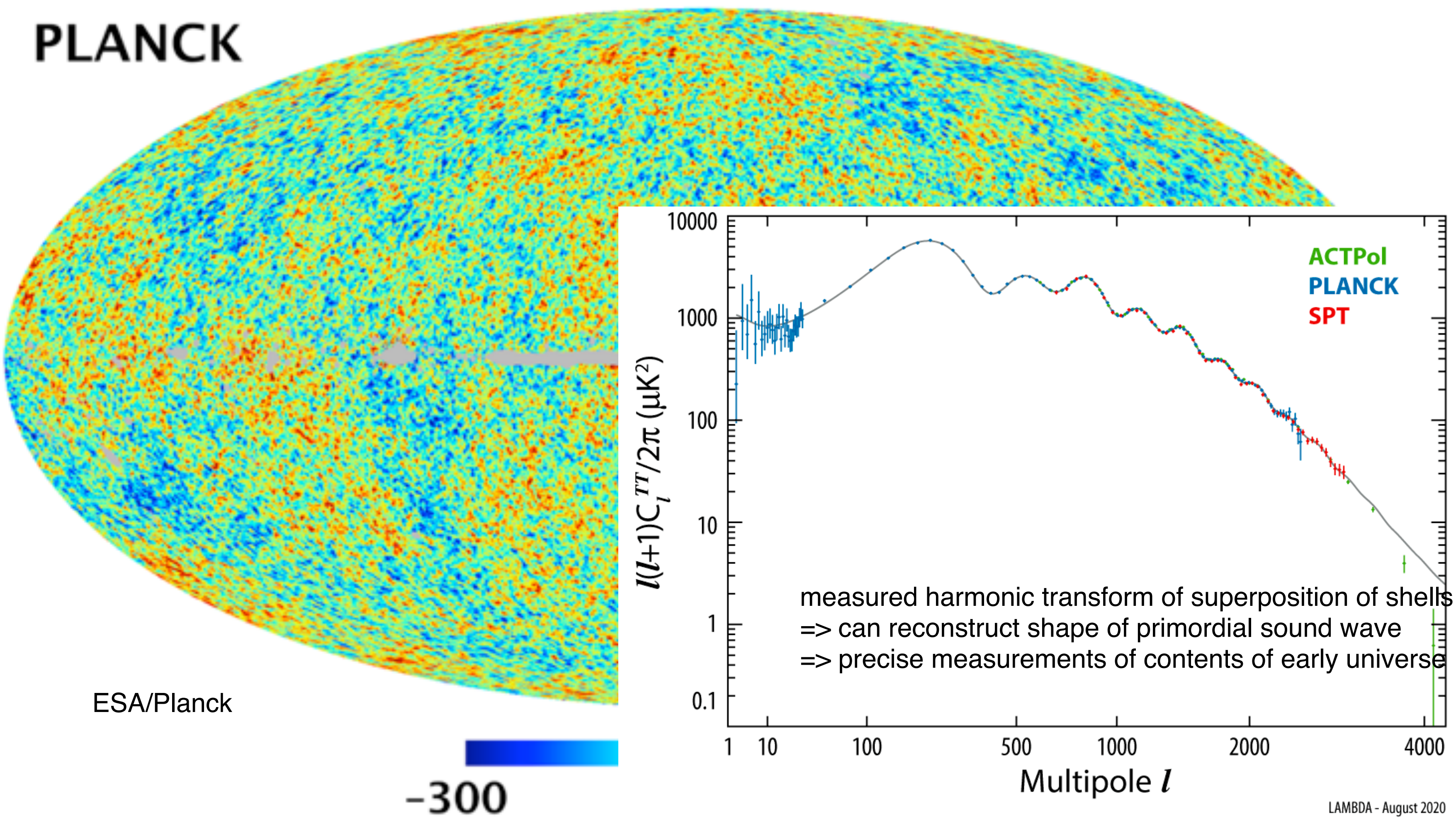


measured harmonic transform of superposition of shells
=> can reconstruct shape of primordial sound wave
=> precise measurements of contents of early universe

ESA/Planck

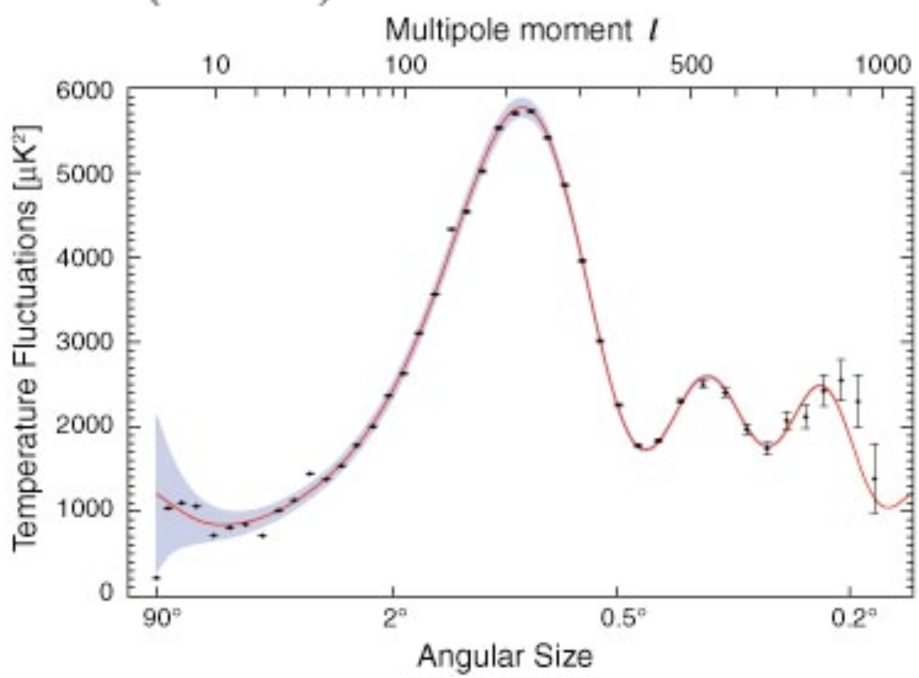
https://lambda.gsfc.nasa.gov/graphics/tt_spectrum/tt_spectrum_2020aug_1024.png

Spherical Harmonics

$$T(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi).$$

$$Y_\ell^m(\theta, \varphi) = \sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\, P_\ell^m(\cos\theta)\, e^{im\varphi}$$

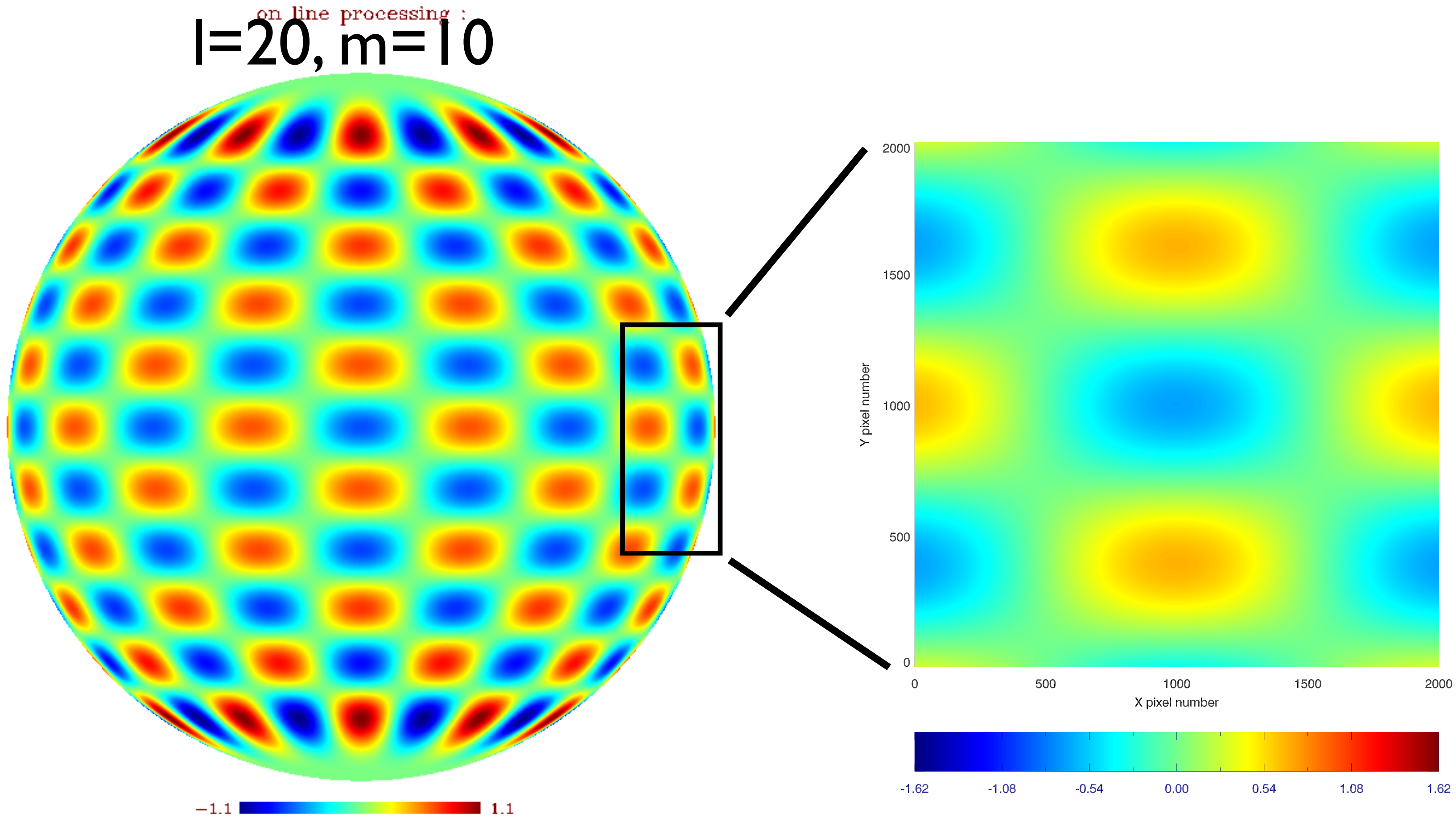
$$\hat{C}_l = \frac{1}{2l+1}\sum_m |\hat{a}_{lm}|^2.$$



Power spectrum Uncertainties

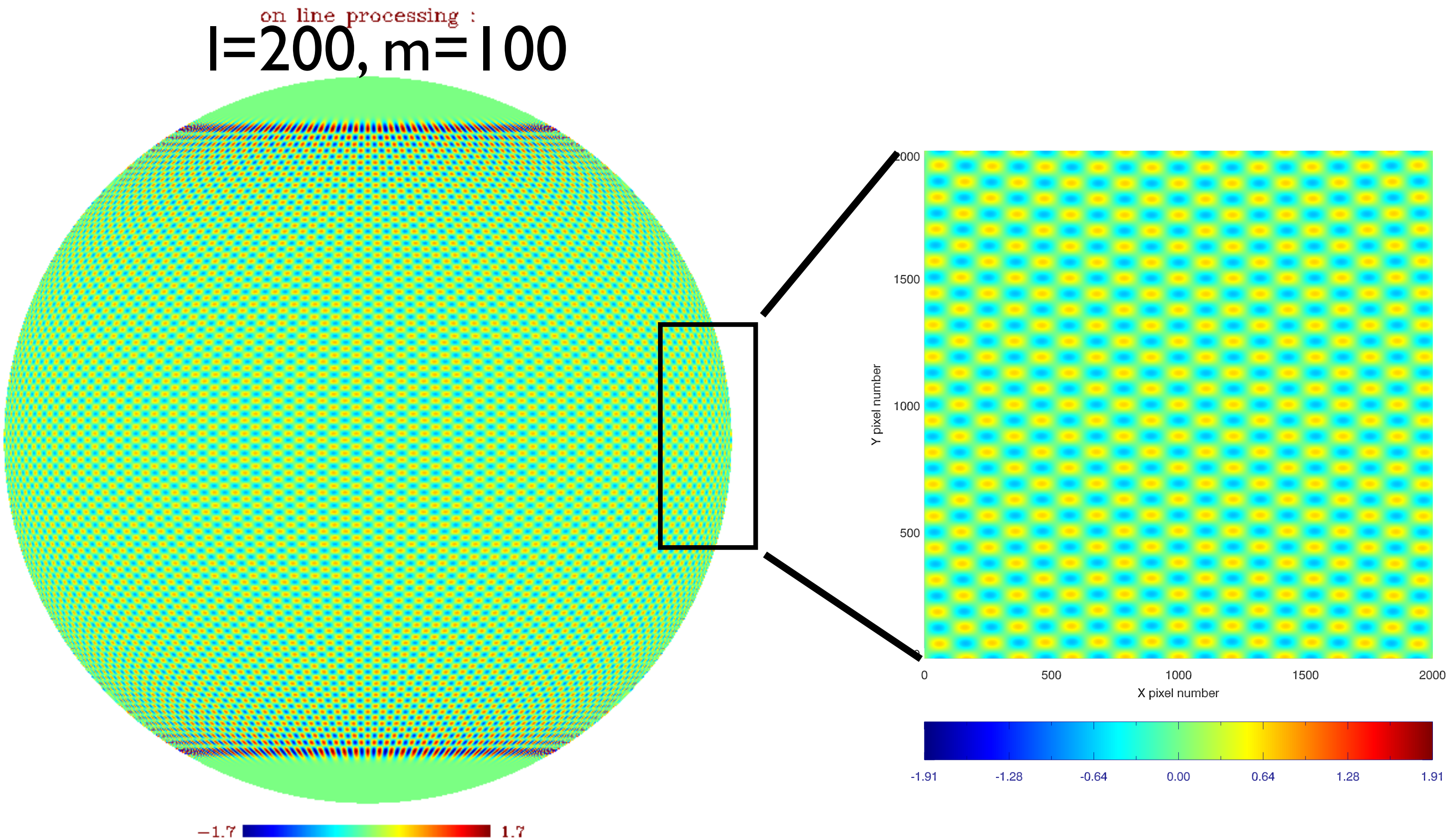
- fundamentally limited by number of independent measurements, noise
- $C_{l;meas}=C_{l;true}+C_{l;noise}$ *in any single map you can't tell the difference*
- $Var(C_l) \sim (2/n_{meas})C_l^2$ ***"sample variance"***
- more modes means better measurement of $C_{l;true}+C_{l;noise}$
- lower noise gives better measure of $C_{l;true}$

Projecting a_{lm}



Projecting a_{lm}

l=200, m=100



From a_{lm} to $a_{k_x k_y}$

Equation satisfied by $P_{lm}(x)$:

$$(1-x^2)\,y'' - 2xy' + \left(\ell[\ell+1] - \frac{m^2}{1-x^2}\right) y = 0,$$

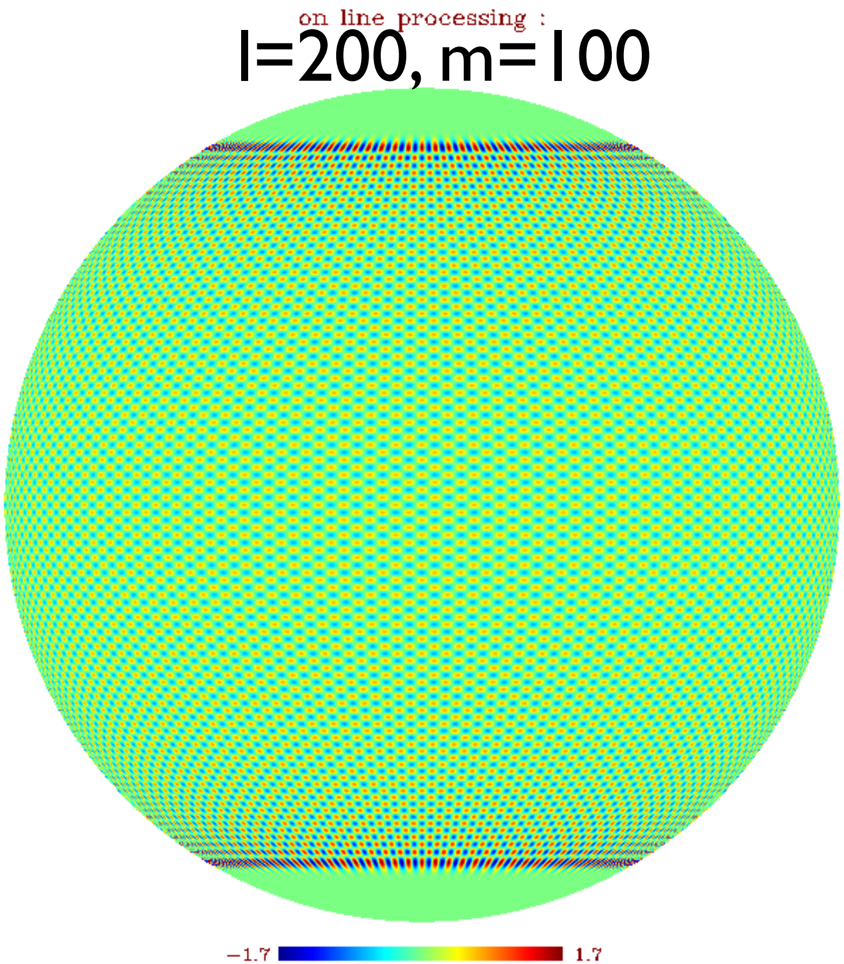
For x~0:

$$(1-\cancel{x^2})\,y'' - 2\cancel{x}y' + \left(\ell[\ell+1] - \frac{m^2}{1-\cancel{x^2}}\right) y = 0,$$

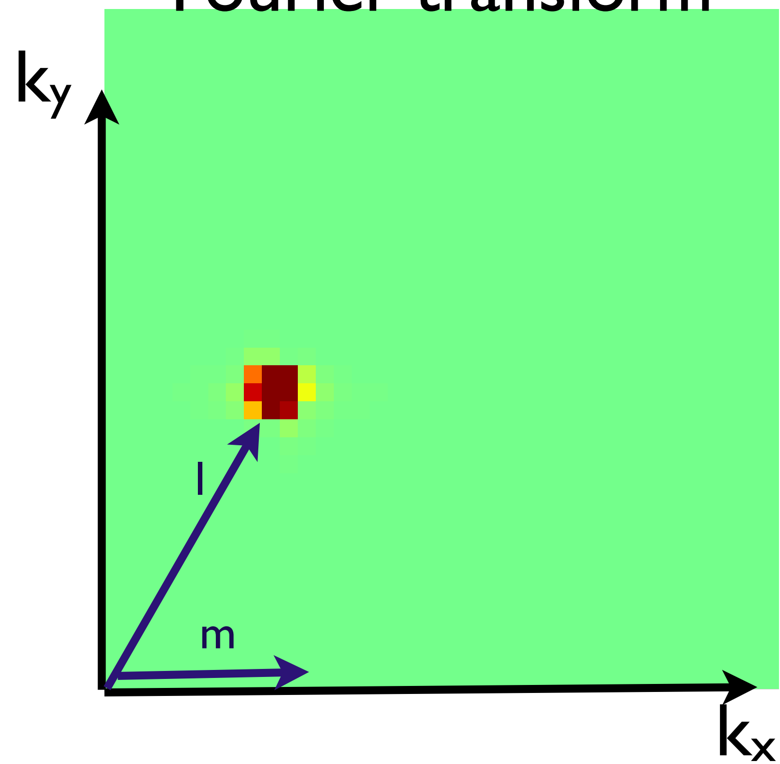
Harmonic Oscillator with ***k=l(l+1)-m²***

(Fourier modes!)

Projecting a_{lm}



Fourier transform

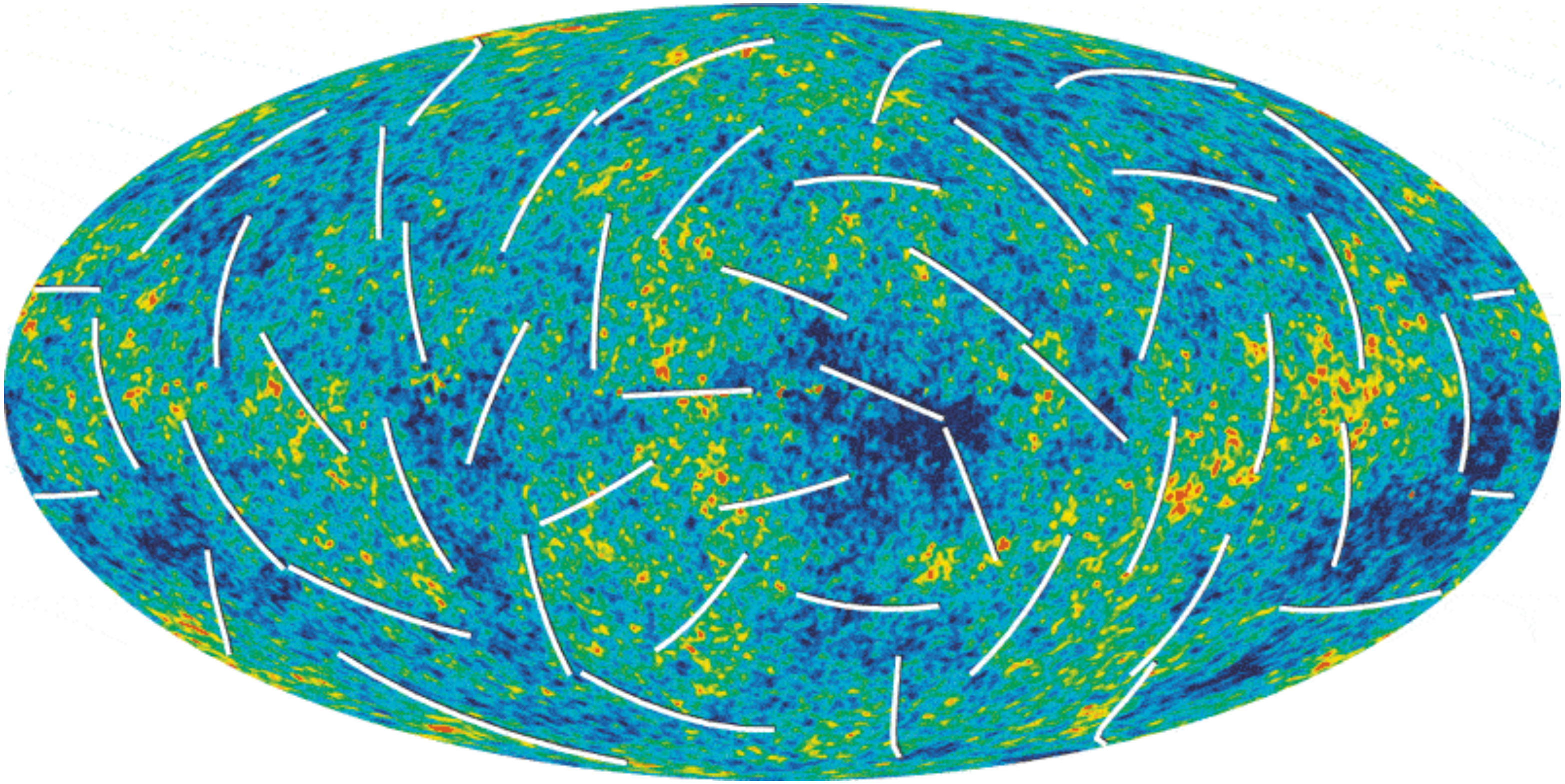


Cross Spectra

(all quantities are Fourier space!)

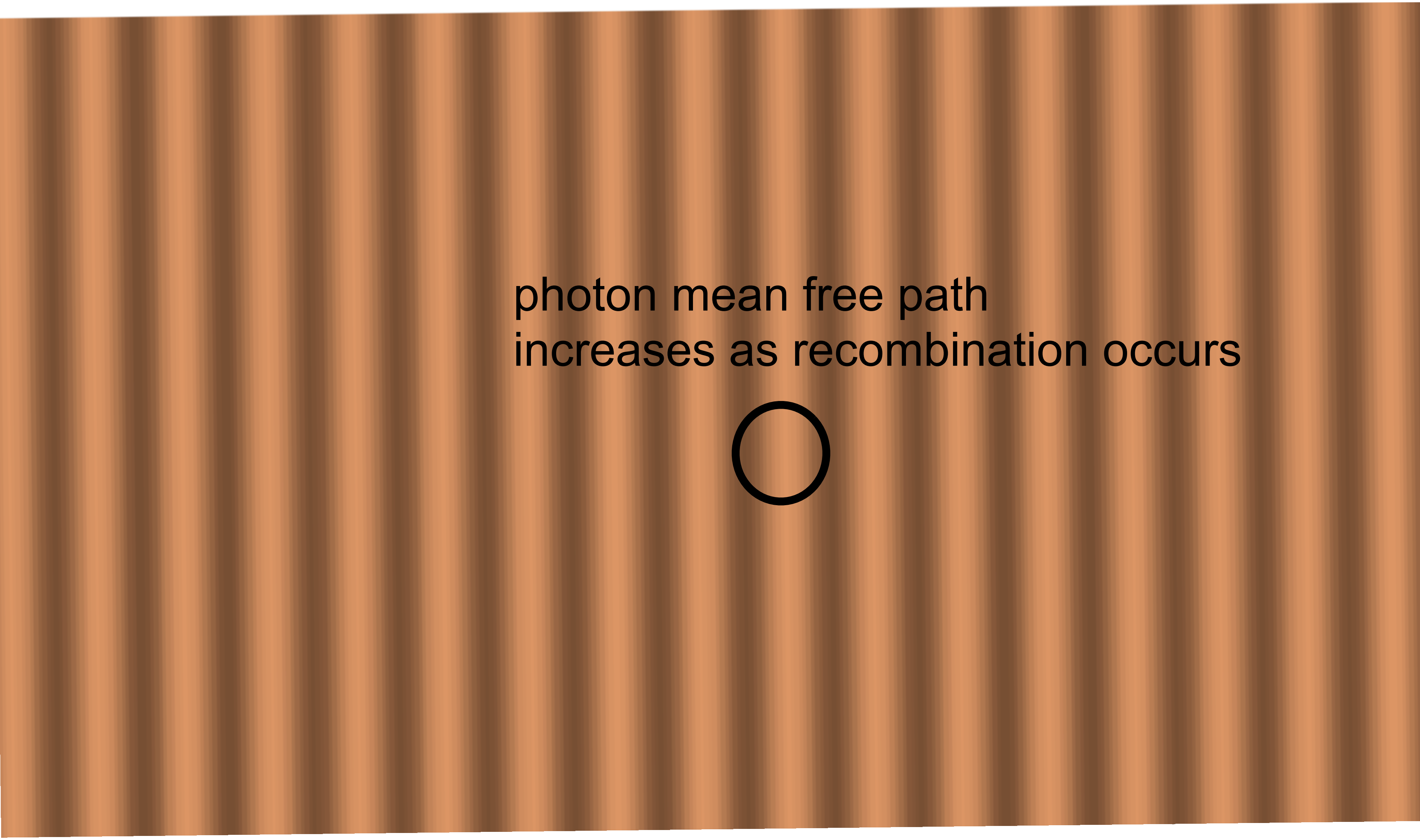
- $T_m = T + n$
- $\langle T_{1;m} T_{2;m} \rangle = \langle T_1 T_2 \rangle + \langle n_1 n_2 \rangle + \langle T_1 n_2 + T_2 n_1) \rangle$
- for $1=2$ (map auto power spectrum), $\langle n_1 n_2 \rangle = s^2$
- if $1 \neq 2$, $\langle n_1 n_2 \rangle = 0$, so no bias
- quirks in your noise model don't affect cross spectrum!

CMB Polarization



- CMB fluctuations are relatively strongly polarized (~10%)

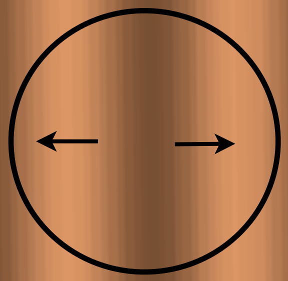
Polarization from Anisotropy



Recall: Scattering Quadrupole Intensity Leads to Linear Polarization in Preferred Directions

Two reasons for local photon quadrupole anisotropy

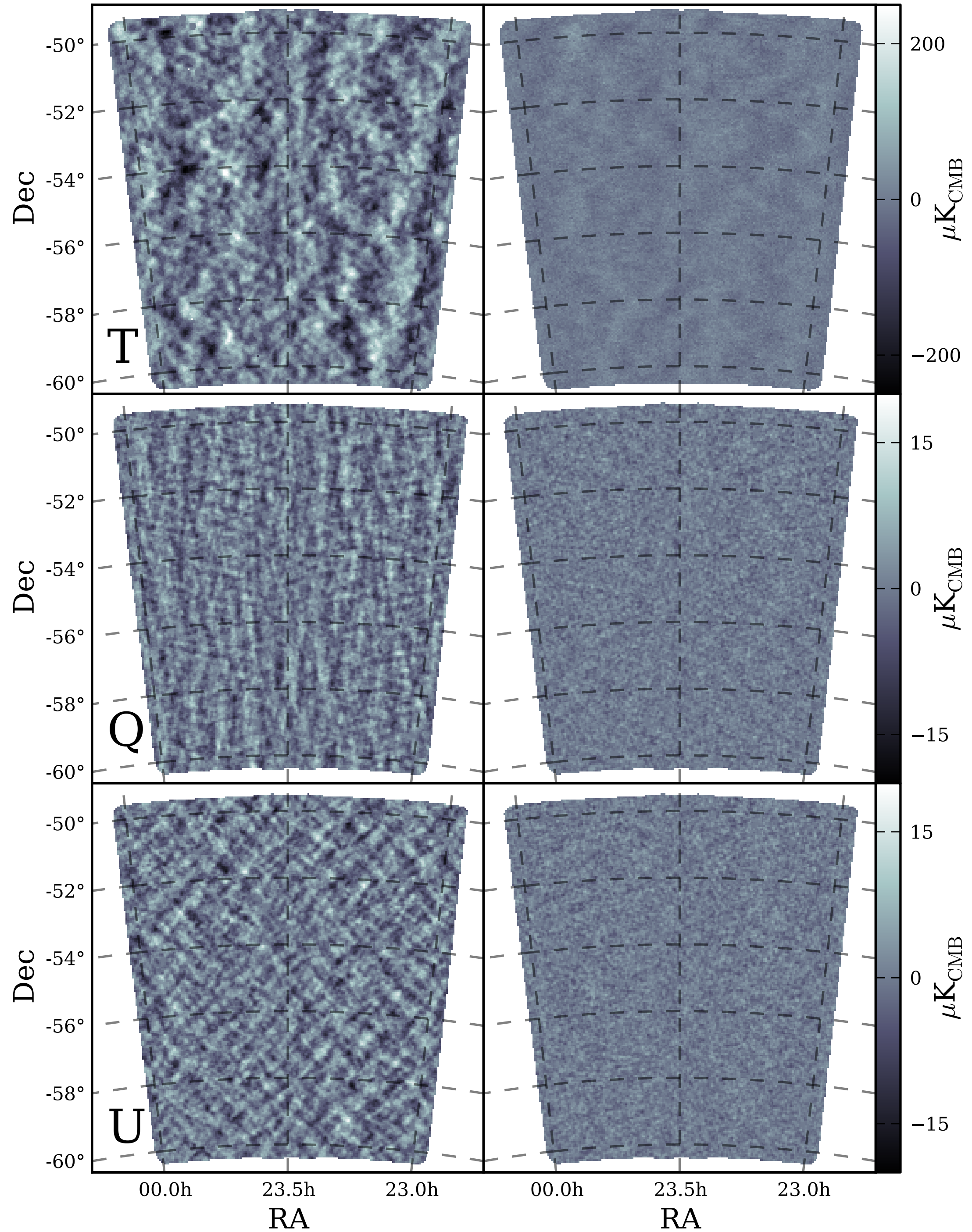
1. quadrupole in local Temperature
2. shear in Doppler shift from velocities



SPTpol Maps

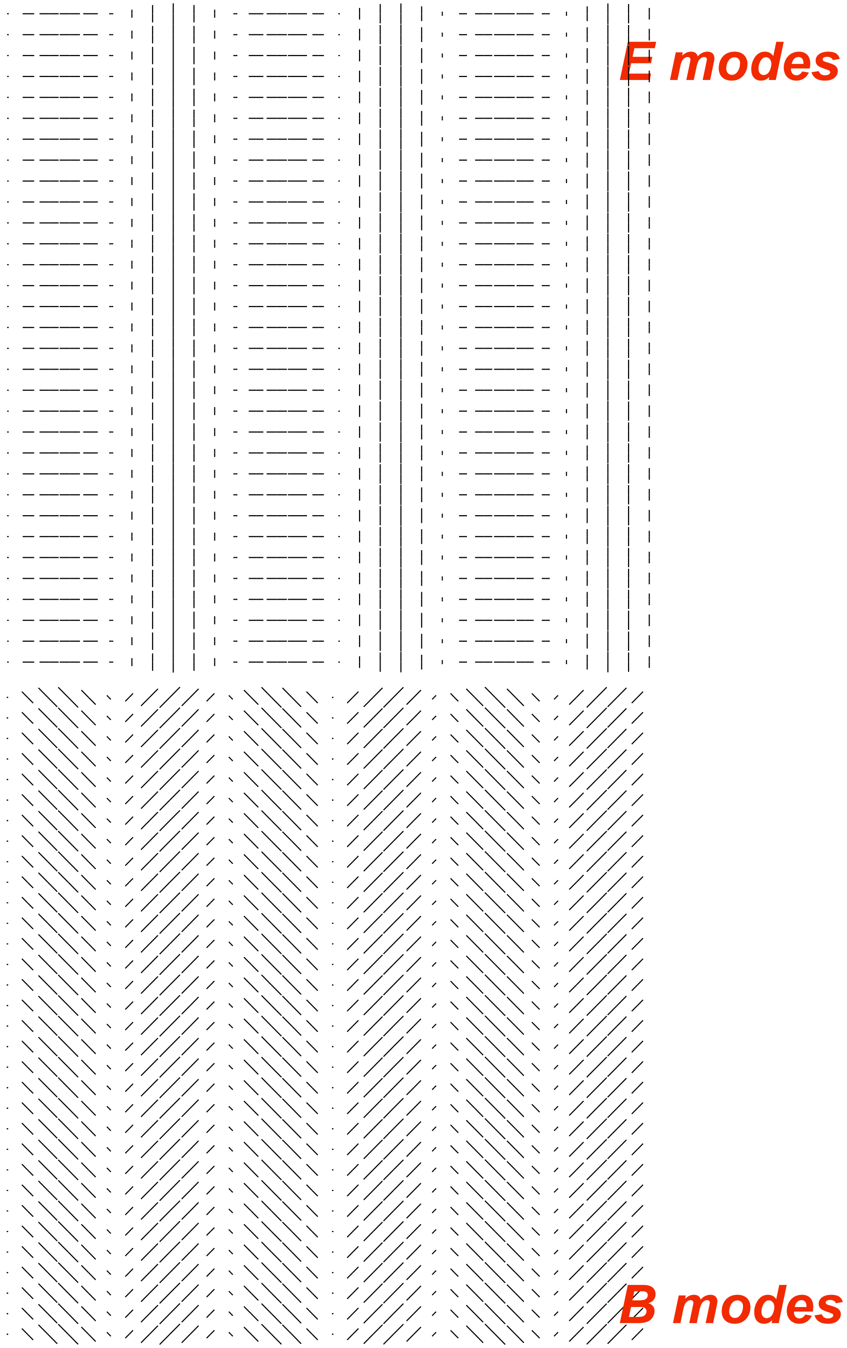
Stokes Q and U maps have boxiness to them because generated by fluctuations in gravitational potential at last scattering: "E-modes"

Crites et al 2015



E-modes/B-modes

- E-modes vary spatially parallel or perpedicular to polarization direction
- B-modes vary spatially at 45 degrees
- CMB
    - scalar perturbations only generate *only* E
    - vector and **tensor** perturbations generate both E and B



Bunn

Stokes Q/U Rotated

- Stokes Q/U are tied to coordinate system
- rotate coordinates, Q/U are changed
- polarization is spin-2

$$Q' = Q\cos 2\psi + U\sin 2\psi$$
$$U' = -Q\sin 2\psi + U\cos 2\psi$$

$$(Q \pm iU)'(\hat{n}) = e^{\mp 2i\psi}(Q \pm iU)(\hat{n}),$$

The nature of the E-B decomposition of CMB polarization

Matias Zaldarriaga
Physics Department, New York University, 4 Washington Place, New York, NY 10003
(May 10 2001. Submitted to Phys. Rev. D.)

E-modes and B-modes

$$Q(\boldsymbol{l}) = [E(\boldsymbol{l})\cos(2\phi_l) - B(\boldsymbol{l})\sin(2\phi_l)]$$
$$U(\boldsymbol{l}) = [E(\boldsymbol{l})\sin(2\phi_l) + B(\boldsymbol{l})\cos(2\phi_l)].$$

- E/B is a different way to express polarization field
- easy to understand in flat-sky limit (i.e. Fourier modes)

Full-Sky E/B: Spin-2 Spherical Harmonics

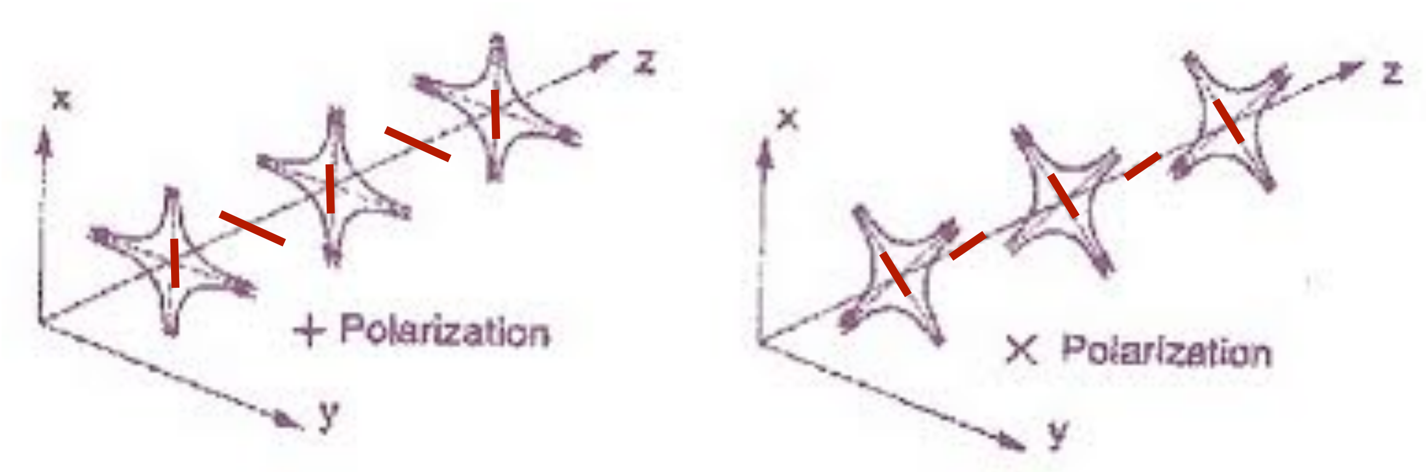
$$(Q \pm iU)(\hat{n}) = \sum_{lm} a_{\pm 2,lm} \, {}_{\pm 2}Y_{lm}(\hat{n}).$$

$$a^E_{lm} = -(a_{2,lm} + a_{-2,lm})/2$$

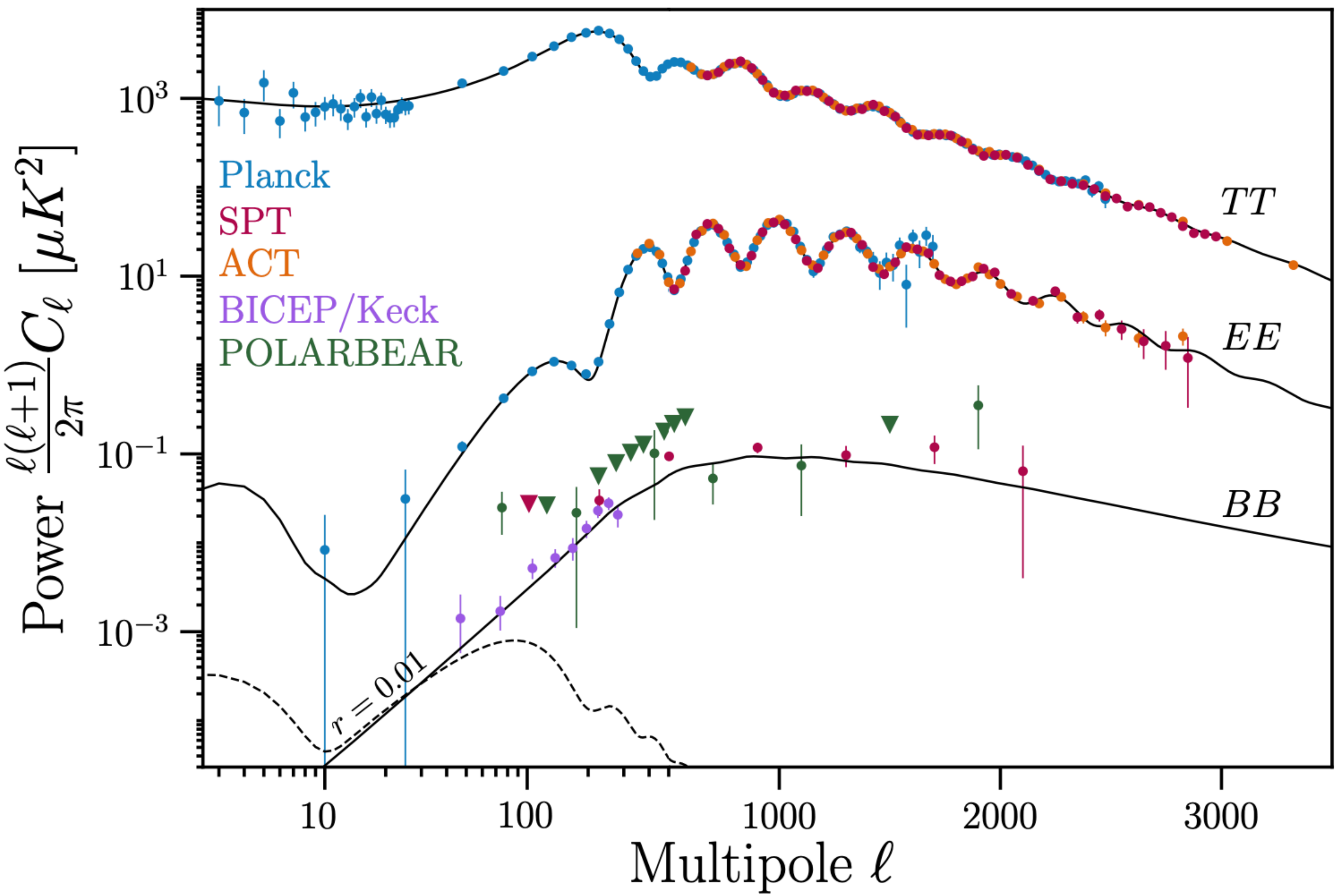
$$a^B_{lm} = i(a_{2,lm} - a_{-2,lm})/2.$$

- spin-2 S.H. easily derived from regular old S.H. through second derivatives

Gravitational Waves Generate E and B



B modes are a great probe of gravitational radiation in the early universe!!



Snowmass CMB Measurements white paper 2203.07638

The Angular 2-point Correlation function

$$\mathcal{C}(\theta) = \langle \Delta T(\mathbf{n_1}) \Delta T(\mathbf{n_2}) \rangle, \qquad \mathbf{n_1} \cdot \mathbf{n_2} = \cos\theta.$$

$$\mathcal{C}(\theta) = T_0^2 \sum_{\ell} \frac{2\ell + 1}{4\pi} C_\ell P_\ell(\cos\theta),$$

Paolo Cea

- position space analog of the power spectrum
- often used for galaxy surveys because of complex survey masks
- let's calculate some! https://colab.research.google.com/drive/1eTdIY2EUTv1WDJIHs_vdOZ3dsVQV8mXa?usp=sharing